
A Language for the Sociological Description of
Pedagogic Texts with Particular Reference to the
Secondary School Mathematics Scheme *SMP 11-16*

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Abstract

A Language for the Sociological Description of Pedagogic Texts with Particular Reference to the Secondary School Mathematics Scheme *SMP 11-16*

The thesis is concerned with the production of a language for the systematic sociological description of pedagogic texts and with the application of this mechanism to two series of textbooks within the secondary school mathematics scheme, *SMP 11-16*. One series is targeted at the upper end of the 'ability' range, the other is intended for 'low ability' pupils.

The thesis opens with a discussion of two prominent positions within mathematics education, concluding that they both 'mythologise' mathematical knowledge by abstracting it from the social bases of its elaboration. A search of the literature on the analysis of textbooks reveals that the majority of sociologically-oriented work entails either ideological analysis or the analysis of the representation of one or more particular categories, most frequently gender and/or race. None of this research combines a theoretically coherent position with a set of derived principles for the detailed analysis of text.

Chapter 3 presents a general methodological position in relation to three themes. These are, the distinction between the abstract and the concrete, the construction of subjectivity, and the contextualising and recontextualising of practices. The principal resources in this discussion are the works of Basil Bernstein, Pierre Bourdieu, Umberto Eco, Michel Foucault, and Valerie Walkerdine. Out of a critical discussion of this work, ten Theoretical Propositions are derived. These propositions form the general methodological basis of the 'language of description' which is derived from them in Chapter 4.

The following five Chapters comprise an introductory description and a detailed analysis of the two series of textbooks. The analysis is predominantly qualitative in nature, but also incorporates a quantitative component. The latter focuses, in particular, on the modes of signification (icon, index, symbol) that are incorporated in the textbooks. The principal findings that emerge from the analysis describe the ways in which the texts select and construct apprenticed and alienated ideal readers. The

differentiation between the apprenticed and alienated ideal readers is, primarily, constructed in terms of social class.

The concluding Chapter includes an overview of the thesis and a discussion of the limitations of and possibilities arising from the language of description and its application. The concluding Section works more freely with the language and with the findings of the analysis in developing a theoretical speculation in respect of a possible conception of the relationship between sociological research and educational practice.

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Abbreviations

The following abbreviations have been used in this thesis.

AT	Attainment Target (a category used in the National Curriculum for England and Wales)
ATAM	Association for Teaching Aids in Mathematics (now the Association of Teachers of Mathematics, ATM)
B1-5	<i>SMP Books B1 -5</i>
B1TG-B5TG	<i>SMP Teacher's Guide to Books B1 -5</i>
BEd	Bachelor of Education
CSE	Certificate of Secondary Education (replaced by GCSE in summer 1988)
CSMS	Concepts in Secondary Mathematics and Science (former research programme based at Chelsea College (now Kings College), University of London)
DES	Department of Education and Science
DIY	Do-It-Yourself
DNA	Deoxyribonucleic Acid
DS ⁺	Discursive Saturation (strong)
DS ⁻	Discursive Saturation (weak)
G1-8	<i>SMP Books G1 -8</i>
G1TG-G8TG	<i>SMP Teacher's Guide to Books G1 -8</i>
GCE	General Certificate in Education (Ordinary-level replaced by GCSE in summer 1988, Advanced-level retained at 18 ⁺)
GCSE	General Certificate in Secondary Education (replaced GCE O-level and CSE in summer 1988)
GSU	Global Semantic Universe (a term coined by Umberto Eco, 1976—also, Global Semantic System)
HMI	Her Majesty's Inspectorate of Schools
ILEA	Inner London Education Authority (dissolved by the Conservative government, on March 1st 1990)

IMPACT	A primary school mathematics project involving parents in their children's mathematical activities—no universally used title other than IMPACT
MIST	Mathematics Involved in Specific Tasks (a category used by the Bath University Study on behalf of the Cockcroft Committee, Bailey, 1981)
NCC	National Curriculum Council
R1-4	<i>SMP Books B1 -4</i>
R1TG-R4TG	<i>SMP Teacher's Guide to Books R1 -4</i>
RNA	Ribonucleic Acid
SEAC	Schools Examination and Assessment Council
SMILE	Secondary Mathematics Individual Learning Experience (mathematics curriculum scheme originally based in the ILEA and organised on an individualised learning approach)
SMP	School Mathematics Project
SMSG	School Mathematics Study Group (USA)
STIM	Specific Tasks Involving Mathematics (a category used by the Bath University Study on behalf of the Cockcroft Committee, Bailey, 1981)
TP1-10	Theoretical Propositions 1-10 (see Chapter 3)
Y1-5	<i>SMP Books Y1 -5</i>
Y1TG-Y5TG	<i>SMP Teacher's Guide to Books Y1 -5</i>
YE1-2	<i>SMP Y Extension Books 1-2</i>

Reading Conventions

I have employed the following conventions in producing this thesis.

References to chapters

When referring to Chapters or to Sections within this thesis, I have used initial capitals. When referring to chapters or to sections within chapters of the SMP texts, I have used lower case initials (other than at the beginning of a sentence). Specific chapters in the SMP texts are referred to as Y1.01 (Book Y1, chapter 1), etc.

Extracts from SMP texts

Where I have included extracts from the SMP texts other than as Plates, I have maintained the line endings that obtain in the originals. I have also, where possible, copied the arrangement of the original in respect of emphasis, indentations, line spacing, and diagrams. Deviations from this have been noted.

In referencing extracts from the SMP texts, I have used 'Y1', to stand for Book Y1, 'Y2', for Book Y2, and so on. Teacher's Guides are referred to as 'Y1TG', 'Y2TG', etc.

Other extracts

All emphases, brackets, etc are in the original versions unless otherwise stated. Extracts which start or finish within a sentence are preceeded (or succeeded) by '...'. My ellipses or ammendments within an extract are denoted by square brackets, non-bracketed ellipses are in the original.

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Introduction

The two textbook covers shown in Plates 1.1 and 1.2 are both similar and very different. They are similar in size and shape and they both include the expression, 'SMP 11-16' and a large capital letter next to the numeral, 1, in a ruled-off section at the top. The books appear to be elements of the same system, but we cannot be sure about the nature of the system—or even that these are textbooks—unless we happen to know what is signified by 'SMP 11-16'. The colours are different: one is fawn with 'Y1' in yellow; the other has an orange background with 'G1' in green. The images on the covers are also very different, but neither are unambiguous indicators of the contents. The G1 cover is fairly straightforward, presenting three, ordinary-looking timepieces, all showing the same time. A rather mundane image. Y1 is different. It appears to be a contour map of a face. This is an unexpected, perhaps enigmatic image. Which book would you open first? Why? What would you expect to find?

If we do open the books and look rather carefully, we find that they relate to the 'School Mathematics Project' and that Y1 and G1 are part of the 'yellow series' and 'green series', respectively. The fact that G1 has a chapter called 'Time' seems to account for its cover illustration. This particular illustration of 'time' might suggest that the topic is to be treated in a rather mundane way. Section E of chapter 11 in Y1 is called 'Contours and profiles'. This explains the contour map on the cover, but not why the map is in the form of a human face. A humanising of mathematics, perhaps, or perhaps a mathematising of humanity: an iconic chiasmus. In any event, something is left unexplained about the illustration. We are invited to resolve the mystery by reading the book. The G book will tell us about watches and clocks, in case we didn't know already.

It is very easy to announce these obvious differences between the two textbook covers. It is rather more difficult to describe the differences in sociological terms. It might even be difficult to believe that they have any sociological significance: they are, after all, only cover illustrations. Nevertheless, the proposition to be explored in this thesis is that these two books, and the series of which they are a part, can be described as participating in a sociocultural nexus such that they serve to produce and

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Plate 1.2
(G1: front cover)

Plate 1.1
(Y1: front cover)

reproduce patterns of social relations by the differential distribution of mathematical knowledge and by the construction of different forms of articulation between the mathematical and the non-mathematical. In exploring this proposition, I shall derive and present a theoretical language for the systematic sociological reading of pedagogic texts. This language will be applied to the two series of textbooks of which Y1 and G1 are initiators.

In the remainder of this introductory Chapter, I shall make some brief comments regarding the kind of analysis that will be presented and about the manner of its presentation. I shall then give a brief, schematic guide to the thesis as a whole.

1.1 The analysis

As I have indicated, my project is the description of the obvious. However, the problem is not simply to say that *A* differs from *B* in the following respects. The real problem is to produce an analysis which is, firstly, sociological and, secondly, exhaustive—within the terms laid out by the general methodological position and within the constraints of a PhD thesis.

A search of the literature (see Chapter 2) has revealed an absence of systematic sociological readings of pedagogic texts. This thesis represents an attempt to generate a theoretical framework, or model, to facilitate such readings. At the stage of its presentation, the framework constitutes an approach towards a coherent deductive structure which can be applied to the analysis of pedagogic texts (within the constraints discussed in Chapter 11). However, the making of the model involved both deductive and inductive processes. That is, the formulation of a general methodology proceeded alongside an immersion in the empirical texts in a dialogical manner. The culmination of this dialogue between the texts and an emergent theory was the production of both a language of description, which attempts deductive coherence, and an analysis of the textbooks. This dialogical process cannot adequately be represented by its product, this thesis. I have, however, attempted to give some of the flavour of the inductive process in the form of presentation of the analysis in Chapters 6 and 7.

At the point of its application, the relationship of the language to the text is, for the most part, such that the concepts of the former are indicated by the elaboration of exemplars from the latter, rather than by fixed markers. This does not reduce the value of the analysis; as Paul Atkinson argues, in relation to ethnographic work: ‘...

even the most “scientific” of accounts depends upon rhetorical, persuasive features’ (Atkinson, 1990; p. 2). However, this entails that the reliability of the description can be affirmed only on the basis of socialisation into the language itself and into its application. I have attempted to provide for this, in part, by quoting extensively from the texts and by including a substantial number of plates which reproduce pages from the texts. Of necessity, this has increased, somewhat, the overall length of the thesis.

The fact that the theory was developed, at least partly, within the context of an empirical analysis has had consequences for both the theory and the analysis. These are discussed in the concluding Chapter. However, one point should be made at the outset. This is that the analysis has treated the SMP scheme as a closed text. By this I mean that there has been no attempt to negotiate the meaning of the text either with its empirical authors or with its users. This does not pose a problem, from a methodological point of view, because there is no *a priori* reason to privilege either authors’ interpretations of their own texts or the interpretations of particular groups of readers. All empirical research must mark out its data base in some way. I have chosen to do so by focusing on the physical boundaries of the textbooks. The reason for this choice is, essentially, one of exigency in the context of a theoretically and empirically original project. The choice entails that, unless otherwise specified, all subjectivities that are referred to are constructions of the text (more correctly, constructions of the reading of the text). In particular, students and teachers are constructed as ideal and not empirical readers and authors of pedagogic texts.

Nevertheless, in order to avoid any undue tormenting of language, forms of words may be used that appear to attribute agency to these textual constructions. It is asserted, here, that such reification is not intended. Nor is it intended to reify any other structure or element as exhibiting causal agency or existential choice. The theoretical development is directed at systematic sociological description, not at the location of social motors.

Finally, it is acknowledged that the fact that the empirical object of the analysis is a mathematics text places constraints on both the language of description that has been generated and the analysis that it has produced. Nevertheless, it is asserted that the language is capable of more general application. These constraints and possibilities will be examined in the concluding Chapter.

1.2 A schematic guide to the thesis

The work of the thesis begins in Chapter 2, which is intended to locate the project empirically. There are three components to Chapter 2. Firstly, I shall discuss two prominent epistemological positions in the field of mathematics education. These are important, in the context of the thesis, because they are concerned with the relationship between school mathematical knowledge and the ‘world’ beyond the mathematics classroom. A form of this relationship is indexed by the illustrations on the textbook covers in Plates 1.1 and 1.2 and is clearly of central importance to a sociology of school mathematics.

The two epistemological positions are both concerned with mathematics in the world, but they both effect an abstraction of mathematical knowledge from the material conditions of the practices that they describe. My initial response to this, described in the second part of Chapter 2, was to construct a map of social locations for these practices. The map was constructed during an early phase of the research (Dowling, 1989) and was found to be inadequately theorised for the task in hand. Nevertheless, it established an empirical location for the project. Within this location—the recontextualising field of the school site—a narrower focus is placed upon school mathematics textbooks. This sets up the context for the third part of the Chapter, which reviews the literature on the analysis of textbooks.

Chapter 3 is concerned with the general methodological location of the thesis. In this Chapter, I engage, at the theoretical level, in a critical discussion of a body of work which has been particularly influential in the establishing of my language of description. The discussion is presented within three themes. The first concerns a recurring distinction that is made, within the social sciences, between the abstract and the concrete. This distinction has been made in a variety of ways across a range of disciplines and is the basis of a crucial dimension in my own work; this is the concept of *discursive saturation*. The second theme concerns the construction of subjectivity. My starting point is Althusser’s (1971) theory of ideology and, in particular, his interpellation metaphor. I develop my own position from Althusser’s theory of ideology in general, to a description of pedagogic action in respect of ideologies in particular, drawing particularly on Eco’s (1976) conception of the relationship between metaphor and metonymy. Finally, in the third theme, I consider the contextualising and recontextualising of practices. In this theme, in particular, the work of Basil Bernstein is established as foundational to the general methodological position that is being developed.

At the end of Chapter 3, I present ten Theoretical Propositions. These statements of general methodology have arisen out of the engagement with the theoretical issues involved in the three themes which have been discussed in the Chapter. They constitute a statement of the general methodological location of this thesis and form the basis for the theoretical development in the following Chapter.

In Chapter 4, I present the language of description which is the principal theoretical achievement of this thesis. The Chapter opens with an introduction to the concept of a language of description which was originally proposed by Bernstein. The main structure of the language is then presented as a derivation from the Theoretical Propositions which were presented in the previous Chapter. An exhaustive presentation of the language, at this point, would result in an unduly complex Chapter. Furthermore, since illustrations of the various terms of the language would have to be included, this would also entail a substantial amount of overlap with the work in the analysis Chapters. Chapter 4 is, therefore, to be understood as an introduction to the main structure and principal terms of the language of description.

Chapter 5 turns the focus onto the application of the language of description and onto the empirical achievements of the thesis. It opens with a brief introduction to the School Mathematics Project and to the *SMP 11-16* scheme which is the empirical object of the thesis. Following this introduction, the language of description is used to perform an initial coarse (and essentially quantitative) analysis of the Y and G series of books. The purpose of this analysis is to motivate the sampling of the materials. It reveals two mathematical topics, algebra and probability, as exhibiting appropriately contrasting treatment within the series. This initial analysis also reveals important differences between the two series which constitute 'findings', in themselves.

Chapters 6 and 7 analyse chapters in each textbook series dealing, respectively, with the mathematical topics that were indexed in the previous Chapter, that is, algebra and probability. I have presented this analysis in the form of close, comparative readings of the initial textbook chapter in each series dealing with the relevant mathematical topic. The intention is to suggest the inductive part of the process of the development of the language of description, in order to complement the deductive form of Chapters 3 and 4. However, the result is that there is a certain amount of repetition of findings. This is particularly the case between Chapters 6 and

7. That is, although they reveal different treatment by the texts of algebra as compared with probability, they introduce, with one important exception, essentially the same features of the language of description. This is justified, firstly, by the sampling decisions taken as a result of the analysis in Chapter 5. Secondly, these two pairs of SMP chapters represent comparatively rare instances of the two series of books dealing with what are nominally the same topics. It was felt that this offered a valuable opportunity for comparison.

Chapters 8, 9 and 10 are arranged in terms of three major categories of the language of description. That is, they describe the Y and G textbooks in different ways, illustrating different features of the language of description and generating substantially different kinds of finding. Essentially, I conclude that the textbooks construct 'ability' via the distribution of different kinds of mathematical knowledge to ideal readers which are characterised in terms of social class. The selection of illustrative extracts has been made on the basis of that which will best facilitate a comparison between the two series and in order to cover as wide a range of the textbooks as is possible in the available space. There is, of course, a degree of arbitrariness about some (but not all) of these selections. However, it is asserted that the analysis is exhaustive insofar as the selection of alternative extracts would not have generated fundamentally different findings. This assertion is supported by the form of analysis presented in Chapters 6 and 7, which covers all of the text in each of the relevant Chapters (as well as some additional material).

The final Chapter of the thesis, Chapter 11, is in three parts. Firstly, I provide an overview of the thesis as a whole, including a summary of the language of description and of the findings of the empirical analysis. Secondly, I have explored the limitations of the research in respect of four areas: the fundamental assumptions of the language of description; the application of the language; the specificities of school mathematics; the specificities of the *SMP 11-16* texts. The discussion in relation to each of these areas suggests areas for further research and these are also discussed in this part of the Chapter. I have also indexed four pieces of research which are currently underway or which have recently begun and which involve the application and development of my language of description within different areas of mathematics education.

In the final Section of Chapter 11, I work more freely with the language of description and with some of the findings in order to generate a theoretical speculation on a possible conception of a productive relationship between sociological research

and educational practice. As a sociological description which is not concerned to locate and tune, nor even to specify the existence of, the motors of social change, it may be said that the thesis presents a disturbing image of a small aspect of the educational world. As a structured resource for the organisation and interrogation of educational practice, however, it may generate a more radical potential.

Chapter 2

Mathematics Education & Textbook Analysis: an empirical focus

My purpose in this Chapter is to establish the empirical focus of the thesis and to give some consideration to previous work which has had a similar focus. Although the disciplinary basis of the thesis lies within sociology, it clearly has a bearing on mathematics education. A search of the literature on the sociology of mathematics and mathematics education reveals an almost total void¹. In the first Section, therefore, I shall describe, briefly, two epistemological positions which are widely represented in the professional and research literature within the general field of mathematics education. From the point of view of this thesis, these positions are of importance because they are concerned with establishing a relationship between mathematics and practices which are, in some sense, beyond mathematics. I shall argue that, from the perspective of sociological enquiry, a principle failing of each of these positions is a lack of any adequate attention to the level of the social—to this extent they are revealed as ‘mythical’. I shall then introduce a map of social contexts for mathematical practices which will enable me to locate the specific interest of this thesis. Of the various possible focuses within this location, I have chosen to look at a particular scheme of school mathematics textbooks. Therefore, I shall conclude the Chapter with a consideration of other work which has taken the school textbook as its object.

¹ Exceptions are the work of Barry Cooper (1983, 1985) (whose work is discussed briefly in this Chapter and in Chapter 5) and Chris Knee (1983, Farwell & Knee, 1990) whose work on the educational basis of mathematical knowledge is discussed in Dowling (1986). Also discussed in my earlier study are Bloor (1976) and Restivo (1984)—on the sociology of mathematical knowledge—and Spradbery (1976), on pupil resistance to mathematics curriculum innovation. This work is not of direct relevance to the present study. The Marxist analysis of Alfred Sohn-Rethel is of relevance and will be discussed in Chapter 3. There is a considerable body of work on the gendering of mathematics education (for example: Brown (1984); Buerk (1982); Burton (1986, 1988); Burton et al (1986); Cresswell & Gubb (1987); Ernest (1988); Fennema (1985); Isaacson (1988); Leder (1987); Lee (1992); McBride (1989); Scott-Hodgetts (1986); Shuard (1986); Walden & Walkerdine (1982, 1985); Walkerdine (1988, 1989)). This work is of sociological interest in that it focuses on a dimension of social inequality. With the exception of Walkerdine’s work (to be discussed in Chapter 3, although not in relation to gender), however, it is tangential to the present enquiry.

2.1 Two mythical epistemologies in mathematics education

The two positions that I shall describe will be referred to as ‘utilitarianism’ and ‘mathematical anthropology’¹. In neither of these areas am I laying claim to an exhaustive treatment, either of the literature, or of possible criticisms of it. My intention is, primarily, to demonstrate the existence of these positions within the mathematics education literature and to establish my sociological position in relation to it.

2.1.1 Utilitarianism

‘Utilitarianism’ refers to that view of mathematical knowledge which represents it as having a general functional value. Jean Lave’s description of ‘functionalist psychological theory’ introduces an appropriate metaphor:

... mind and its contents have been treated rather like a well-filled toolbox. Knowledge is conceived as a set of tools stored in memory, carried around by individuals who take the tools (eg ‘foolproof’ arithmetic algorithms) out and use them, the more often and appropriately the better, after which they are stowed away again without change at any time during the process. The metaphor is especially apt given that tools are designed to resist change or destruction through the conditions of their use.

(Lave, 1988; p. 24)

This view of mathematics is pervasive within the ‘official’ school curriculum. Her Majesty’s Inspectorate of Schools (HMI), for example, have made use of the same metaphor:

The aim is to encourage the effective use of mathematics as a tool in a wide range of activities within both school and adult life.

(DES, 1985; p. 3)

The first (of five) ‘Attainment Targets’ (ATs) of the current mathematics National Curriculum in England and Wales is entitled ‘Using and applying mathematics’. This AT includes the following preamble:

Pupils should choose and make use of knowledge, skills and understanding outlined in the programmes of study in practical tasks, in real-life problems and to investigate within mathematics itself. Pupils would be expected to use with confidence the appropriate mathematical content specified in the programmes of study relating to other attainment targets.

(DES, 1991; p. 2)

¹ These terms were introduced in this context in Dowling (1989) (see, also, Brown & Dowling, 1989).

The utilitarian value of mathematics is, thus, an important (although not the exclusive) justification of its place on the school curriculum; as the National Curriculum Mathematics Working Group noted:

The power and pervasiveness of mathematics accounts for its pre-eminent position, alongside English, in the school curriculum.

(DES, 1988; p. 3)

An important document within the context of the current English and Welsh mathematics curriculum is the report of the Cockcroft Committee of Enquiry. The Committee was set up in 1978, in the wake of Prime Minister Callaghan's Ruskin College speech and the 'Great Debate'. These events are, themselves, seen as responses to widespread criticisms of state education, especially from the engineering industry and the writers of the Black Papers (see Dowling, 1986; Kogan, 1978; Lawton, 1980; Salter & Tapper, 1981). The Cockcroft Committee was instructed to pay:

... particular regard to the mathematics required in further and higher education, employment and adult life generally ...

(Cockcroft et al, 1982, p. ix)

As part of their response, the Committee set up three research studies. Two of these looked at mathematics and employment and were carried out at the Universities of Nottingham (Shell Centre, nd) and Bath (Bailey et al, 1981). The third was a smaller scale study focusing on 'adult life generally' (Sewell, 1981). I have discussed these three reports elsewhere at some length (Dowling, 1986, 1989). Here, I shall refer to brief extracts from each of these studies to illustrate the main point that I want to make concerning the utilitarian position. The first is from Sewell's study, *Uses of Mathematics by Adults in Everyday Life*:

Percentages play an ever increasing part in the dissemination of information, both through the news media and from central government. An understanding of the national economy assumes a sophisticated comprehension of percentages, as does much of the discussion about pay rises. For the shopper, the ability to estimate 10 per cent can be a valuable 'key' to checking other percentages—even if a precise answer seems too difficult. Since the currency became decimalised, it is a trivial matter to work out 10 per cent of a sum of money, and this can easily be used to estimate other percentages. Those who lack the skill even to calculate 10 per cent are surely handicapped when attempting to understand the affairs of society.

(Sewell, 1981; p. 17)

Here, a scenario is presented in which mathematics¹ has been incorporated into 'everyday' practices so that the individual who lacks mathematics is effectively

¹ Mathematics in the form of percentages, in this instance. Sewell placed considerable importance upon this particular topic.

'handicapped' by their own ignorance. The response is to teach percentages; Sewell, herself, held the post of 'numeracy coordinator' for Reading.

Clearly, it is the case that 'percentage' is incorporated into the practices that Sewell mentions and many others. It is likely that some individuals perform calculations within these contexts¹. However, Sewell's assertion that a lack of mathematical skills constitutes a handicap within these contexts is untenable: in fact, mathematical skill is neither necessary nor sufficient for optimum participation within these practices. Firstly, in respect of the necessity of mathematics, there are always alternative resources (price labels, tables showing loan repayments or the effects of tax increases, etc, shop assistants, and so on). Jean Lave (1988; Lave et al, 1984), in a rather more sophisticated study, finds that shoppers are highly successful at making best buy decisions and rather less successful at carrying out apparently similar calculations within a mathematical context.

Secondly, Sewell's assertion that '[a]n understanding of the national economy assumes a sophisticated comprehension of percentages' is a strong statement of utilitarianism only if it implies that this 'sophisticated comprehension' is sufficient to such an understanding. In fact, this is clearly very far from the case. Whilst Jean Lave's study entailed accompanying shoppers to see what they actually did, Sewell's study involved 'presenting [her] interviewees with real life situations' of her own devising (Sewell, 1981; p. 5). Some of these 'real life' situations appeared to concern the 'national economy':

On the news recently it was said that the annual rate of inflation had fallen from 17.4% to 17.2%. What effect do you think this will have on prices? (If answer 'none') What do you think ought to happen if it had fallen to, say, 12%?

(Sewell, 1981; p. 33)

In order to make any sense of this in terms of the effect on an individual's financial situation (ie which prices), they would have to be in possession of and comprehend details of (among other things) the particular type of weighted average upon which the retail prices index is computed. It is also not entirely clear how one might interpret, 'what effect do you think this will have on prices?' Sewell's apparent belief that the rate of inflation has some direct effect on prices (rather than the other way around) seems to suggest that her own understanding of the national economy is somewhat less than sophisticated; this has to do with economics, but not mathematics.

¹ There are also everyday 'uses' of percentages within which calculations would be impossible or meaningless: I'm ninety-nine percent certain that this is the case.

A third extract from the Cockcroft research studies—this time from the Bath University study—is instructive. This study focused on the mathematical requirements of the working practices of sixteen-to-nineteen-year-olds. The researchers collected data which they categorised as ‘specific tasks incorporating mathematics’ (STIM) and ‘mathematics incorporated in specific tasks’ (MIST). They found that a great many young employees—a ‘vast army of people’—did not appear to require any formal mathematics, not even counting or recording numbers. Nevertheless, they asserted that ‘all these occupations involve actions which could be described in mathematical terms’ (Bailey et al, 1981; p. 12). The researchers presented a list of these mathematical terms (MIST) as follows:

A set, dis-joint sets.

Mappings, one-to-one, one-to many, many-to-one correspondences.

Symmetry, bilateral and rotational.

Rotation, reflection, translation and combinations of these[.] Tessellating patterns.

Logical sequences (if then).

(Bailey et al , 1981; p. 26)

The terminology used here is very much out of the ‘modern mathematics’ tradition of the era. The tasks (STIM) to which these MIST items correspond are:

- (i) Articles are sorted into separate collections for packing or on an accept/reject basis.
- (ii) Articles are moved into particular orientations involving moving sideways, turning over or round.
- (iii) Articles, such as wine glasses[,] are checked for uniformity of shape.
- (iv) Packed articles form regular patterns.
- (v) Assembly tasks can involve matching parts, such as connecting wires to correct terminals.
- (vi) Tasks often have to be carried out in particular orders sometimes requiring simple decisions, but which would not often be verbalised. For example, a creeler in a carpet factory: ‘Is the spool empty? Yes! Replace with another of the same colour’ Awareness of the consequences of not following the prescribed order may be important.

(ibid; pp. 25-6)

In this case, the researchers have explicitly denied that mathematics is necessary or sufficient for the successful completion of these tasks. Nevertheless, they have been able to describe them in terms of the current (at that time) school mathematics syllabus. My point is that both Sewell and the Bath team are projecting school mathematical signs (in the semiotic sense) onto practices which are constituted within sets of social relationships which are very different from those obtaining in the school. In Sewell’s case, however, there is no recognition of the possibility that

moving between two social contexts might have a reconstituting effect upon practices that appear the same only when viewed from the perspective of one of these contexts.

The final extract is from field notes made by the Nottingham University researcher, Robert Lindsay¹. The extract reports an interview with a Construction Industries Training Board advisor, John:

... every employer he spoke to said the lads don't fill in their time-sheets correctly and therefore find difficulty in seeing how their total money is arrived at. John noted, however, that in the building site dart games these same people can always add and subtract and find out how many more they need to score without any apparent difficulty. They also have a reasonably good idea of how much change they should get, so John infers that the basic ability must be there and it probably hasn't been exercised enough at school.

(Shell Centre, nd; p. 585)

It is John rather than the researcher, who is pathologising 'the lads' as lacking in mathematical experience. This time, there are three contexts which are referred to: the completion of time sheets within a working context; the computation of darts scores within a popular leisure context; the practising of arithmetic within a school context. John is quite ready to suture these fundamentally different contexts via the assumption of a set of transferable skills. An alternative analysis might have investigated the different social conditions obtaining within the contexts through which these 'lads' moved, taking note, perhaps, of another study about another group of 'lads' (Willis, 1977). Lindsay's own notes also mark out clear distinctions between the Further Education curriculum and working experiences of apprentices (Dowling, 1986, 1989).

These three research studies and the Cockcroft Report (1982) for which purpose they were commissioned exhibit a utilitarian epistemology in enquiring into the 'mathematics required' in working and domestic life. This position presents a 'toolbox' image of mathematics which fails to attribute any significance to the specificities of the social relations obtaining in different contexts: the toolbox can simply be carried around from school to work to the shop to the home and so on². These research studies certainly fail to provide any evidence which would justify such

¹ Lindsay unfortunately retired and subsequently died before he was able to analyse his field notes. The notes were published in raw form by the Shell Centre for Mathematical Education.

² Not all work exhibiting a 'utilitarian' epistemology corresponds, like that described above, perhaps, with the aims of what Williams (1961) referred to as the 'industrial trainers'. Some more recent work (for example, Norton, 1993) is, within the same schema, more properly associated with Williams' 'public educators' in its democratic, emancipatory and even critical (Brown & Dowling, 1989; Frankenstein, 1989; Skovsmose, 1985, 1988, 1993) intentions.

idealism¹. Rather more sophisticated learning-transfer research has, similarly failed to produce strong evidence of transfer (Lave, 1988). The work of Jean Lave (1988; Lave et al, 1984) and others (for example, Carraher, D., 1991; Carraher, T.N. et al, 1985; Carraher, T.N., et al, 1987, 1988; Scribner, 1984) provides rather more convincing evidence that any simple notion of the transfer of mathematical skills between school and other contexts is an inappropriate suturing of the social. As Lave points out, the central characteristics of learning transfer research²:

... include the separation of cognition from the social world, the separation of form and content implied in the practice of investigating isomorphic problem solving, and a strictly cognitive explanation for continuity in activity across situations. All of these *dissociate* cognition from its contexts, and help to account for the absence of theorizing about experiments as social situations and cognition as socially situated activity. The enterprise also rests on the assumption of cultural uniformity which is entailed in the concept of knowledge domains.

(Lave, 1988; p. 43)

My intention, however, is not to argue for the absolute boundedness of socially defined contexts (no more is Lave's). Rather, that there exists a widely held position within the field of mathematics education which defines mathematics as having a pervasive, if not universal, utilitarian value. This position is justified by a, generally unquestioned, 'recognition' of school mathematics practices well beyond the classroom door. At the very least, such projections of school mathematics are gross reductions of intra-societal cultural and social diversity and justify the description of utilitarianism as 'mythical'.

2.1.2 Mathematical anthropology

The utilitarian myth 'recognises' mathematical tools in diverse practices, but it constructs a role for mathematics education, in providing the toolbox, and a pathological lack on the part of the yet-to-be-tutored. The term 'mathematical anthropology', on the other hand, refers to a growing body of work which seeks to celebrate an alreadiness of mathematical content within the practices of different

¹ A more recent government sponsored study which claims to show evidence of transfer of skills in relation to the workplace (Wolf et al, 1990) in fact defines 'context' in terms of the nominal content of the training and test exercises. There was no attempt to investigate the performances of the subjects (Youth Training Scheme trainees) within workplace settings. Thus there was an assumption that the 'context' of an activity is determined by its formulation in abstraction from specific social relations.

² To a certain extent, Lave appears, herself, to be guilty of pancontextualism (at least at the level of the signifier) in her justification of her interest in 'arithmetic': 'Arithmetic is a sympathetic "medium" for the researcher who wishes to study activity in open-ended situations, for it has a highly structured and incorrigible lexicon, easily recognizable in the course of ongoing activity.' (Lave, 1988; p. 5). However, her own model of 'cognition in practice' would seem to contradict this.

cultural groups. Within the context of mathematics education, this work has come to be referred to as 'ethnomathematics'—a term originally coined by Ubiratan D'Ambrosio (1985, 1990). I have discussed this work elsewhere (Dowling, 1989, 1992a, 1993b, 1993c, in press, Brown & Dowling 1989). Here, I shall introduce sufficient of the work to establish my principal argument.

Paulus Gerdes (1985, 1986, 1988a, 1988b), for example, is an originally Dutch educator now working in Mozambique. He has studied non-industrialised production processes in Mozambique—the building of huts, the weaving of fishing baskets and buttons, etc—and traditional sand drawing in Angola. Like the Cockcroft researchers, he has described these practices in terms associated with secondary school mathematics. A pattern in a woven button, for example, signifies Pythagoras' theorem (1988b). However, unlike the utilitarians, Gerdes does not pathologise the unschooled Mozambicans. On the contrary, he celebrates a mathematical knowledge which is already there, which has been 'frozen' into the button as a kind of 'dead mathematical labour' (to revive Marx's expression). This discovery, Gerdes suggests, might be incorporated into mathematics lessons to stimulate cultural confidence:

'Had Pythagoras not ... *we* would have discovered it'. The debate starts. 'Could our ancestors have discovered the "Theorem of Pythagoras"?' 'Did they?' ... 'Why don't we know it?' ... 'Slavery, colonialism ...'. By 'defrosting frozen mathematical thinking' one stimulates a reflection on the impact of colonialism, on the historical and political dimensions of mathematics (education).

(Gerdes, 1988b; p. 152)

Reflexion on the impact of colonialism is no bad thing in the Mozambican or, indeed, in any other curriculum. The difficulty is that it appears that a European is needed to reveal to the African students the value inherent in their own culture. When he does so, of course, he does it in European terms, even referring to a European 'mathematician' (Pythagoras)¹. The African culture, in other words, is not being allowed to speak for itself.

Gerdes intention is not simply to stimulate cultural confidence, but also to teach mathematics. The mathematics that he wants to teach is very much in the tradition of the contemporary European school. However, he refers to this mathematics as 'world-mathematics' (1988a). He is able to do this via his process of 'defrosting':

¹ Decontextualising 'the theorem', it appears to have been known outside of Europe well before the time of Pythagoras (Joseph, 1991), so that the attribution itself is eurocentric.

The artisan who imitates a known production technique is—generally—not doing mathematics. But the artisan(s) who discovered the techniques, *did* mathematics, *developed* mathematics, was (were) thinking mathematically.

(Gerdes, 1986; p. 12)

Gerdes' 'defrosting' actually denies cultural value (in mathematical terms) to contemporary Mozambicans. At the same time, he is postulating a mathematical originator who created the technology. Gerdes is thus projecting onto African culture not only 'European' mathematics, but also a 'European' capitalist model of 'Fordist' production, in which the worker has no access to the principles of her/his productive processes and is denied creativity (Braverman, 1974; Dowling, 1991b; Hales, 1980; Matthews, 1989), and a 'European' 'great man' model of history which is a distortion even in 'European' terms¹.

Gerdes describes mathematics as 'panhuman' (1988a). Alan Bishop (1988a, 1988b), similarly, describes mathematics as a 'pan-human phenomenon' and marks out six 'fundamental activities' that he describes as 'both universal, in that they appear to be carried out by every cultural group ever studied, and also necessary and sufficient for the development of mathematical knowledge' (1988a, p. 182). These 'fundamental activities' are: counting, locating, measuring, designing, playing, and explaining. I shall refer to just one of these, locating (although similar evidence could be presented in relation to each). Pam Harris has noted that the Aboriginal people with whom she has worked make use of the points of the compass far more frequently than left and right in orienting themselves in space. This preference extends to circumstances in which no European would ever use the points of the compass:

Even in the most personal and intimate situations related to a person's own body, the Warlpiri still use compass terms and not left and right. Laughren told me of a woman at Yuendumu who, when giving birth to a child in hospital, said that she had a cramp 'warnarri kakarrara'; that is, in the 'east leg'.

(Harris, 1991; p. 24)

The Warlpiri knowledge of compass orientations pre-dated the European invasion of Australia. The sensitivity of individuals of this society to their personal orientation with respect to the world is suggestive of quite a fundamental break with the Cartesian epistemology. The latter locates the knowing subject as the frame of reference for the external world; Warlpiri epistemology² appears to inscribe the

¹ See, for example, Cockburn (1983, 1985a, 1985b, 1985c) and MacKenzie & Wajcman (1985) for more sociologically aware histories of technology.

² I am, of course, applying the European term 'epistemology' to a non-European culture. However, my intention is to use this European expression to highlight rather than to background differences.

individual within the world. This being the case, the classification of both European and Aboriginal practices as 'location' is clearly a gross reduction¹.

Mathematical anthropology is an area which is now attracting increasing interest on a global scale (see in addition to those mentioned above, for example: Abraham & Bibby, 1992; Fasheh, 1991, 1993; Knijnik, 1993; Maier, 1980; Vithal, 1993). To varying degrees, all of this work succeeds in celebrating non-European cultural practices only by describing them in European mathematical terms, that is, by depriving them of their social and cultural specificity². These authors are able to recognise a practice as mathematical only by virtue of recognition principles which derive from their own enculturation into European mathematics³. I shall, however, offer a final extract from what was possibly the first piece of work in the 'ethnomathematics' tradition:

The African adapts his [sic] home admirably to his means of subsistence, to the available materials, and to the requirements of the climate. The circular house in its many versions is found throughout the continent. The circle, of all closed geometric shapes in the plane, encompasses the greatest area within a given perimeter. Confronted by a scarcity of building materials and by the urgency to erect a shelter without professional assistance and in the shortest possible time, the African choses the circle as the most economical form. He is not unique; round houses are constructed in the Arctic as well as at the Equator.

(Zaslavsky, 1973; p. 155-6)

This extract is from Claudia Zaslavsky's book, *Africa Counts*—a title which is patronising in both interpretations of the pun. Unlike most of the other writers in this

¹ There exists a body of work on education and Aboriginal cultures which does not reduce Aboriginal cultures to mere exemplars of a global (and essentially European) culture, see, for example: Carroll (1991); Harris, P. (1991); Harris, S. (1988); Ilyatjar (1991); Kearins (1991); McTaggart (1989); Parish (1991); Watson (1987, 1988); Wunungmurra (1988). Some, however (for example, Carroll, 1991; Currie et al, 1992; Guider, 1991; Kearins, 1991) still seems to prioritise the 'European' school curriculum and focuses on difficulties arising out of cultural difference.

² Mary Harris (1993) argues that my (1989) criticism 'seems harsh' in respect of D'Ambrosio who 'classes western academic mathematics as merely one form of ethnomathematics' (Harris, 1993; no page nos). However, my point is not to accuse D'Ambrosio of an explicit elevation of European mathematics above other practices. Rather, I am maintaining that he implicitly does so by classifying this diversity as mathematics at all. This is because he continues, unreflexively, to use (implicit) recognition principles which derive from his own positioning within European mathematics. Harris's own later work (1985a, 1985b, 1987, 1988a, 1988b) is somewhat less susceptible to (but not exempt from) this criticism than most insofar as she foregrounds and celebrates culturally (and industrially) based technologies which are presented as curriculum materials for European students. Most of the other work in the ethnomathematics category is more clearly intended to be *for* non-Europeans.

³ This point is recognised by Bishop in his qualification of his position: 'Perhaps a safer label would in any case be 'culturo-centric universals' ie universals from *our* culturo-centric position, since we are describing the phenomena as 'counting', etc this then makes it plain that one can never *establish* the universality of phenomena, one is merely choosing to describe a highly extensive set of similarities in a certain way' (Bishop, 1988b; p. 55). However, this (idealist) admission of cultural relativism is negated by the general line of argument and intentions revealed in his book (Dowling, 1989).

tradition, Zaslavsky is addressing middle class America. But perhaps this renders the objectification of *the* African even more excruciating. The myth of mathematical anthropology is a celebration of the mathematical practices of non-European societies. However, it achieves this in the same way as the utilitarian position. European mathematical practices constitute recognition principles which are projected onto the other so that mathematics can be ‘discovered’ as if it was there already. In both cases, the recognition of self in the other abstracts descriptions of practices from the constitution of the practices themselves within distinct sets of social relations. The difference lies only in the extent to which the projector constructs an explicit role for itself. The utilitarian myth pathologises the yet-to-be-schooled, but announces the treatment. The myth of mathematical anthropology denies that there is anything wrong, but implicitly retains the prerogative and principles of diagnosis.

2.2 A map of social contexts for mathematical practices

In this Section I shall give a brief presentation of a map of social contexts which has been introduced at greater length elsewhere (Dowling, 1989). The generation of this map was an important stage in the development of this thesis, although the work subsequently changed direction, both theoretically and empirically. The theoretical developments will be described in Chapters 4 and 5. I have included the map here, however, because it enables me to begin to locate the empirical focus of the main project of the thesis.

I want to propose that (European mathematical) knowledge is: produced as academic practices; recontextualised as official pedagogic practices; transmitted as local pedagogic practices; and acquired as operationalised mathematical practices¹. The structuring of mathematical practices in this way is intended to reflect an assumption that practices must always be referred to the specificities of the social relations of the context within which they are elaborated. Thus, we might describe the production of mathematics within an academic field as practices which are validated within what is officially a peer group. However, stratification within the ‘peer’ group is likely to have implications for the authorship and dissemination of new mathematics². Whatever their specific form, these social relationships will not carry over, unchanged, to the context within which mathematical knowledge is recontextualised as official pedagogic practices. The production of textbooks, for

¹ This structure is a simplification of Bernstein’s model (1985).

² See, for example, Bibby (1988) on the relative success of vector analysis in comparison with quaternions in his ‘social history of mathematics’.

example, is contextualised within a sphere which intersects with the economic domain of publishing (see: Apple, 1986, 1989; Lorrimer & Keeney, 1989). These two contexts I have referred to, respectively, as the fields of 'production' and 'recontextualisation' of mathematical practices.

Although Apple (1986, 1989) describes the school textbook as having an 'immense' impact on the social relationships of the classroom (p. 85), these social relationships are clearly different again from those obtaining in textbook production¹. The classroom itself, however, might be resolved into two contexts for the elaboration of mathematical practices. These contexts are the fields of 'reproduction', where the practices are produced for or on behalf of the teacher, and 'operationalisation', where they are produced by the students. The resolution of the classroom into two fields is justified on the assumption that teachers' and students' practices are each influenced by relationships which are at least partially opaque to the other, as well as by the more obvious relationships between teachers and students².

Thus, I have defined, very coarsely, four 'fields' for the production of mathematical practices. However, practices that we might want to describe in mathematical terms—such as those mythologised by utilitarianism and mathematical anthropology—arise in different institutional settings or 'sites'. Clearly, all four fields may produce mathematical practices within an academic institution such as a university. Equally clearly, the school (primary, secondary, tertiary) is an institution which is separate from the university. Utilitarianism and mathematical anthropology make reference to the domains of working and popular (domestic and leisure) practices which are, again, institutionally distinct from the university and the school. I can thus, again very coarsely, refer to at least four 'sites': academic, school, work, and popular. Taking the cartesian product of 'field' with 'site', generates the map of social contexts shown in Figure 2.1.

In the earlier paper (Dowling, 1989) I engaged in some discussion of this map³. However, its value, here, is less analytic than illustrative and that which is to be illustrated is, in this case, a potential empirical space rather than a theoretical space. I will, therefore, not include further discussion of the map in this thesis. My intention

¹ And there is evidence suggesting that textbooks may not have quite the impact that Apple suggests (Freeman et al, 1983a, 1983b, 1983c; Freeman & Porter, 1989), although other studies have found them to be important (for example, Desforges & Cockburn, 1987).

² See, for example, Paula Ensor's (1991, 1993) discussion of her finding that some black girls in London schools are diligent with their homework whilst being uncooperative at school.

³ In the original formulation of this map of contexts (Dowling, 1989), I used the term 'career' for the category that I am here referring to as 'site'.

is simply to begin to mark out the specific empirical focus of this dissertation which is within cell F of Figure 2.1, that is, in the recontextualising field of school mathematics. This field generates the official pedagogic practices of school mathematics which are represented in textbooks, examination syllabuses, and in the government documents that were cited in the previous Section of this Chapter. Elsewhere, I have discussed some of the governmental documents (Dowling, 1990b, 1990c; Dowling & Noss, 1990; Noss & Dowling, 1990). For the remainder of this thesis I shall be concerned, empirically, with the sociological analysis of pedagogic texts, specifically school mathematics textbooks. The remainder of this Chapter, therefore, will comprise a discussion of the research literature in this area.

Map of Contexts of Mathematical Practices

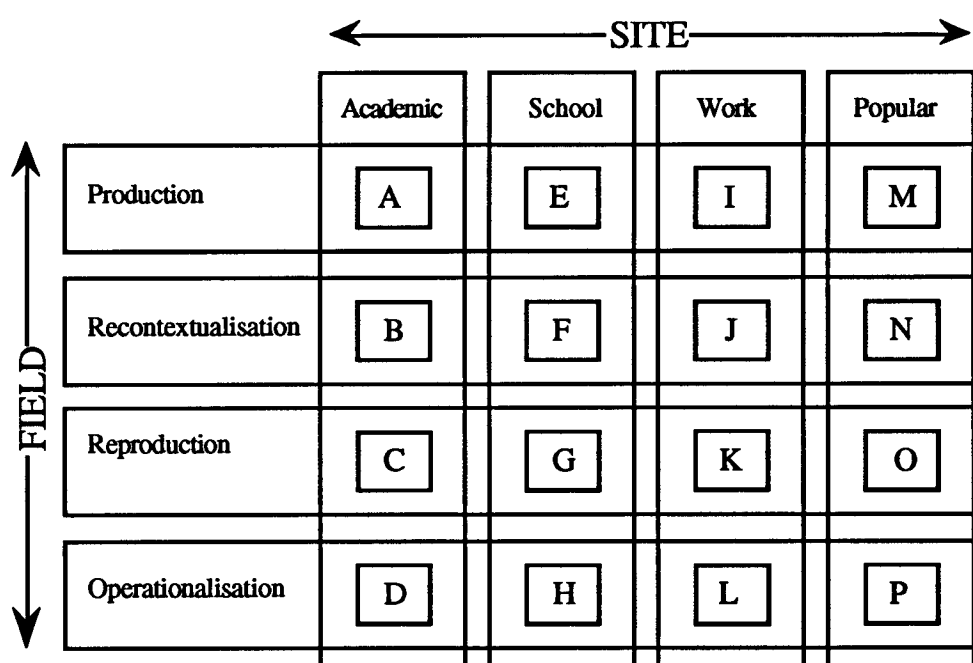


Figure 2.1

2.3 The research literature on the sociological analysis of school textbooks

A search of the English language literature reveals very little interest in the sociological analysis of school textbooks. The analysis, in general, of textbooks is, however, a very large area. Within this capacious category would fall readability and evaluation studies, for example. Almost none of this work is of direct relevance to the work of this thesis. I have, therefore, decided to consider, firstly, all analyses of

mathematics textbooks that were discovered in the search. Secondly, I shall consider 'sociologically-orientated' analyses of textbooks in other curriculum areas. This second category includes, for example, analyses in terms of gender, ethnicity and class and ideological analyses of textbooks. The work in these two categories bears, at most, an indirect relevance to the project of this thesis. Finally, I shall discuss, in rather more detail, two studies of science textbooks which seem, in different ways, to come closest to my own work. I have included, as Appendix 1 to this thesis, references and brief comments relating to a number of items which were discovered in the literature search, but which were found to be of very marginal relevance to my principal areas of concern.

2.3.1 Mathematics textbooks

A search of the research literature reveals very little interest in this area. Analysis of specifically mathematics textbooks is almost non-existent and none of the work that has been produced enters the particular sociological field of interest of this thesis. An earlier search (Garcia et al, 1990) looking for work investigating the portrayal of 'females and minorities' in mathematics texts notes that:

What was surprising in conducting this search was the absence of major studies examining the portrayal of females and minorities in elementary and secondary mathematics books.

(Garcia et al, 1990; p. 3)

In fact, Garcia et al found only one study (Nibbelink et al, 1986—see below) in respect of the representation of females and none relating to the portrayal of ethnic minorities. Their own study looked at illustrations and 'story problems' in elementary mathematics texts. They concluded that there had been a reduction, over a period of a decade, in the degree to which texts could be described as 'sexist' and that the textbooks that they analysed were appropriately described as 'multicultural'. Their main criticism of the texts concerned a common lack in the featuring of careers in mathematics.

My own search has revealed a small number of other studies focusing on the gendering of mathematics texts. Jean Northam (1982), for example, analysed textbooks published between 1970 and 1978. She concluded that '[t]here is a clear tendency in the books studied to define mathematics as the province of males, and especially adult males' (p. 14). Northam points to stereotypical images of males and females, the virtual absence of adult women from the texts and a steady decline in the representation of girls as the target age range moved from 3 to 13 years. Another

study of sex-role stereotyping in mathematics textbooks was produced by the New Zealand Department of Education (1980). This study looked at six primary school textbooks and twenty-five secondary school textbooks and, again, concluded that there was strong evidence of bias in favour of males. Maggie McBride (1989) includes a brief discussion of stereotyping in mathematics textbooks in her 'Foucauldian analysis of mathematical discourse', although without presenting any systematic analysis.

In a paper which incorporated a response to McBride, I (Dowling, 1991c) illustrated three 'textual strategies' in the gendering of mathematics texts. The first strategy reinforces a 'gender code' via stereotypical constructions; the second excludes the feminine; and the third 'defuses' antisexist challenges by introducing the feminine in a tokenist way. This paper also included a discussion of social class and mathematics textbooks which adumbrated some of the findings reported in Chapter 10 of this thesis (see, also, Dowling, 1990a, 1990b, 1991a, 1992a, 1992b, 1993a, in press).

The analysis reported in Northam's (1982) paper was undertaken as part of a mathematics diploma course for primary teachers. Work looking at representations of gender in mathematics textbooks is commonly directed at practitioners rather than at researchers. The discussion in Burton et al (1986) of gender stereotyping in mathematics textbooks, for example, appears in a pack for teachers produced collaboratively by the Open University and the Inner London Education Authority (ILEA). The ILEA's Centre for Learning Resources (1985) produced its own publication looking for bias in primary mathematics materials. Morehead's (1984) consideration of the illustrations in three textbook series appears in the UK professional journal, *Mathematics in School*, and Nibbelink's (1986) analysis of problems in elementary school mathematics textbooks appears in the US professional journal *The Arithmetic Teacher*.

The ILEA publication (1985) also addresses the issue of the representation of ethnic minority groups in mathematics texts. In this area: Hudson (1985) looks at 'militarist', sexist and ethnocentrist bias in mathematics textbooks and computer software; Rivers (1990) considers the portrayal of females and ethnic minorities in first year algebra textbooks; the Garcia paper (op cit) has already been mentioned.

Other studies of mathematics textbooks have focused on issues of mathematical subject content or problem-solving. Remillard (1991a, 1991b), for example,

evaluates the conceptions of problem-solving in elementary mathematics texts and Donald Freeman and others (Freeman et al 1983a, 1983b, 1983c; Freeman & Porter, 1989) have analysed elementary mathematics textbooks in terms of mathematical content and pedagogic intention; these analyses were compared with analyses of lessons and of standardised tests. Cairns (1984) has produced a content analysis of *SMP 7-13* in comparison with two other schemes in terms of subject content and sequencing. Kim (1993) has compared, briefly, the measurement and geometry content in American and Republic of Korean textbooks, seeking some explanation for outperforming of American students by Korean students in the Second International Mathematics Study.

In his PhD thesis, Wing (1989) draws on theoretical resources from the fields of semiotics (Eco and de Saussure) and philosophy (Heidegger) and on his own experience in using school mathematical texts as well as interview data with primary school children. Wing argues that, in their use, these texts necessarily become authoritarian in character, so as to preserve, impart or emphasise a ‘textual’ mathematics. Although this work draws on some of the semiotic resources that inform the language of description used in my own thesis, it cannot be described as a sociological study.

Finally, in relation to mathematics textbooks, the studies of Barry Cooper (1983, 1985) and Bob Moon (1986) should be mentioned. Cooper’s work is concerned with the development of ‘modern mathematics’ and specifically with the success of the School Mathematics Project in the secondary school curriculum. This work clearly relates to textbooks, but does not, itself, constitute an analysis of textbooks. Similarly, Moon’s study is concerned with the ‘new maths curriculum controversy’ in elementary education. This work is directly relevant to the development of mathematics textbooks in the UK, specifically, the Nuffield Project texts. However, as is the case with Cooper’s work, there is no analysis of the texts themselves.

2.3.2 Other curriculum areas—‘sociological’ analyses

Moving beyond mathematics as a curriculum area, textbooks have attracted some attention in the area that might be broadly described as ‘sociologically-orientated’ studies. Notably, Michael Apple (1986, 1989) considers the conditions of the production of textbooks at a higher level of analysis even than obtains in the studies by Cooper and Moon. Apple’s answer to the question how is “‘legitimate” knowledge made available in schools?” is:

By and large it is made available through something to which we have paid too little attention—the textbook. Whether we like it or not, the curriculum in most American schools is not defined by courses of study or suggested programs, but by one particular artifact, the standardized, grade-level-specific text in mathematics, reading, social studies, science (when it is even taught), and so on. The impact of this on the social relations of the classroom is immense. It is estimated, for example, that 75 per cent of the time elementary and secondary students are in classrooms and 90 per cent of their time on homework is spent with text materials [...]. Yet even given the ubiquitous character of textbooks, they are one of the things we know least about.

(Apple, 1986; p. 85)

However, Apple concentrates on the ‘ideological, political and economic sources of the production, distribution and reception’ (ibid, p. 86) of school textbooks and does not analyse any empirical texts. Lorimer & Keeney (1989) are also interested in the macroeconomic context of textbook production. They draw on analyses of the thematic and ideological content of elementary language art readers and of science textbooks in their description of the role of multinational publishing companies in ‘defining’ the Canadian school curriculum. Jean Anyon (1979, 1981a, 1981b, 1983) has also conducted ideological analyses of empirical textbooks. She looked at seventeen ‘well-known, secondary school United States history textbooks’ (1981a, p. 21-2). She analysed the content of the textbooks in the area of economic and labour union developments from the US Civil War to World War I ‘to determine whether the information they contained was in any way biased’ (ibid). She concludes that:

The analysis of history textbooks here suggests that the priorities of specific groups now powerful in the United States industrial hierarchy are expressed as well by a hidden structure of interests in the social studies curriculum. Although obscured by the claims of objectivity, a set of ideological judgements and beliefs can be identified that provide support for the activities of powerful groups.

(Anyon, 1981b; p. 32)

However, Anyon fails to present the principles of her description of these texts or of the social which, in terms of ‘interests’, is being represented by them. In this sense, she is not detecting bias so much as introducing it through her treatment of the texts as transparent with respect to her own interpretations.

Joel Taxel (1981, 1983) analysed the content and narrative structure of thirty-two children’s novels about the American Revolution, published in four periods between 1899 and 1976. He concluded that a ‘simplistic, selective, and conservative conception of the Revolution’ (1983, p. 74) was also reinforced by the narrative structure of the novels, for example, in the coding of the characters along the good/bad opposition. Taxel also compares his own analysis with Wright’s analysis of Western films and finds ‘remarkable similarities’ in the results. Taxel argues that the

novels and films serve to legitimate the structure of socioeconomic relationships. He rejects a determinist model in favour of a 'sensitivity to the historic specificity of culture and classroom life' (ibid; p. 85). However, although he genuflects to theorists such as Bernstein, Bourdieu and Willis, Taxel fails to provide any structuring to his 'sensitivity', while Anyon needs her analysis of textbooks to provide her with an image of the social:

An examination of school knowledge as a social product suggests a great deal about the society that produces and uses it. It reveals which groups have power, and demonstrates that the views of these groups are expressed and legitimized in the school curriculum. It can also identify social groups that are not empowered by the economic and social patterns in the society: they do not have their views, activities, and priorities represented in the school curriculum. The present analysis suggests that the United States working class is one such group, [...] the poor may be another. Omissions, stereotypes, and distortions that remain in 'up-dated' social studies textbook accounts of Native Americans, blacks, and women reflect the relative powerlessness of these groups.

(Anyon, 1983; p. 49)

My thesis adopts the position that, at least in its final form, a sociological analysis of texts should present a coherent theoretical account of those aspects of the social that are to be described and should generate a language which enables the movement between social structure and textual reading. The approach, here, should also be distanced from the apparent ontological commitments of ideological analysts, such as Anyon and Taxel; as Rob Gilbert describes 'structuralism':

... structuralism as a method does not require a transcendental generating force beyond the structure itself. No necessary ontology need be implied in the construction of a structuralist reading of text. To construct a model of the rhetorical elements and relations of a text need not be to reify that structure, or to deny that the model itself could be subjected to a similar analysis. The exercise can just as easily be seen as a translation of one discourse into another, the notion of structure being a means of representation rather than a reality beyond discourse.

(Gilbert, 1989; p. 67)

Gilbert refers to his own position as 'in some respect post-structuralist' (ibid; p. 71). I shall refer to my analysis as '(post)structuralist' to indicate that it is not non-structuralist, but that it has learned from the criticisms of much structuralist work as essentialist (see, for example, Laclau & Mouffe, 1985). Thus, in contrast to ideological textual analysis, I am not seeking to uncover bias, but to translate pedagogic texts into a sociological language. The bases and elements of this language are presented in Chapters 3 and 4.

A number of other 'ideological' analyses of textbooks were discovered in the literature search. None of these, however, proved to have any direct bearing on this thesis. These items, together with brief descriptions of their contents are included in

Appendix 1 to the thesis. Also included in Appendix 1 are similar listings of work focusing on the representation of gender and/or ethnicity. These studies concentrate on the detection of bias in textbooks. This thesis will address gender and class representations in terms of how such representations are implicated in the construction of the student. This is a very different approach from that adopted by all of the studies found which analyse textbooks in terms of gender and/or ethnicity representation.

2.3.3 Two studies

The specific interest of this thesis may be described as the development of a systematic language for the sociological description of pedagogic texts in terms of their regulation of 'who' can say or do 'what'. There is a very small amount of literature which, on the face of it, is related to this project. In particular, I shall discuss two pieces of work in greater detail.

In the literature search, some attention was paid to textbooks in the field of 'English for Specific Purposes'. A review of the articles published in the journal of that name reveals that most are concerned with the difficulties of students having a language other than English as their first language in working with textbooks in the medium of English (Chimombo, 1989; Love, 1991; Marshall & Gilmour, 1993). These articles are clearly directed towards the amelioration of these difficulties. In this respect, they are adopting an 'engineering' approach to textual production. In a slightly different vein, Kuo (1993) considers possible principles for the selection and development of materials in English for Science and Technology for use at university level in Taiwan. Tadros (1989) presents a model of discourse analysis which is used to analyse the instance of predictive categories in university law textbooks. Myers (1992), however, offers an analysis of university science textbooks which explicitly invokes sociology; he asserts that:

To understand what makes textbooks different from other academic texts, and perhaps what makes them problematic for our students, we need to see what they do in the social structure of academic disciplines, how writing and reading them reproduces knowledge and reproduces academics. We need to ask some sociological questions.

(Myers, 1992; p. 3)

Myers first considers the role of textbooks in the life histories of scientists. He rejects a position (which he attributes to Thomas Kuhn) in which 'early training in textbooks makes alternative views of phenomena unthinkable, so that there can be no rational comparison of competing paradigms' (ibid; p. 5). This view, he argues, ignores the importance of craft knowledge in the work of a scientist and also the

critical relationship to textbooks which is adopted by some scientists (exemplified by James Watson, one of the discoverers of the structure of DNA)¹.

Myers then considers a 'more useful' line of thinking in which textbooks are seen 'as the end of the development of a fact, not as the beginning of the development of a scientist' (ibid; p. 6). Here, Myers contrasts the textbook with the research journal article. Journals constitute an arena for conflicting views and claims. The textbooks, however, establishes a selection of these claims as 'facts' which are incorporated into a particular kind of order. Myers produces an analysis of two short sections of text which are related in terms of content, one from a university textbook on genetics and the other from a journal article on RNA processing. The texts are compared in terms of their respective uses of personal and impersonal subjects, verbal tense, modality, cohesion, references to other texts, and illustrations. The following is an extract from the comparison of modality:

Nonscientists may be surprised to find just how often the statements in the scientific article are modified.

Analogous intervening sequences *appear* to interrupt ... (2)
The possibility that this precursor represents a transcript of the beta-globin gene including intervening sequences is *attractive* because ... (6)

I have argued [...] that almost all new claims are hedged. In contrast, the textbook represents these claims as accredited facts that need no hedging:

The coding region of most eukaryotic structural genes is interrupted by non-coding sectors ... (1)

The textbook uses some hedges too (although fewer), but it usually reserves them for matters of representation about which there is not yet a clear consensus:

perhaps most appropriately called precursor mRNA ... (8)

Other hedges indicate remaining uncertainty:

they *appear* never to leave the nucleus ... (11)

A student who knows only the way textbooks use hedges for uncertainty is unprepared for the ways articles use them in polite statements of claims.

(ibid; p. 11)

Myers summarises his comparisons:

... the textbook generally makes it easier for the student reader than the article, but in doing so it makes it harder for those students who later encounter (or write) research articles.

(ibid; p. 12)

¹ Kuhn's (1970) argument also ignores any empirical reference to the readers of textbooks or to teachers.

Myers is, essentially, comparing texts which are produced within what I have described (in Section 2.2) as the field of production and the field of recontextualisation of academic science, that is, to cells A and B in Figure 2.1. Myers' intention is clearly to encourage the production of textbooks which will best facilitate career development between these two fields. However, he pays no regard to the specificities of the social relations obtaining within these two fields. Furthermore, he also elides the distinction between the field of textbook production (cell B) and the fields of textbook use in the lecture room, that is, the fields of reproduction and operationalisation of academic knowledge (cells C and D).

It is arguably possible to reconcile Myers' view of the textbook¹ as in some sense, 'the end of the development of a fact' with Bernstein's (1990) concept of recontextualisation². Unlike Bernstein, however, Myers makes no attempt to produce a sociological account of the 'development' of knowledge. Bernstein has contrasted the 'primary context' of the production of scientific knowledge with the 'secondary context' of its reproduction³. The relationship between 'discourse' within the primary context of production and discourse within the secondary context of reproduction is described by Bernstein as 'a complex transformation from an original [ie unmediated] to a virtual/imaginary discourse' (p. 185). This involves the delocation and relocation of scientific knowledge via the process of 'recontextualisation'. Essentially, the recontextualising of primary discourse is achieved by 'pedagogic discourse'. Pedagogic discourse is constituted by the recontextualising principles of the 'pedagogic device', which constitutes and sustains the relations of power and principles of control within a pedagogic relationship, effecting differential transmission and acquisition.

It would be appropriate to describe the texts that Myers analyses as differentially implicating certain resources in certain strategies which are constitutive of reader-author relationships which are appropriate to their respective contexts. The strong modality of the textbook sustains its authority. The weakening of modality within the journal article is no mere politeness. On the contrary, within the context of the production of knowledge, the validity of all claims is to be judged by the academic community which constitutes the 'peer' group readership of research articles. The modality of the text is weakened because the authority of the author is weakened in favour of that of the reader. The vector of evaluation is reversed.

¹ More correctly, the incorporation of the textbook into empirical classroom practices.

² Bernstein's concept and my own use of the term are discussed in Chapters 3 and 4.

³ Although Bernstein is referring, primarily, to the school as the site of reproduction.

The linguistic tools that Myers uses are of value in displaying part of the reservoir of resources for textual production and analysis. However, these indicators are abstracted from any sociological context.

Like Myers, Lynn Mulkey (1987) also describes her work as 'sociological'. She has produced a content analysis of 187 science textbooks used in middle and working class districts of New York at different age-grade levels. The analysis assesses the 'functional/dysfunctional role [of textbooks] in socialization for scientific careers' (p. 511). The general hypothesis to be tested was that:

... textbook content for middle-class districts and higher grades would be organized so that knowledge was more facilitative of the acquisition of the intellectual and emotional characteristics of scientists than for working-class districts and lower grades. The consequences are that if children in working-class districts are being socialized into nonparticipant roles they are less likely to contribute to scientific advancement than those children in middle-class districts who are being socialized into participant roles.

(Mulkey, 1987; pp. 512-3)

The concept 'textbook content' consisted of six dependent variables which Mulkey describes as follows:

... (1) orientation to cognitive flexibility (knowledge supportive of the ability to make connections between ideas in novel ways (2) orientation to abstract thinking (knowledge supportive of the ability to apply rational ideas to empirical ends to produce generalized and systematic schemes (3) orientation to communicative fluency (knowledge supportive of the ability to acquire the social and cognitive conventions of science (4) orientation to autonomy (knowledge facilitative of self-direction in new situations (5) orientation to goal achieving (knowledge supportive of the ability for personal mastery of an event), and (6) orientation to positive imagery (knowledge which influences the perceptions of students about scientists so that entry into the scientific professions results). The degree of 'participant organization' (grouping of content conducive to the acquisition and development of the cognitive and personality precursors of scientific roles) versus 'non-participant organization' (grouping of content that is not conducive to the acquisition and development of the cognitive and personality precursors of scientific roles) was measured on a variable nine-point scale.

(ibid; pp. 513-4; brackets as in original)

The results of the analysis did not produce statistically significant differences, in general, between textbooks used in middle class districts in comparison with those used in working class districts, although differences were found between the grade levels (higher grade textbooks provided higher degrees of participant organisation). The results also offered 'minimal support' for the hypothesis that the greatest degree of participant organisation would be provided by upper grade middle class textbooks¹.

¹ Although the differences were statistically significant only in respect of 'orientation to cognitive flexibility'.

Mulkey compared her findings to those of another study (by Harrington) which found differences between middle class and working class political science texts, but found less pervasive differences between grade levels. Mulkey postulates that the differences between the studies are produced by the differences between the disciplines with which they are concerned. Scientific ideas, she contends, 'retain a universality uncommon in other areas of human enterprise and are unprecedented except by mathematical ideas' (ibid; p. 519). Thus, science textbook writers might be expected to have a consistency of approach which is relatively impervious to the effects of social class. On the other hand, 'the complex cognitive skills and accompanying motivational factors that are required in order to do science' (ibid) would necessitate the grade-development in all aspects of orientation to participation in science books; presumably, these skills and factors are not understood to conceptualise political science.

Mulkey's explanation for her finding that the highest degree of orientation to participation is provided by upper grade middle class textbooks is predicated upon the assumption of a model of cultural deprivation on the part of textbook writers and selectors. It is suggested that these agents presume a greater degree of preparedness for orientation to cognitive flexibility on the part of older middle class students than would be the case for working class students of the same age.

In terms of specific research methodology, Mulkey's analysis gives a very high-level description. The unit of analysis is the individual textbook which is assigned a single value on each of the dependent variable indicator scales described above. Each textbook was also assigned a value on the two independent variable scales, that is, middle class/working class¹ and grade level. In the light of her use of such a coarse instrument of analysis, her failure to find very much may not be surprising, despite the fact that there is a sense in which she knew what she was looking for. My interest is in the production of a much closer form of textual reading and, to this extent, my project is closer to that of Myers. Myers attempts to conjure sociology out of a linguistic bricolage. My intention has been to generate a purpose-built language of description out of an engagement between a general sociological orientation and an empirical text. Unfortunately, the dynamics of this production are, of necessity,

¹ The exclusivity of the class value of the textbooks seems odd: 'Of the total 187 books reported used extensively and occasionally by middle-and working-class districts, the three middle-class districts accounted for 64% ($n = 119$) and the working-class districts only 36% ($n = 68$).' (Mulkey, 1987; p. 515).

elided to a certain extent in the presentation of the language and the analysis as artefacts.

The intention of Mulkey's analysis is to provide an instrument of evaluation to assist with the production and selection of 'functional' as opposed to 'dysfunctional' textbooks. This general methodological perspective of Mulkey's work is inconsistent with that to be adopted in the present thesis. A distancing is to be announced in relation to both the 'engineering' intention of Mulkey's analysis, which coincides with that of Myers, and its explicit functionalism. The position to be adopted in this thesis is that the textbook participates, tacitly, in the reproduction of a basically stratified society. The analysis of the textbook entails the description of the nature of this participation. That is, the analysis in this thesis addresses the question: 'how does the textbook select and construct apprenticeship and alienation and how does it construct the "to-be-apprenticed" and the "not-to-be-apprenticed"?'

Both Myers and Mulkey seem to treat that which ought to be transmitted by the textbook as basically unproblematic; this is the case despite Myers' affiliation to the 'sociology of scientific knowledge'. Mulkey's ideal textbook seems to have something of the quality of a superconductor, whilst Myers appears to want the textbook to mirror research papers. In both cases, the production of appropriate materials is essentially a technical problem. To assert that the transformation of social structure is a technical problem, however, is to deny the contingent specificity of the social which inscribes educational researchers and sociologists as fully as it does textbook writers and teachers. My position, to be elaborated in the thesis, is that the textbook constructs both its message and its receiver in a form which is consistent with the social conditions within which it exists.

2.4 Summary

In this Chapter, I have tried to mark out the empirical focus of the thesis through a discussion of work in the areas of mathematics education and textbook analysis. I have argued that the utilitarian and mathematical anthropological epistemologies mythologise mathematics as immanent in diverse practices. They achieve this by abstracting mathematical projections from any social context. The utilitarian myth pathologises the yet-to-be-schooled and, indeed, the inadequately schooled, as Jean Lave puts it:

The most common view distinguishes successful alumni from the unsuccessful, attributing constant and skilled use of school knowledge to the former, and rare, often erroneous, use to the latter.

(Lave, 1988; p. 5)

The utilitarian myth, therefore, constructs a role for school mathematics in the production of well-tutored users of mathematical tools. The mathematical anthropological myth announces that mathematical practices are always already there in human activities, even in advance of formal schooling. However, mathematical anthropology claims for mathematics education the ownership of the principles of recognition of mathematics.

In charting the empirical sociological space within which mathematical practices (including the mythical mathematics of utilitarianism and mathematical anthropology) might be considered to take place (Figure 2.1), I was able to locate my own empirical interest in the recontextualising field of the school site. The introduction of this map is of relevance because of its significance in the development of the work which has produced this thesis. However, following the work which led to the production of the map, my research took a theoretical and an empirical turn. The theoretical starting points are discussed in the next Chapter.

In reviewing the literature on the analysis of school textbooks, it has been found that there is very little which is of direct relevance to my project in this thesis. Most analyses of textbooks have no sociological interest. Some of the work which is, broadly, sociological in focus, does not produce empirical analysis of texts (eg Apple, Cooper). Other broadly sociological work has tended to concentrate on describing bias in terms of one or more macro-dimension of social inequality. Whilst these dimensions are most certainly of interest in this thesis, my project is not to uncover bias, but to produce a systematic description of the texts in terms of their construction of mathematical practices and of associated subject positions. None of the work which was discovered in the search generated a language of description in the terms to be discussed in Chapter 4. In Chapter 3, I shall move on to a consideration of the general methodological basis of the thesis and derive ten Theoretical Propositions from which the language of description will be derived in Chapter 4.



Chapter 3

General Methodology: the generation of Theoretical Propositions

This Chapter and the one that follows will derive and lay out the language of description which is the central achievement of this thesis. In subsequent Chapters, this language will be used to present an analysis of a set of school mathematics texts. The language of description has been generated, firstly, via a consideration of theoretical issues and, secondly, through an extensive and intensive engagement with the empirical texts. Thus, both deductive and inductive processes have been involved. In this exposition, however, the language will be presented in deductive form. The work of this Chapter, therefore, is to generate a set of Theoretical Propositions from which the language of description will be derived in Chapter 4.

The Theoretical Propositions will be generated out of a discussion of work which has been central in providing inspiration for this thesis as a whole. Clearly, there are problems associated with the presentation of such a discussion, both in terms of selection and ordering. The need to limit space has a bearing on each of these problem areas. The selection of work has been made on the basis of my perception of the importance of the work in the development of the thesis. In ordering the Chapter, it would have been possible to have organised the discussion around each of the Theoretical Propositions (which are given at the end of this Chapter). However, this would have resulted in a very much longer Chapter and a certain measure of redundancy (or an unmanageable amount of cross-referencing). I have, therefore, decided to organise the discussion around three interrelated themes.

The first theme concerns the distinction between the abstract and the concrete. This is an opposition which has received a great deal of attention across a range of disciplines in the social sciences during the twentieth century and it corresponds to a distinction which is crucial in this thesis. The elaboration of this theme will open with a brief consideration of the discursive/non-discursive distinction indexed by Michel Foucault. I will then move to a discussion of some of the ways in which the modality of the discursive has been treated in the literature. Out of this discussion, I will establish a scaling of practices which will constitute a fundamental dimension in my language of description.

The second theme concerns the production of subjectivity. In this discussion, and throughout this Chapter, I shall make use of the term 'ideology', which is a familiar expression. In the construction of my language of description, I shall be defining an analytic space which will enable me to describe the empirical as constituted by the division of labour in general. Within the empirical domain, this space is always 'ideologised'. That is, it is always inscribed within a particular division of labour. In referring to my analytic space, therefore, I shall use an alternative term ('activity'). This term will be defined in Chapter 4, but will be used in the formulation of the Theoretical Propositions at the end of this Chapter.

The third and final theme to be discussed in this Chapter is concerned with the contextualising and recontextualising of practices. Out of this discussion, I will establish a second fundamental dimension of the language of description. The Chapter will close with the Theoretical Propositions from which the language of description will be derived in Chapter 4.

3.1 Abstract & concrete practice

3.1.1 The discursive & the non-discursive

Michel Foucault is rather ambivalent about the distinction that I want to make, here, that is between the discursive and the non-discursive. In his later work, for example, he introduced the term 'apparatus' (*dispositif*) which he described in an interview in the following terms:

What I am trying to pick out with this term is, firstly, a thoroughly heterogeneous ensemble consisting of discourses, institutions, architectural forms, regulatory decisions, laws, administrative measures, scientific statements, philosophical, moral and philanthropic propositions—in short, the said as much as the unsaid. The apparatus itself is the system of relations that can be established between these elements.

(Foucault, 1980; p. 194)

Foucault's interviewers subsequently press him on the issue of the non-discursive:

J.-A. MILLER: With the introduction of 'apparatuses' you want to get beyond discourse. But these new ensembles which articulate together so many different elements remain nonetheless *signifying* ensembles. I can't quite see how you could be getting at a 'non-discursive' domain.

FOUCAULT: In trying to identify an apparatus, I look for the elements which participate in a rationality, a given form of co-ordination, except that

J.-A. MILLER: One shouldn't say rationality, or we would be back with the *episteme* again.

[...]

FOUCAULT: The term 'institution' is generally applied to every kind of more-or-less constrained, learned behaviour. Everything which functions in a society as a system of constraint and which isn't an utterance, in short, all the field of the non-discursive social, is an institution.

J.-A. MILLER: But clearly the institution is itself discursive.

FOUCAULT: Yes, if you like, but it doesn't much matter for my notion of the apparatus to be able to say that this is discursive and that isn't. If you take Gabriel's architectural plan for the Military School together with the actual construction of the School, how is one to say what is discursive and what institutional? That would only interest me if the building didn't conform with the plan. But I don't think it's very important to be able to make that distinction, given that my problem isn't a linguistic one.

(ibid, pp. 197-8)

Such nonchalance is, perhaps, to be expected in one who rarely makes explicit the principles of his descriptions. Descriptions which would thereby lay claim to a certain transparency of data, were it not for their stunning originality. Foucault clearly needs to index a discursive/non-discursive differentiation, because therein lies the inevitability of the 'failure' of 'programmes' which are realised in purely discursive terms. These programmes are associated with 'technologies' which extend beyond the discursive and, therefore, beyond its control. The result is the subjectless 'strategies' discussed by Foucault in the interview cited above¹. Foucault's originality lies precisely in his use of such terms. The breadth of these concepts, however, of necessity allows a great deal of scope for idiosyncratic interpretation. Foucault's originality resides in his organising strategies rather than in the precision of his histories. His ultimate refusal to establish a clear distinction between the discursive and the non-discursive is, possibly, a consequence of his recognition of a paradox: were he to provide such a distinction, then he would either have defined the limits of the discursive within discourse itself², or he would have rendered the non-discursive discursive.

In recognition of this paradox, the discursive/non-discursive distinction can only be a heuristic distinction and it is in this sense that it will be made in this thesis. Thus, we can assert that there always exists an excess of human practices over the strictly linguistic; an excess which corresponds to Heidegger's 'background'. This excess can never be fully realised in language, although its extent will vary between different aspects and instances of practice. The importance of this heuristic proposition is that it enables me to differentiate between different modes of practice within the discursive in terms of the extensiveness of the non-discursive excess. Thus, practices which minimise the non-discursive excess are, by definition, those which are most fully realisable within language. Such practices must tend to make explicit the principles of

¹ See Gordon's discussion in the afterword to the same volume (Gordon, 1980).

² An impossibility also recognised by Wittgenstein (1961).

their regulation in order to minimise reliance upon the unsayable. On the other hand, practices which exhibit a comparatively high degree of non-discursive excess are less capable of making explicit their regulating principles; they are, substantially, non-discursively regulated. These modes of practice will be described as exhibiting high and low *discursive saturation* respectively. In order to give more substance to this distinction and to acknowledge its intellectual pedigree, I shall now consider some other work which has made a corresponding differentiation of practice within the discursive level.

3.1.2 *The modality of the discursive*

There is, throughout the social sciences, a persistent dichotomising of modes of practice or of cognition which can be formulated at the level of discourse. An early example is to be found in the work of Lucien Lévy-Bruhl who 'was the first to point out the qualitative features of primitive [sic] thought and the first to treat logical processes as products of historical development' (Luria, 1976). Lévy-Bruhl was thereby making a distinction between the modes of practice in societies at different stages of development, 'primitive' and modern. Yu.M. Lotman distinguishes between 'grammar-oriented' and 'text-oriented' societies which are described by Umberto Eco as follows:

There are cultures governed by a *system* of rules and there are cultures governed by a *repertoire* of texts imposing models of behaviour. In the former category texts are generated by combinations of discrete units and are judged correct or incorrect according to their conformity to the combinatorial rules; in the latter category society directly generates texts, these constituting macro-units from which rules could eventually be inferred, but that first and foremost propose models to be followed and imitated.

(Eco, 1976; p. 137-8)

There is also a developmental relationship between these two forms, Eco continues:

Lotman suggests that text-oriented societies are at the same time expression-oriented ones, while grammar-oriented societies are content-oriented. The reason for such a definition becomes clear when one considers the fact that a culture which has evolved a highly differentiated content system has also provided expression-units corresponding to the content-units, and may therefore establish a so-called 'grammatical' system—this simply being a highly articulated code. On the contrary a culture which has not yet differentiated its content-units expresses (through macroscopic expressive groupings: the texts) a sort of *content-nebula*.

(Eco, 1976; p. 138)

This distinction between rule-governed practices and practices which draw upon a repertoire of exemplary texts, or perhaps a repertoire of techniques, resonates with my distinction between high and low discursive saturation. My differentiation,

however, is intended to distinguish modes of practice within a society rather than to classify societies in total. Furthermore, there are questions to be addressed relating to both the acquisition and the application of either rules or exemplars, that is, to the issues of pedagogy and context. These issues are discussed in other sections of this Chapter.

Alfred Sohn-Rethel (1973, 1975, 1978) introduces a differentiation which is intra-societal. He focuses his attention on the familiar distinction between intellectual and manual labour as a fundamental division in class societies:

Clearly the division between the labour of head and hand stretches in one form or another throughout the whole history of class society and economic exploitation. It is one of the phenomena of alienation on which exploitation feeds.

(Sohn-Rethel, 1978; p. 4)

For Sohn-Rethel, intellectual labour is necessarily social insofar as it is predicated upon 'necessary abstractions' from the form of social relationships. For example, the introduction of coinage inaugurates an abstraction whereby the object of the coin must stand for something other than itself. Sohn-Rethel's project is to demonstrate that 'a true identity exists between the formal elements of [this] social synthesis and the formal constituents of cognition' (ibid; p. 7). Sohn-Rethel argues that his materialist theory of cognition 'accounts for the historical emergence of the clear-cut division of intellectual and manual labour associated with commodity production' (ibid). His study of the genesis of this division is intended to reveal the preconditions for its disappearance in the advent of a socialist mode:

When we distinguished 'societies of production' and 'societies of appropriation' we made the point that on the basis of primitive communal modes of production, as they preceded commodity production, the social practice was rational but the theory was irrational (mythological and anthropomorphic), while on the basis of commodity production the relation was reversed; namely, the social practice has turned irrational (out of man's control) but his mode of thinking has assumed rational forms. What Marx has in his mind's eye [...] is man's historical potentiality of achieving a rational practice and a rational theory combined, which is simply another way of speaking of communism.

(ibid; pp. 133-4)

In this extract, however, we can see evidence of another distinction which resonates with that of Lévy-Bruhl, that is, between irrational, mythological and anthropomorphic 'theory' and rational 'theory', elsewhere referred to as 'science' (the 'logic' of which is 'mathematics'). The cognition of 'primitive' people is irrational as are the practices of manual labourers¹. Viewed from the perspective of

¹ The attribution of rationality (at least, in a global sense) to the social practice of 'primitive' people also comes into conflict with the Bourdieu's description of such practice as 'polythetic' (see the discussion below).

Sohn-Rethel's particular version of marxism, this may be the case. 'Rationality', however, has meaning only within discourse. 'Rational' intellectual labour and 'irrational' manual labour thus have some correspondence with high and low discursive saturation. But this correspondence is not an identity: the whole of social practice must, for Sohn-Rethel, finally become realisable in discourse in the ultimate rationality of communism. In this sense, Sohn-Rethel certainly denies the ontology of the non-discursive¹ and apparently prioritises the intellectual over the manual. This priority is, however, undermined by his historicising of rationality and by his apparent rejection of the rational in the last sentence of his book:

Above all it must be seen that it is not the recourse to the acclaimed neutrality of intellect and intellectual judgement but, on the contrary, the revolutionary commitment of our exposition that yields the truth.

(ibid; p. 204)

A third position is adopted in this thesis which neither lays claim to detached neutrality nor predicates its validity on faith. Rather, the intention is to elaborate an empirical description alongside the principles that make the description possible. Sohn-Rethel's developmental model is securely grounded in very high level theory. His engagement with the empirical, however, has a predigested quality which obscures its own genesis. Nevertheless, the association of the modality of practice with the division of labour is an important advance on the non-sociological classification of Lévy-Bruhl and is reflected in the model being developed here.

Lev Vygotsky (1978, 1986) and his colleagues, Alexander Luria (1976) and Aleksei Leont'ev (1978, 1979) also adopted a marxist methodology in their studies of cognition. For Vygotsky, ontogenesis is achieved as thought appropriates speech and progressively structures itself. However, speech is a social phenomenon the level of development of which is contingent on the level of development of society. The social thus facilitates and delimits cognitive development. A major distinction is made by Vygotsky between thinking in 'complexes' and thinking in 'concepts'². Vygotsky argues that the former has been found to characterise the thinking of 'primitive' people (by Lévy-Bruhl), of 'the insane' (by Storch) and of children (by Piaget). Each of these groups displays the trait of 'participation', whereby objects are classified

¹ Ontology is clearly important if one is attempting to change the world in a prescribed way.

² Another distinction made by Vygotsky is that between 'spontaneous' and 'non-spontaneous' (or 'scientific') concepts. Insofar as the former are acquired in use and the latter in formal instruction—ie via definitions—this distinction resonates with that made by Bourdieu between practical and formal logic (see below). In Vygotsky, however, the relationship between the two is dialogic and developmental, whereas, for Bourdieu, the objectivism of formal logic is inconsistent with practical logic.

together on the basis of 'bonds unacceptable to adult logic' (Vygotsky, 1986; p. 129). Thus, Vygotsky argues:

... the child, primitive man, and the insane, much as their thought processes may differ in other important respects, all manifest participation—a symptom of primitive complex thinking and of the function of words as family names.

(ibid; p. 129-30)

Whereas Vygotsky's own empirical work focused on the cognitive development of the 'normal', modern individual, Luria (op cit) carried out some work in peasant (ie comparatively 'primitive') societies. Most of Luria's subjects were unschooled and illiterate. Luria drew on the work of Vygotsky, Goldstein and others to distinguish between different kinds of cognitive action. Thus:

In abstract or categorial classification, the normal subject forms a distinct category by selecting objects corresponding to an abstract concept. This kind of classification yields instances of abstract categories such as *vessels, tools, animals, or plants* in an appropriate group, no matter whether the particular objects are ever encountered together.

(Luria, 1976; p. 48)

Such classification, Luria argues, exploits the higher capacities of language associated with literacy. By contrast:

Subjects who gravitate towards [concrete or situational thinking] do not sort objects into logical categories but incorporate them into graphic-functional situations drawn from life and reproduced from memory. These subjects group together objects such as a table, a tablecloth, a plate, a knife, a fork, bread, meat, and an apple, thereby reconstructing a 'meal' situation in which these objects have some use.

(ibid; p. 49)

In his empirical work, Luria found that the illiterate subjects did indeed tend to employ situational thinking in classification activities, whereas those who had received even a small amount of elementary schooling tended to classify objects taxonomically. Responses to questions involving syllogisms also separated his subjects. Thus the literate individuals generally recognised the logical connexion between the major and minor premises of a syllogism and drew the correct conclusion independently of any reference to their own practical knowledge. Luria's illiterate subjects most frequently referred back to their own experience or lack of it. For example, Luria presented a thirty-seven-year-old illiterate peasant with the following problem: 'Cotton can grow only where it is hot and dry. In England it is cold and damp. Can cotton grow there?' (ibid; p. 108). The subject first said that he didn't know, that he had never travelled outside of his region. When pressed, he drew on his practical knowledge of growing crops:

If the land is good, cotton will grow there, but if it is damp and poor, it won't grow. If it's like the Kashgar country, it will grow there too. If the soil is loose, it can grow there too, of course.

(ibid)

When asked, finally, 'what do my words suggest?', the peasant replied:

Well, we Moslems, we Kashgars, we're ignorant people; we've never been anywhere, so we don't know if it's hot or cold there.

(ibid)

Luria concludes that the subject is unable to make use of the higher facilities of language which enable a distancing from immediate experience. For Luria, syllogistic reasoning and categorial thinking are facilities which are natural in the sense that they are a function of the most highly advanced form of social structure. This is a materialist epistemology as is that of Sohn-Rethel. It is apparent, however, that both the Vygotsky/Luria schema and that of Sohn-Rethel have limited affinities with Marx. Neither seeks to specify a form of consciousness which is unique to the capitalist mode of production. Sohn-Rethel refers his necessary abstraction to the commodity form and to coinage. Luria's empirical differentiation is predominantly made on the basis of elementary schooling. Luria's work does, however, more closely approach the current project in that it moves to a focus on language and, therefore, towards the possibility of empirical description.

The distinction between the context-dependency of concrete thinking (in terms of 'graphic-functional situations') and the comparative context-independency of abstract thinking (things can be classified together even if they are never encountered together) is also crucial in the development of the expression, discursive saturation. Practices which exhibit low discursive saturation are, of necessity, context dependent, since they do not incorporate explicit regulatory principles. On the other hand, the availability of such principles in practices which exhibit high discursive saturation, renders them comparatively independent of any immediate context¹.

The work of Valerie Walkerdine has been highly influential, both in this project and in the wider field of mathematics education, to which this project relates. I shall discuss, at some length, an early paper (1982) in which she adopts a distinction between 'formal' and 'practical' reasoning which is similar to Luria's differentiation².

¹ My use of 'context', here, refers to the unrepeatability of an event or its sedimentation as a 'graphic-functional situation'.

² In *The Mastery of Reason*, Walkerdine distinguishes between 'instrumental discourse' and 'pedagogic discourse' in analysing 'mother-initiated exchanges' between mothers and young children: 'Instrumental referred to tasks in which the main focus and goal of the task was a practical accomplishment and in which numbers were an incidental feature of the task, for example in cake-

In this paper, Walkerdine turns her attention to a critique of developmental psychology as represented by Margaret Donaldson (1978)¹. Walkerdine argues that it is inappropriate for Donaldson to draw conclusions about children's abilities to operate with logical relations on the basis of tasks which 'call up' familiar practices involving games with teddy bears etc. She argues that:

In practical reasoning we determine the truth or validity of a statement in terms of its correspondence to the rules of a practice, whereas in formal reasoning truth is determined in terms of the internal relations of the statement itself.

(Walkerdine, 1982; p. 138)

Formal reasoning, is to be understood as 'an act performed upon language; it is a peculiar one which is not in any sense of the term "natural", and we do not have to seek explanations in terms of the structures of the child's mind' (ibid; p. 140). From Walkerdine's theoretical perspective (although less obviously from Piaget's²) the challenge on Donaldson is valid. However, a similar criticism could not be made of Luria and Vygotsky. This is because, for them, cognitive development is essentially tied up with language development which constitutes rather than expresses it. Luria's illiterate subjects are cognitively limited because they have had no access to the higher capacities of language. For Vygotsky, the development associated with such access is necessarily spread out in time, so that a developmental sequence is inevitable. Walkerdine does not aim her criticism at Luria or at Vygotsky. However, it is only by backgrounding the possibility of a materialist analysis, such as theirs, that she is able to claim that formal reasoning is 'not *in any sense of the term* "natural"'.

Nevertheless, all three would agree that the acquisition of formal reasoning necessarily entails instruction. For Walkerdine, the instruction involves the disembedding of mathematical tasks which are originally embedded within other more familiar 'discourses' via a stripping away of the 'metaphoric' associations of the non-mathematical context and a maintenance of the 'metonymic' structure of the

making, in which the number *two* might feature in relation to the number of eggs needed and so on. In the pedagogic tasks numbers featured in quite a different way: that is, numbers were the explicit focus of the task' (Walkerdine, 1988; p. 81). This distinction is clearly related to the formal/practical opposition. It also has some resonance with Leont'ev's hierarchy, in activity theory, between goal-directed 'actions' and means-oriented 'operations' (1978, 1979; see also Zinchenko, 1979). There is also a resonance with Heath's (1986) study of school and home language. In relation to the mode of participation of numbers in the discourses and tasks described by Walkerdine and by Zinchenko, this work also relates to the strategy/resource hierarchy that I shall introduce in the next Chapter.

¹ Paul Light (1986) offers a critique of Donaldson and of some of his own work which clearly draws on this paper by Walkerdine.

² It can, however, be argued that Donaldson and her colleagues committed a certain amount of violence to Piaget's empirical work by transforming what were originally clinical interviews into tests.

mathematical statements. This is a useful image in relation to pedagogy and informs my own conception of this process. However, I should also note, now, that I intend to move from a conception of metaphor and metonym in Jakobson's sense as constituting orthogonal axes to a simplification of Eco's (1976, 1979) construction in which metaphors are facilitated by subjacent chains of metonyms. I shall describe an action of pedagogy as the construction of metonymic chains between the non-mathematical and the mathematical, between the student and the teacher. Equally, metonymic chains may be constructed within mathematics. Metaphors are to be understood as 'shortcircuiting' metonymic chains.

Walkerdine uses the term 'discourse' in a way which is different from that of her source for the term, Michel Foucault (1972, 1977b)¹. Walkerdine distinguishes between the discursive and the material, the former constituting a form of realisation of the latter; 'discourse' can also refer to everyday speech, thus:

Children do not have raw experiences of concrete objects: meaning is created at the intersection of the material and the discursive, the fusing of the signified and signifier to produce a sign. These meanings are located in, and understood in terms of, actual social practices, represented in speech as discourse. It is by analysing the form and content of discourse, the processes of selection and combination, of metaphor and metonymy, that we can account for the origins and processes of reasoning. Young children are able to shift in and out of discourses from an extremely young age and I have examined some of the ways in which they adopt different discursive positions.

(Walkerdine, 1982; p. 153)

Foucault is not concerned with everyday utterances which, in any event, would generally be unavailable in the context of historical studies. Rather, he is concerned with the conditions of existence of what Dreyfus and Rabinow (1982) have translated as 'serious speech acts' (archaeology) or in diachronic discontinuities in practices (genealogy). Foucault's objects of study are always very large scale affairs: 'a history of insanity in the age of reason' (1965); the human sciences (1970); clinical practices (1973); 'the birth of the prison' (1977b); sexuality (1978, 1984, 1986). These all seem a long way from playing with teddy bears (which might constitute a 'discourse' in Walkerdine's study).

The distinction is important because the small scale 'discourses' that Walkerdine is interested in are not regulated or regulating in the same way as are the discursive formations and apparatuses which are of concern to Foucault². This is precisely because Walkerdine's discourses are never archived other, that is, than in

¹ Which is not to say that Foucault is entirely consistent in his use of the term and of associated terms.

² Although the distinction is, perhaps, less striking in respect of Foucault's work on sexuality.

recontextualised forms in research such as her own. Rather, these local practices are negotiated within the context of their immediate elaboration. This is not to say, of course, that such practices are anarchic. Practices such as shopping, for example, are always elaborated within the context of a physical and linguistic matrix. This is also the case in respect of children playing at shopping. However, discourse associated with shopping is unlikely to be appropriately described as regulated to the same extent as, for example, discourse associated with school mathematics¹, the latter being institutionalised in a way that the former is not². It is precisely this sort of distinction that I want to make in this thesis.

Thus, 'formal reasoning' corresponds to high discursive saturation and 'practical reasoning' to low discursive saturation. The distinction rests on the extensiveness of the excess of the non-discursive over the discursive, that is, on the extent to which the regulation of the practice lies within or outside the linguistic. In my terms, playing with teddy bears is not a discourse to the extent that its principles are always context dependent and so non-explicit. School mathematics, on the other hand, is more discursive, because its principles are comparatively explicit and context independent. However, this does not render the former 'natural' and the latter unnatural.

Walkerdine, in this paper, also differs from Foucault in her conception of the non-discursive. As I noted above, Walkerdine is conceptualising discourse as a form of realisation of material actions. However, she appears to refer to the words that are uttered (or diagrams that are drawn) as 'signifiers' for physical actions which are the corresponding 'signifieds'. She also refers to this relationship as metaphorical. Thus in an interaction between a teacher and a young child, the teacher:

... puts the blocks in two piles on the table, and as she says 'put them all together' she moves them together with her hands. This her first relation of signified (moving the blocks) and signifier (saying the words). The task is practised a second time and then the teacher makes an interesting move in discourse:

T: Good boy, let's count them altogether. One-two-three-four-five-six-seven. So Nicola had four, Debbie had three, so three and four make ... (*she puts the blocks together*)

Ch: Seven.

She repeats the same exercise of putting the blocks together, but this time the phrase that she uses refers to the blocks *implicitly* but makes no reference in language to them, so that she introduces the children to the 'disembedded' form of the statement: 'three and four make ...'

(Walkerdine, 1982; p. 146)

¹ See Lave (1988; Lave et al, 1984) and Fiske (1989) which very differently describe negotiation in shopping practices.

² Elsewhere, Walkerdine herself describes some of the regularities of school mathematics, in relation to developmental psychology (1984; Corran & Walkerdine, 1981) and in relation to gender (1989; Walden & Walkerdine, 1982, 1985).

Thus 'discourse' appears, here, to be being represented as comprising only signifiers and physical actions are signifieds¹. Clearly, there is a need of some modification of Saussure (1983)² since Walkerdine is focusing her attention upon speech (*parole*) rather than on language (*langue*), which was Saussure's object of study. Nevertheless, such a sundering of the sign seems too radical. Volosinov, for example, who also prioritises *parole* over *langue*, argues that the notion that 'the expressible is something that can somehow take shape and exist apart from expression; that it exists first in one form and then switches to another form' (Volosinov, 1973; p. 84) is untenable, that the dualism is invalid, 'after all, there is no such thing as experience outside of embodiment in signs' (ibid, p. 85)³.

Walkerdine needs to distinguish between the discursive and the non-discursive because she wants to make use of more delicate linguistic tools in her analysis (metaphor, metonymy, signifier, signified) and to locate formal reasoning as 'an act performed upon language'. However, this has still not resulted in the production of a language of description which is adequate to the task to be undertaken in this thesis. One move might be to refer to all utterances (spoken and written) to interpretants⁴, so that all texts are to be interpreted in relation to interpretive bases which themselves are to be described. This move indexes the discussion of context in Section 3.3.

Despite her separation of the material from the discursive, Walkerdine nevertheless describes practical reasoning as rule-governed as is illustrated by the first extract from her paper cited above. This raises the question of whether the rules are in the minds of the participants or whether they are *post hoc* constructions (or potential constructions) in Walkerdine's own (potential) analysis. This question is addressed in Pierre Bourdieu's critique of the objectivism of structuralism (one of several recurring themes in his work):

The generative formula which enables one to reproduce the essential features of the practices treated as an *opus operatum* is not the generative principle of the practices, the *modus operandi*. If the opposite were the case, and if practices had as their principle the

¹ Walkerdine is not entirely consistent in this. Elsewhere in the article, she states that 'formal reasoning draws its validity from, and depends entirely upon, reflection on the metonymic axis—on the relations between signs and not on their metaphoric content' (p. 141). Nevertheless, there remains a suggestion that the discursive is exclusively governed by syntagmatic principles and that the semantic is alienated. The signified is thus present, but in an impoverished form.

² Walkerdine cites the 1974 edition.

³ This last statement would be true if we were to assume that the sign was stable. My heuristic distinction between the discursive and the non-discursive, however, asserts an inevitable degree of context-dependency which denies the final closure of any signifying system: there is always an excess of material contingency.

⁴ In Peirce's sense (1931-8). See, also, Eco (1973, 1976, 1984, 1990).

generative principle which has to be constructed in order to account for them, that is, a set of independent and coherent axioms, then the practices produced everything according to perfectly conscious generative rules would be stripped of everything that defines them distinctively as practices, that is, the uncertainty and 'fuzziness' resulting from the fact that they have as their principle not a set of conscious, constant rules, but practical schemes, opaque to their possessors, varying according to the logic of the situation, the almost invariably partial viewpoint which it imposes etc. Thus, the producers of practical logic are rarely entirely coherent and rarely entirely incoherent.

(Bourdieu, 1990; p. 12)

More simply (and less precisely) put:

The science of myth is entitled to describe the syntax of myth, but only so long as it is not forgotten that, when it ceases to be seen as a convenient translation, this language destroys the truth that it makes accessible. One can say that gymnastics is geometry so long as this is not taken to mean that the gymnast is a geometer.

(ibid; p. 93)

For Bourdieu (1977)¹, structuralist accounts of, for example, gift exchange remove the necessary temporal dimension within which such practices are embedded. Pure reciprocity can only ever be available if the exchange is instantaneous. That this is never the case introduces a degree of uncertainty regarding the circumstances under which reciprocal action will take place, or even if it will take place at all. Practices of this kind are more properly understood as following a 'practical logic'. This has the quality of being 'polythetic', that is, it cannot be reduced to a consistent structure because it relates to involvement in practices which are fundamentally corporal and oral and local and thereby incommensurable: they are context-dependent. The individual's apprenticeship into such polythetic practices, the *habitus*², constitutes the embodying of the structures of the world, 'that is, the appropriating by the world of a body thus enabled to appropriate the world' (Bourdieu, 1977; p. 89).

¹ See also Robbins (1991).

² *Habitus* serves a similar purpose for Bourdieu as does the concept of 'structuration' for Giddens (1984; Cohen, 1987), by allowing for both dispositions and creativity. There are also clear resonances between this term and Wittgenstein's (1958) notion of a 'language game', Gadamer's (1976) structure of 'prejudices' and Dasein in Heidegger (1962). It is curious that Bourdieu nowhere appears to cite the previous use of this unusual term by Marcel Mauss, whose paper was first published in the *Journal de Psychologie Normale et Pathologique* 35^e année, 1935, pp. 271-93. Bourdieu does refer to a publication in which the paper is included, but in fact acknowledges Mauss in a very restricted way and nowhere in relation to *habitus*. Mauss states: '... I have had this notion of the social nature of the *'habitus'* for many years. Please note that I use the Latin word—it should be understood in France—*habitus*. The word translates infinitely better than *'habitude'* (habit or custom), the *'exis'*, the 'acquired ability' and 'faculty' of Aristotle (who was a psychologist). It does not designate those metaphysical *habitudes*, that mysterious 'memory', the subjects of volumes or short and famous theses. These 'habits' do not vary just with individuals and their imitations; they vary especially between societies, educations, proprieties and fashions, prestiges. In them we should see the techniques and work of collective and individual practical reason rather than, in the ordinary way, merely the soul and its repetitive faculties' (Mauss, 1979; p. 101). This extract is from a section of Mauss's book entitled 'Body Techniques'. *Habitus* is here very much associated with physical dispositions—gait, posture, etc. It thus suggests a differentiation between discursive and non-discursive regulation of dispositions. This is a distinction which is, perhaps, lost in Bourdieu's later use of Mauss's expression which becomes less of a concept and more of an arena for Bourdieu's perpetual reinterpretations.

The regulation of 'polythetic practices' is more non-discursive than discursive and, in that sense, it is appropriate to refer to them as exhibiting low discursive saturation. The acquisition of such practices is, according to Bourdieu, mimetic¹:

So long as the work of education is not clearly institutionalized as a specific autonomous practice, and it is a whole group and a whole symbolically structured environment, without specialized agents or specific moments, which exerts an anonymous, pervasive pedagogic action, then the essential part of the *modus operandi* which defines practical mastery is transmitted in practice, in its practical state, without attaining the level of discourse. The child imitates not 'models' but [sic] other people's actions.

(Bourdieu, 1977; p. 87)

Opposed to practical logic, however, is not one, but three 'modes of theoretical knowledge' (ibid; p. 3) which 'may be described in a dialectical advance towards adequate knowledge' (ibid). He describes the first two of these modes in the following terms:

The knowledge we shall call *phenomenological* [...] sets out to make explicit the truth of primary experience of the social world, ie all that is inscribed in the relationship of *familiarity* with the familiar environment, the unquestioning apprehension of the social world which, by definition, does not reflect on itself and excludes the question of the conditions of its own possibility. The knowledge we shall term *objectivist* [...] constructs the objective relations (eg economic or linguistic) which structure practice and representations of practice, ie, in particular, primary knowledge, practical and tacit, of the familiar world. This construction presupposes a break with primary knowledge, whose tacitly assumed presuppositions give the social world its self-evident, natural character [...]. It is only on condition that it poses the question which the *doxic* experience of the social world excludes by definition—the question of the (particular) conditions making that experience possible—that objectivist knowledge can establish both the structures of the social world and the objective truth of primary experience as experience denied *explicit* knowledge of those structures.

(Bourdieu, 1977; p. 3)

'Phenomenological' knowledge cannot reflect on the conditions of its own possibility. 'Objectivism' does reflect on the conditions of primary experience in setting out 'to establish objective regularities [...] independent of individual consciousnesses and wills [and so] introducing a radical discontinuity between theoretical and practical knowledge' (1990; p. 26). The distinction between polythetic, practical logic, on the one hand, and objectivist thinking, on the other, is made in terms of the modality of the action of power. Polythetic practices are constituted within the context of individualised and localised power operating through the play of personal strategies exercised in primary contact. Objectivist thinking, on

¹ Although the notion of 'imitation' seems, perversely, to exclude the social and to exclude creative potential. A similar point was made, in Chapter 2, regarding Gerdes (1986) description of non-industrialised production processes.

the other hand, is predicated upon the establishment of institutions which minimise personal power in favour of a more generalised power action. However, objectivism cannot interrogate these conditions of its own possibility. Bourdieu, therefore, proposes 'a second break, which is needed in order to grasp the limits of objectivist knowledge' (ibid) to constitute an 'adequate science of practices' (ibid):

Just as objectivist knowledge poses the question of the conditions of the possibility of primary experience, thereby revealing that this experience (or the phenomenological analysis of it) is fundamentally defined as *not* posing this question, so the theory of practice puts objectivist knowledge back on its feet by posing the question of the (theoretical and also social) conditions which make such knowledge possible. Because it produces its science of the social world against the implicit presuppositions of practical knowledge of the social world, objectivist knowledge is diverted from construction of the theory of practical knowledge of the social world, of which it at least produces the lack.

(Bourdieu, 1977; p. 4)

We might conclude that Bourdieu is hovering dangerously between the abyss of the infinite regress and the annihilatory self-deconstruction of the postmodern. Derek Robbins describes Bourdieu's position in rather more concrete terms:

Bourdieu's main problem during the 1980s has been to sustain his symbolic power whilst simultaneously undermining the scientificity on which it was originally founded. Some would say that he has tied the noose around his own neck and kicked away the stool from beneath his feet.

(Robbins, 1991; p. 150)

However, this ignores the plausibility of a division of labour within sociology between the construction and deconstruction of ('objectivist') interpretive frameworks. Such a division clearly penetrates most concrete sociologists dialogically and might be described as the partial trammelling of what Feyerabend (1975) has referred to as 'epistemological anarchy'. Bourdieu's work, perhaps better than most, exemplifies the productivity of such dialogue. However, his insistent critique of 'objectivism' denies him the possibility of constructing a language of sufficient precision and stability for the purposes of textual analysis. Basil Bernstein (1971b, 1977, 1990; see also Atkinson, 1985) gets much closer to what is required here in his work on speech codes:

The simpler the social division of labour, and the more specific and local the relation between an agent and its material base, the more direct the relation between meanings and a specific material base, and the greater the probability of a restricted coding orientation. The more complex the social division of labour, and the less specific and local the relation between an agent and its material base, the more indirect the relation between meanings and a specific material base, and the greater the probability of an elaborated coding orientation.

(Bernstein, 1990; p. 20)

The distinction draws on Durkheim's distinction between mechanical and organic solidarity (1984) which exhibit relatively simple and relatively complex division of labour respectively. The empirical realisation of coding orientations often bears a strong resemblance to the responses of Luria's subjects. Thus, when asked to classify different types of food, lower working class children had a tendency to reconstruct meals, whilst middle class children favoured taxonomic classification. Bernstein, however, achieves the provision of a material base for the different modes of response in terms of the Durkheimian model. Furthermore, Bernstein does not characterise society as a whole as exhibiting mechanical or organic solidarity. Rather, he uses these concepts to describe different forms of the division of labour which coexist within societies. Individuals within such a hybrid configuration are likely to be located predominantly within one form of the division of labour, but may be expected to move routinely between locations. The relationship between the individual and speech codes, therefore, is one of orientation and not confinement. Thus the fundamental distinction between restricted and elaborated code is not so much developmental (as is Durkheim's own work and the 'marxist' account of Luria) as locational; orientation is, essentially, distributed by social class.

An orientation towards restricted code means an orientation towards context-dependent meanings¹. That is, the meaning of an utterance if given only in the enactment of the practice within which it occurs. The meaning would be radically altered (and almost certainly ambiguous) if, for example, the utterance were to be recorded and played-back under different material circumstances. Restricted codes are highly context-dependent, highly localised. Elaborated codes, by contrast, generate utterances which are more explicit at the level of language, so that they are less context-dependent and more generalised.

Because of their relationship to different forms of the division of labour, Bernstein's speech codes are of considerable heuristic value in the present sociological account. However, as I shall discuss in the next Section, my emphasis is on the analysis of (pedagogic) text and upon subjectivity as constructed by texts. It would, therefore, be inappropriate to graft these concepts onto my own model (which would inevitably transform them). My own conceptual framework will be specified in the following Chapter.

A series of oppositions has been discussed in this Section. Each opposition marks out, in a particular way, a distinction between the concrete and the abstract within the

¹ In relation to contexts, which are defined in terms of 'classification' and 'framing'; see Section 3.3.

level of the discursive. The specificity of 'primitive' thought as distinct from that of 'modern' individuals, described by Lévy-Bruhl. Lotman distinguishes text- from grammar-oriented societies. Sohn-Rethel marks out manual from intellectual labour. Vygotsky and Luria distinguish between thinking in complexes and thinking in concepts via the notion of participation. Walkerdine describes practical and formal reasoning. Bourdieu differentiates practical logic from three modes of theoretical knowledge. Bernstein constructs restricted and elaborated codes. Corresponding oppositions are also made, for example, by Lévi-Strauss (1972), in the distinction between *bricolage* and science and by Freud (1973a, 1973b) in that between the id and the ego. Each of these oppositions clearly displays a uniqueness which derives from the theoretical framework within which it is embedded. Each construction is also more or less adequately fitted to its empirical or (in the case of Sohn-Rethel) political purpose. Nevertheless, each opposition resonates with the modality of the discursive which is being defined in this thesis for its own specific purposes. Each of these oppositions, in other words, can be mapped onto the scaling of discursive saturation as low or high (this mapping is summarised in Table 3.1). Practices exhibiting high discursive saturation are associated with a degree of context independency or generalisation; practices exhibiting low discursive saturation are associated with comparative context dependency or localisation.

The Dual Modality of Practice

Author	Abstract Context-independent Generalisation DS+	Concrete Context-dependent Localisation DS-
Bernstein Bourdieu Foucault Freud Lévi-Strauss Lévy-Bruhl Lotman Luria Sohn-Rethel Vygotsky Walkerdine	elaborated code formal/theoretical logic programmes ego science modern thinking rule-governed practice abstract thinking intellectual conceptual thinking formal reasoning	restricted code practical logic technologies id bricolage primitive thinking repertoire of exemplary texts situational thinking manual complex thinking practical reasoning

Table 3.1

Mathematics is clearly a case of high discursive saturation, a practice which is highly organised at the level of discourse and so produces generalised utterances. The development of such practices is, as Bernstein suggests, indicative of a complex

division of labour. Domestic and manual practices are examples of low discursive saturation, because they are not generally highly organised at the level of discourse and so they produce localised utterances¹. These practices exhibit a simple division of labour. Of necessity, no practice can be fully realised within discourse. If there really were nothing but discourse, it would not be possible for the pre-linguistic child ever to enter the domain of the linguistic. Even higher mathematics is dependent upon what I might (temporarily) refer to as a mathematical component of *habitus*, as Livingston (1986) has illustrated. This *habitus* consists of the 'yet-to-be-discursive'. However, Livingston's discursive indexing of the tacit assumptions in Gödel's inconsistency theorem² still cannot exhaust the practice; there is always an excess of the material over the discursive. The distinction is one of relative saturation of a material practice by discourse.

3.2 *The production of subjectivity*

In the previous Section, my discussion focused attention on the modality of practices rather than on the subjects of those practices³. I now need to consider the nature of subjectivity and its production via pedagogic action. I want to adopt a concept of subjectivity in which the subject is inscribed within, or constructed by, social practices and relations. This is broadly consistent with the notion of the subject in the work of Foucault (see, also, Henriques et al, 1984; Laclau, 1984; Laclau & Mouffe, 1985). As I announced at the beginning of this Chapter, I shall, for the time being, make use of the term 'ideology' to identify that which is constitutive of subjectivity. In developing the concept of ideology and the production of subjectivity via pedagogic action, I shall have need of heuristic models which I shall draw from Althusser and Eco in the first three Sub-sections of this Section. In the fourth Sub-section, I shall introduce the concept of the textual subject which is necessary for the form of textual analysis to be adopted in the thesis.

¹ Other examples of comparatively low saturation in a discursive practice are to be found in spectator sports. In cricket, for example, even apparently technical terms such as 'wicket' and even 'bat' and 'ball' have no unambiguous meaning.

² In his introduction, Livingston indexes tacit assumptions in the more widely familiar proof concerning the relationship between the angles subtended at the centre and at the circumference of a circle. See, also, Knee (1983) who points to similar tacit assumptions in Euclid's *Elements*.

³ Although subjectivity is clearly not ignored by the authors whose work was discussed.

3.2.1 Althusser & ideology in general

Ideology, for Althusser, is material:

... an ideology always exists in an apparatus and its practice, or practices. This existence is material.

(Althusser, 1971; p. 156)

The material is ultimately rooted in physical matter, in human terms, the body and its physical environment. This does not entail a prioritising of the physical over the intellectual, rather, an insistence that ideology, cast in these terms, exceeds the discursive. This is clearly consistent with the position adopted in Section 3.1. For Althusser, ideology constitutes individuals as subjects via the 'process' of interpellation. Althusser introduces a metaphorical scenario in which 'interpellation' is described as a hailing in the street; this metaphor is worthy of some consideration:

Assuming that the theoretical scene I have imagined [the interpellation of concrete individuals by ideology] takes place in the street, the hailed individual will then turn round. By this mere one-hundred-and-eighty-degree physical conversion, he [sic] becomes a *subject*. Why? Because he has recognised that the hail was 'really' addressed to him, and that 'it was *really him* who was hailed' (and not someone else).

(ibid; p. 163)

This interpellation of the individual as subject entails their subjection to the Subject, the metaphor for which is God, the transcendental signified. Becoming a subject of necessity entails subjection, as Foucault puts it:

There are two meanings of the word *subject*: subject to someone else by control and dependence, and tied to his [sic] own identity by a conscience or self-knowledge. Both meanings suggest a form of power which subjugates and makes subject to.

(Foucault, 1982; p. 212)

For Althusser, who is describing ideology *in general* and not specific ideologies, '*individuals are always-already subjects*' (op cit; p. 164). Thus, the metaphor, 'interpellation', entails a heuristic introduction of an imaginary temporal dimension. Subjects in general, however, are not always-already subjects in particular. Specifically, a pedagogic relationship must (at least as an ideal type¹) comprise different subjects/subjectivities. Initial recognition of the pedagogising subjectivity by the to-be-pedagogised subjectivity must be of otherness: they confront one another. At the completion of pedagogy, they must stand together. There is still a one-hundred-and-eighty-degree turn by the pedagogised. However, this is now of necessity extended in real time and, furthermore, Althusser's directions are reversed: the subject becomes rather than worships her/his God.

¹ I use the term 'ideal type' in the Weberian sense (Gerth & Mills, 1948).

The 'interpellation' of ideology in general, then, is to be replaced by 'pedagogic action' which effects a 'symbolic violence' in the imposition of a cultural arbitrary (Bourdieu & Passeron, 1977) which might be thought of as a specific ideology. This is not the original establishing of the subject, but the apprenticing of an always-already subject into a new subjectivity. It is through this process that, for example, a school student becomes a mathematician. The question now to be addressed is, what is the nature of this subjectivity, this specific ideology?

3.2.2 Ideology in particular

For purely heuristic purposes, I want to postulate (drawing inspiration from Eco, 1976, 1979) the notion of a *Global Semantic Universe* (GSU). The universe should be thought of as comprising all possible cultural units as proto-signs. Conceived of in this way, the universe is the imaginary space in which semiosis proceeds. The GSU, itself, is devoid of semantic expression or content, which are only to be understood in relational terms, that is, within the context of semiotic action. Within this space, an 's-code' (Eco, 1976) is (always-already) established as the articulation of units as a relational totality. Where the units are signifiers (Saussure, 1983¹) the s-code constitutes a plane of expression; where they are signifieds, it defines a plane of content. A code is a specific correlation of an expression plane with a content plane, that is, a correlation of two s-codes (Eco, 1976)². It is this 'code' which, in my interpretation, corresponds to ideology in particular. Eco describes *langue* as follows:

A code as '*langue*' must therefore be understood as a sum of notions (some concerning the combinatorial rules of the expression items, or syntactic markers; some concerning the combinatorial rules of the content items, or semantic markers) which can be viewed as the *competence* of the speaker. However, in reality this competence is the sum of the individual competences that constitute the code as a collective convention. What was called 'the code' is thus better viewed as a *complex network of subcodes* which goes far beyond such categories as 'grammar', however comprehensive they may be. One might therefore call it a *hypercode* [...] which gathers together various subcodes, some of which are strong and stable, while others are weak and transient, such as a lot of peripheral connotative couplings. In the same way the codes themselves gather together various systems, some strong and stable (like the phonological one, which lasts unchanged for centuries), others weak and transient (such as a lot of semantic fields and axes).

(Eco, 1976; pp. 125-6)

¹ This translation of Saussure inexplicably replaces the conventional terms 'signifier' and 'signified' with 'signal' and 'signification', respectively. I intend to retain the more familiar terms.

² I am not making an ontological distinction between expression and content planes, although expression and content can always be distinguished in context.

Eco's lack of sensitivity to sociological structuring is apparent in his use of the term 'sum' to conceptualise *langue* as the aggregate of individual competences and in his description of the 'hypercode' as that which 'gathers together' various subcodes. The potential for describing relationships between individuals and groups would seem to be very limited within this schema. This does not pose a problem for Eco, whose principal interest lies in the exploration of the workings of what are essentially discursive systems, Thus:

... it is not up to semiotics to establish whether [factual judgements] are true or false, but it is up to semiotics to establish whether or not they are socially acceptable. Many factual judgements seem unacceptable, not because they are false, but rather because to accept them would mean to impose a restructuration of the Global Semantic System or large parts of it. This explains why, under particular historical conditions, physical proof of the truth of certain judgements could not stand up before the social necessity of rejecting these same judgements. Galileo was condemned not for logical reasons (in terms of True or False) but for semiotic reasons—inasmuch as the falsity of his factual judgements is proved by recourse to contrary semiotic judgements of the type 'this does not correspond to what is said in the Bible'

(Eco, 1979; pp. 84-5)

Eco has no language adequate to the description of the social in sociological terms. Nevertheless, via a reinterpretation of *langue* as an ideology in particular, a workable structure begins to emerge. That is, an ideology (in particular) is to be understood as a specific articulation of cultural elements. Cultural elements are here understood to extend beyond the discursive to include what has been referred to as *habitus*. The ideology is also understood to comprise subordinate structures at a level corresponding to that of Eco's 'subcodes', but not necessarily described in the same way. Ideology as '*langue*' must constitute 'speaking' subjects and their 'utterances'. Conceived of in this way, an ideology is instantiated as individual subjectivities and as texts¹. To concretise: school mathematics (say) as an ideology consists of the totality of its texts and the subjectivities of school mathematics teachers and students *qua* school mathematics teachers and students.

The questions now to be addressed concern the production of text and subjectivity by or in ideology. Before I can proceed to an answer, however, I must take a brief diversion, in order to consider the forms of articulation of the elements which comprise ideology. I shall describe these articulations in terms of the tropes 'metaphor' and 'metonymy'.

¹ Because an ideology is to be defined (in the next Section) in sociological terms, the postulation of a collective subjectivity as the totality of its individual subjectivities and texts does not pose the difficulties for sociological description that obtain in the case of Eco's 'hypercode'.

3.2.3 Metaphor & metonymy

I noted in Section 3.1 that Walkerdine¹ was using the relations ‘metaphor’ and ‘metonym’ in the sense given by Jakobson, thus:

The development of a discourse may take place along two different semantic lines: one topic may lead to another either through their similarity or through their contiguity. The metaphoric way would be the most appropriate term for the first case and the metonymic way for the second, since they find their most condensed expression in metaphor and metonymy respectively.

(Jakobson, 1956; p. 76)

Jakobson associates metonymy with Freud’s mechanisms of defence, including ‘displacement’ and ‘condensation’², and metaphor with the Freudian processes, ‘identification’ and ‘symbolism’. With reference to literature, Jakobson describes ‘romanticism’ as being closely linked with metaphor and ‘realism’ with metonymy. Metaphor and metonymy are also seen as the lines of ‘least resistance’ for poetry and prose, respectively. This orthogonal organising of the two tropes also recalls Saussure’s ‘syntagmatic’ and ‘associative’ relations, otherwise, the syntagmatic and paradigmatic axes of language. Useful as this interpretation has undoubtedly been, it has a tendency to prioritise the grammatical, that is, the linearity of written or spoken language, in its description of metonymy as being concerned with relations of contiguity.

This interpretation also has a superficial appeal with reference to school mathematics (which was also the context of Walkerdine’s discussion³). Insofar as mathematical ‘utterances’ are regarded as strings of mathematical symbols, ideally exemplified in a mathematical equation or proof, mathematics would seem to be characterised as ‘metonymic’. On the other hand, school mathematics often involves reference to non-mathematical objects and relations. These seem to stand in ‘metaphorical’ relationship to mathematical objects and relations. However, such a construction is problematic in two respects. Firstly, it seems to demand that there are practices which ‘legitimately’ occur in school mathematics but which are, in fact, not mathematics (this indexes the discussion in the next Section of this Chapter). Secondly, it ignores mathematics as a relational totality, capable of producing strings

¹ See also the discussions in Henriques et al (1984) on this interpretation of these tropes. See also Atkinson (1990) on the use of metaphor and metonymy in ethnographic writing.

² The latter being described as synecdoche, interpretable as a species of metonymy.

³ See, also, Corran & Walkerdine (1981). ‘Metaphor’ has been widely discussed with reference to mathematics education; see, for example: Brown (1981); Janvier (1987); Liebeck (1986); Otte (1983, 1986); Pimm (1986, 1987, 1990); Rotman (1985); Tahta (1985).

as utterances, but not confined to such instances. Ideology is not, ultimately, reducible to the syntagmatic.

Alternatively, we might consider what constitutes the condition for a metaphorical relationship. To take a specific instance that I have used elsewhere (Dowling, 1993a): how can I interpret the statement, 'her face was an aspirin'? This is undoubtedly a metaphor. However, it can only work as a metaphor because 'her face' and 'aspirin' have shared associations: whiteness or pallor; roundness; sickliness; hardness, perhaps. These associations stand in metonymic relationship with 'her face' and with 'aspirin'. The metaphor achieves a selection of possible qualities or associations of the two terms, that is, it is conditional upon metonymy. It seems inappropriate to describe dependent tropes in a perpendicular relationship (which is a geometric condition for independence). Eco describes an alternative arrangement:

A metaphor can be invented because language, in its process of unlimited semiosis, constitutes a multidimensional network of metonymies, each of which is explained by a cultural convention rather than by an original resemblance. The imagination would be incapable of inventing (or recognizing) a metaphor if culture, under the form of a possible structure of the Global Semantic System, did not provide it with the subjacent network of arbitrarily stipulated contiguities. The imagination is nothing other than a ratiocination that traverses the paths of the semantic labyrinth in a hurry and, in its haste, loses the sense of their rigid structure. The 'creative' imagination can perform such dangerous exercises only because there exist 'Swedish stall-bars' which support it and which suggest movements to it, thanks to their grill of parallel and perpendicular bars. The Swedish stall-bars are Language { *langue* }. On them plays Speech { *parole* }, performing the competence.

(Eco, 1979; p. 78)

Eco also describes metonymy as a relationship of contiguity. However, this is not limited to syntagmatic contiguity, but can include semantic or even phonetic proximity. The relationship between metaphor and metonymy in this conception is one in which the former constitutes a 'short-circuiting' of the latter: metaphors are always 'provable' by metonymic substitution. As Eco notes, relations of metonymy are cultural conventions. In the schema which is being developed in this thesis, these relations are regulated, which is to say 'rarified' (see Foucault, 1972) within particular ideologies. In this sense, then, ideology is the condition of possibility of metaphor.

The articulation of the signifying elements which comprise an ideology can thus be described as metonymic or as metaphorical. I want to move on, now, to consider the production of ideological realisations in texts and subjectivities.

3.2.4 *Text, the reader & pedagogy*

Hodge & Kress describe the relation between text and system as 'dialectical':

Terms in a system have value by virtue of their place in that system. At the same time, a system is constantly being reproduced and reconstituted in texts. Otherwise it would cease to exist. So texts are both the material realization of systems of signs, and also the site where change continually takes place.

(Hodge & Kress, 1988; p. 6)

Ideology is realised in texts which are themselves the sites of both the reproduction and the transformation of ideology: texts produce and reproduce ideology. In order to sustain the dynamic intention of this definition, I shall make use of the expression '(re)production' in referring to this relationship. I should make two observations before proceeding. Firstly, I must re-emphasise that ideology and therefore text is understood to exceed the discursive. Texts, therefore, are not to be understood purely in linguistic terms. Secondly, in order to foreground the transformative potential of the text, I could, in the present study, have chosen to focus my attention upon empirically contested readings. Hodge & Kress (1988), for example, provide a description of a cigarette advertisement which has been paid some attention by an oppositional campaign. The wording of the advertisement, for example, has been transformed by politically intentioned graffitists from 'New. Mild. And Marlboro.' to 'New. Vile. And a bore.' Within the context of school mathematics texts, I could clearly have looked at readings produced by different groups of students, teachers, etc. I have chosen not to do this, but to produce an analysis of a text from a single perspective. This is because the principal task that I have set myself in this thesis is the production of a language for the sociological description of pedagogic texts which are, in this case, mathematical. As the principles of the description are sociological, the reading of the text is made from a position which is outside of the context of the production of the text as a work (see Barthes, 1981a). Thus, the language of description itself constitutes the text as a site of contestation: the analysis is of necessity critical.

Texts in general and pedagogic texts in particular must, of necessity, presuppose a reader, in Eco's terms:

To organize a text, its author has to rely upon a series of codes that assign given contents to the expressions he [sic] uses. To make his text communicative, the author has to assume that the ensemble of codes he relies upon is the same as that shared by his possible reader. The author has thus to foresee a model of the possible reader (hereafter Model Reader) supposedly able to deal interpretatively with the expressions in the same way as the author deals generatively with them.

(Eco, 1979; p. 7)

The analysis of pedagogic texts will thus entail the inference of the model reader. The language of description which is to be developed must provide the terms in which categories of model reader are to be described. Again, this is not to say that the model reader resides within the work, but within the critical reading of the work. Similarly, the text (or, rather, its reading) constructs its author:

In a communicative process there are a sender, a message, and an addressee. Frequently, both sender and addressee are grammatically manifested by the message: 'I tell you that ...'

Dealing with messages with a specific indexical purpose, the addressee is supposed to use the grammatical clues as referential indices (/I/ must designate the empirical subject of that precise instance of utterance, and so on). The same can happen even with very long texts, such as a letter or a private diary, read to get information about the writer.

But as far as a text is focused *qua* text, and especially in cases of texts conceived for a general audience (such as novels, political speeches, scientific instructions, and so on), the sender and the addressee are present in the text, not as mentioned poles of the utterance, but as 'actantial roles' of the sentence (not as *sujet de l'énonciation*, but as *sujet de l'énoncé*) ...

(ibid; p. 10)

Eco describes both 'author' and 'model reader' as 'textual strategies'. It is in this sense that author and reader are to be interpreted in this thesis. Author and reader are the products of the principled analysis of the text. They are thus to be distinguished from the empirical author and reader which are not to be the focus of attention (in line with the decision outlined above). However, it can be noted in passing that, even if interviews with authors and classroom observations etc were to be carried out, the 'empirical' author and reader emerging from the subsequent analysis would be no less fictive in their dependence upon the principles through which they were to be described.

In (re)producing ideology, a text thus constructs textual subjects which are associated with that ideology. It is these textual subjects which are the concern of this thesis and which are to be produced within the textual analysis which is its empirical work. A consideration of the relationship between textual subjects and empirical subjects (actual human beings) involves two issues, one theoretical and one methodological. The theoretical issue concerns the introduction of an interactional dimension. This dimension is suppressed within the model and analysis as presented in this thesis, because the focus is on texts. However, an indication of where this dimension may be introduced is given in the penultimate Section of Chapter 4. The methodological issue relates to questions of validity and reliability of the analysis, which turn on the status of the text in relation to the particular ideology under consideration. This issue will be addressed in Chapter 5. The nature of the

association between the textual subjects and the ideology is contingent upon the nature of the texts which, in this thesis, are pedagogic.

From the perspective of this thesis, a pedagogic text is an utterance within the context of a pedagogic relationship which implicates a pedagogue and a to-be-pedagogised. These are, respectively, the subject and object of pedagogic action. Pedagogue and to-be-pedagogised are thus to be interpreted as, respectively, author and reader of the pedagogic text. In the terms used in this Chapter, pedagogic action entails (ideally) the apprenticing of the to-be-pedagogised into an ideology. The author/pedagogue is the agent of the ideology which is (re)produced by the text¹. The reader/to-be-pedagogised of necessity connotes an otherness with respect to this ideology, yet is the addressee of pedagogic action. Author and reader as subjectivities thus stand in metaphoric relationship to one-another. Pedagogic action can therefore be described as the provision of metonymic links between these subjectivities. It is not supposed that ideology can be acquired all in one go. Apprenticeship therefore entails the provision, extended in time, of metonymic links between pedagogue and to-be-pedagogised. The adept has completed the one-hundred-and-eighty-degree turn via which the apprentice is transcribed from the floor of the classroom to the teacher's desk.

3.3 Context & recontextualisation

What remains to be done is to discuss the multiplicity or the contextualising of ideology. In the preceding discussion I have referred to ideology in the particular rather than in the general sense. In the general sense, the subject is always-already a subject of ideology. This applies to the 'to-be-pedagogised' as much as to the 'pedagogue'. The 'to-be-pedagogised' must, therefore, already be a subject of a particular ideology which is other than that of the pedagogue. The implication is clear, that is, that there exist multiple ideologies. Ideologies are, as discussed above, (re)produced by subjectivities which must, in empirical terms, be instantiated in human individuals. Since human individuals must, of necessity, traverse the ideological space, such individuals must be constituted as multiple subjectivities (see Henriques et al, 1984; Laclau, 1984; Laclau & Mouffe, 1985). This is a necessary feature of my general methodology and is clearly germane to the understanding of pedagogic action and apprenticeship as outlined above. However, since I am concerned with textual subjects, I shall not, at this stage, develop my theory of

¹ This is not to deny contradiction within ideology.

empirical individuals beyond this basic assertion of multiplicity. I shall, however, return to this issue in the Chapter 4.

The notion of the multiplicity of ideologies resonates with Walkerdine's use of 'discourse'. Bakhtin (1981) describes language as 'heteroglossic'. This stands in critical relationship to Saussure's notion of the object of linguistic analysis which Volosinov¹ describes as 'an objective system of incontestable, normatively identical forms' (Volosinov, 1973; p. 67). For Bakhtin/Volosinov the utterance is the proper object of study. However, unlike Saussure's *parole*, the utterance is not an act of individualised will, but reflects the material conditions of its production:

All the diverse areas of human activity involve the use of language. Quite understandably, the nature and forms of this use are just as diverse as the areas of human activity. This, of course, in no way disaffirms the national unity of language [...] Language is realised in the form of individual concrete utterances (oral and written) by participants in the various areas of human activity. These utterances reflect the specific conditions and goals of each such area not only through their content (thematic) and linguistic style, that is, the selection of the lexical, phraseological, and grammatical resources of the language, but above all through their compositional structure. All three of these aspects—thematic content, style, and compositional structure—are inseparably linked to the *whole* of the utterance and are equally determined by the specific nature of the particular sphere of communication. Each separate utterance is individual, of course, but each sphere in which language is used develops its own *relatively stable types* of these utterances. These we may call *speech genres*.

(Bakhtin, 1986; p. 60)

Corresponding to Eco's 'model reader', 'speech genre' exhibits a model addressee:

Thus, addressivity, the quality of turning to someone, is a constitutive feature of the utterance; without it the utterance does not and cannot exist. The various typical forms this addressivity assumes and the various concepts of the addressee are constitutive, definitive features of various speech genres.

(ibid; p. 99)

The materialist critique of Saussure is regarded here as entirely appropriate and the importance of the addressee/reader has already been acknowledged. However, Bakhtin's overriding emphasis on the utterance is as limiting for a sociological study as is Saussure's exclusion of *parole*. For Saussure there is no specificity to the empirical text. For Bakhtin, there is no possibility describing regularity, that is, of specifying context.

¹ 'Volosinov' may or may not have been a 'flag of convenience' for Bakhtin (see the discussion in Hirschkop, 1989). For the purposes of this thesis, however, the works cited constitute a plausibly unitary position.

Foucault (1981) speaks of a contextualised domain of discourse. This domain is regulated by three main kinds of procedures. Firstly, 'procedures of exclusion' include: the prohibition of certain kinds of speech; the opposition between reason and madness; and the opposition between truth and falsehood. Secondly, internal procedures of 'rarefaction' include principles such as: the commentary; authorship; and the discipline. Finally:

There is, I believe, a third group of procedures which permit the control of discourses. This time it is not a matter of mastering their powers or averting the unpredictability of their appearance, but of determining the condition of their application, of imposing a certain number of rules on the individuals who hold them, and thus of not permitting everyone to have access to them. There is a rarefaction, this time, of the speaking subjects; none shall enter the order of discourse if he [sic] does not satisfy certain requirements or if he is not, from the outset, qualified to do so. To be more precise: not all the regions of discourse are equally open and penetrable; some of them are largely forbidden (they are differentiated and differentiating), while others seem to be almost open to the winds and put at the disposal of every speaking subject, without prior restrictions.

(Foucault, 1981; pp. 61-2)

Foucault's gaze is, as usual, cast very broadly. His organising principles are clearly of great value in speaking of the discursive field in general and enable him to mark out specific 'positivities' (1972). It is not at all clear, however, how he might make sense of a single text. Nor is it always clear how he selects the texts upon which he bases his 'histories' (although he claims exhaustive coverage of extant contemporary texts relating to sexuality in ancient Greece (1984)). Nevertheless, his work begins to suggest a way forward for the current project.

Foucault's procedures for the control of discourse clearly operate, firstly, on what can be said and secondly upon who can say it. They also distinguish between relatively public and relatively private regions of discourse. Furthermore, the relatively private discourses are never entirely closed-off: 'they are differentiated and differentiating'. One of the principal mechanisms of this differentiation is formal education:

Although education may well be, by right, the instrument thanks to which any individual in a society like ours can have access to any kind of discourse whatever, this does not prevent it from following, as is well known, in its distribution, in what it allows and what it prevents, the lines marked out by social distances, oppositions and struggles. Any system of education is a political way of maintaining or modifying the appropriation of discourses, along with the knowledges and powers which they carry.

(Foucault, 1981; p. 64)

The project of this thesis is more modest than the analysis of an education system, far less the field of discourse in general. However, Foucault's broad brush strokes at this high level of analysis sketch the outline of two heuristic questions. Firstly, how

do we distinguish between public and private ideologies? Secondly, how can a pedagogic text be described as distributing the content of a specific ideology so as to constitute included and excluded subjectivities? The following extract from Bourdieu addresses the first of these questions and also introduces a third:

Knowledge does not only depend, as an elementary relativism suggests, on the particular viewpoint that a 'situated and dated' observer takes up vis-à-vis the object. A much more fundamental alteration—and a much more pernicious one, because, being constitutive of the operation of knowing, it inevitably remains unnoticed—is performed on practice by the sheer fact of taking up a 'viewpoint' on it and so constituting it as an object (of observation and analysis). And it goes without saying that this sovereign viewpoint is most easily adopted in elevated positions in the social space, where the social world presents itself as a spectacle seen from afar and from above, as a representation.

(Bourdieu, 1990; p. 27)

The additional question concerns the relationship between ideologies: how does one 'view' another? The 'elevated positions' to which Bourdieu refers clearly include those that are elevated by the educational system. Those positions that are characterised, in our society, by formal, objectivist thinking, rather than the polythetic thinking of practical logic. In addressing these questions, I want to refer, again, to Bernstein's work, principally to his concept of 'classification'. The first question, introduced above, refers to the specialising of discourse, Bernstein argues that:

If categories of either agents or discourse are specialized, then each category necessarily has its own specific identity and its own specific boundaries. The speciality of each boundary is created, maintained and reproduced only if the relations between the categories of which a given category is a member are preserved. What is to be preserved? The *insulation between the categories*. It is the strength of the insulation that creates a space in which a category can become specific.

(Bernstein, 1990; p. 23)

The strength of the insulation is referred to as 'classification'¹. Different strengths of classification relate to different principles of the social division of labour because insulation 'presupposes *relations of power* for its creation, reproduction, and legitimation' (ibid; p. 24). As is made clear in the above extract, classification can be used to measure the degree of specialisation of any social category². My interest here concerns education:

¹ See also Bernstein 1971a, 1971b, 1977.

² In an earlier paper I made use of this notion to examine transformation in Cuban society since the Revolution in respect of educational categories and of the categories of gender and race. I concluded that: '... with the possible (and uncertain) exception of race, many of the principles of classification that obtained prior to 1959 remain intact in Revolutionary Cuba, the basis of this conclusion is the maintenance of many pre-Revolutionary boundaries between social categories. This is not to deny the enormous improvements that have been made in the lives of the majority of the population [for example, the abolition of] the conditions that give rise to a culture of poverty' (Dowling, 1985; pp. 9-10).

We can regard the social division of labour of a school to be composed of categories (transmitters and acquirers) and categories of discourse ('voices'). If the coding principle is one of strong classification, then there is strong insulation between educational discourse ('voice') and non-educational discourse ('voices'). Discourses are strongly insulated from one another, each with its own specialized 'voice' so that transmitters and acquirers become specialized categories with specialized 'voices'. Within the category of transmitter there are various 'sub-voices', and within the category of acquirer there are various 'sub-voices': age, gender, 'ability', ethnicity. In the process of acquiring the demarcation markers of categories (agents/discourse) the acquirer is constituted as a specialized category with variable sub-sets of voices depending upon age, gender, 'ability', ethnicity.

(Bernstein, 1990; p. 26)

I have three qualifications to make to Bernstein's conception in moving towards establishing my own use of it. The first is a product of my working within a general framework which is heavily influenced by structural linguistics and it is this. Bernstein refers to classification as a measure of the strength of boundaries or insulation between categories (of agents/discourse). If we refer to school subjects—mathematics, English, history, physics, etc—as the relevant categories, then it is clear that the term 'boundary' has some significance. For example, different subjects are often taught in different physical spaces which are insulated from each other. It is also the case that we can, metaphorically, consider mathematics, for example, as historically having established a space for itself which is distinct from, say, physics. However, this 'space' is here being conceptualised as a particular articulation of a notional Global Semantic Universe. This being the case, the metaphors 'boundary' and 'insulation' are not appropriate (although, of course, physical boundaries may be incorporated, semiotically, into the establishing of the space).

I have described the articulations between signifying elements within an ideology as metonymical and metaphorical. It is possible to describe articulations between ideologies in the same way. That is, an element in one system can be considered to be metonymically, or connotatively, linked with elements in other systems. Where the availability of such connotative links between the elements of two ideologies is high, we can describe the strength of classification between them as low. Where the availability of connotative links is low, or, alternatively, where such connotations are predominantly ones of 'otherness', the strength of classification is high. Concretely, the following mathematical expression minimises non-mathematical connotations:

$$2(P) \equiv (\exists x)(\exists y)[P(x).P(y).x \neq y.(z)(P(z) \supset. z = x \vee z = y)]$$

Clearly, some of the symbols connote signifiers in standard English. However, even these connotations are minimised by the immediate context. This 'text', then, indicates strong classification of mathematics with respect to other ideologies.

The above example leads to the second qualification that I need to make regarding my use of Bernstein's concept of classification. Thus, whilst it is the case that the above mathematical expression minimally connotes the non-mathematical, it is possible to present it in an alternative form which weakens the classification, thus:

... there are two Ps if and only if there are x, y such that x is P and y is P and x is not the same thing as y and for all z , if z is P then z is the same thing as x or z is the same thing as y ...

The denotative content has remained unchanged¹. Now, however, the mode of expression is less specialised, so that there is a slight increase in the availability of non-mathematical connotations: 'thing', for example, is not specific to mathematics. The classification has been weakened. Furthermore, if I note that both expressions constitute a definition of the number 2, classification is weakened still further.

In other words, the strength of classification of mathematics is not a fixed quality of mathematics, but varies, depending upon the particular mathematical content under consideration, or upon the manner in which it might be expressed. Ideologies, generally, are thus presumed to be regionalised in terms of their strength of classification with respect to each other.

The third qualification relates to Bernstein's use of 'sub-voices' which, in the case of the 'acquirer', are exemplified as: age, gender, 'ability', ethnicity. My point here is that these categories are not sociologically equivalent (and Bernstein clearly recognises this in his use of quotation marks around 'ability'). I want to maintain (and shall produce evidence to this effect) that, in the context of school mathematics, age and ability are explicit variables relating to competence and performance. Gender and ethnicity and, indeed, social class, are not. For example, school mathematics generally produces different curricula for students of different ages and abilities; it generally does not (at least, not in the UK at the present time) produce curricula which explicitly index students of different genders and different ethnic groups. Indeed, many contemporary texts often go to great lengths (not always successfully) to be 'politically correct' in their treatment of categories which differ in these respects. The structuring of curricula in gender and ethnic terms is commonly implicit rather than explicit. I shall also argue that social class is an implicit variable.

¹ The 'translation' is provided by Benacerraf & Putnam (1983, p.15).

Gender, ethnicity and social class are, in other words, certainly interpreted by the ideology under examination. Arguably, these fundamental categories are interpreted by all ideologies. In this respect, however, they should be understood as constituting resources in the elaboration of the ideology and not as organising principles. This is a consequence of my construction of an analytic space. There is no doubt that the division of labour is 'ideologised' along class, gender and ethnic dimensions. These are not, however, being construed as the 'motors' of the social, but as describing its contingent, material state. This being the case, they are certain to be implicated within all 'ideologies in particular'. The subjectivities constructed by ideologies are, therefore, implicitly rather than explicitly defined in terms of these major dimensions of social structure. This will be the case other than in very particular cases (for example, gender is likely to be an explicit principle in domestic ideology).

Classification refers to relations between categories. Bernstein introduces the concept 'framing' to refer to the form taken by control within categories. Bernstein defines framing as follows:

Framing refers to the principle regulating the communicative practices of the social relations within the reproduction of discursive resources, that is, between transmitters and acquirers. Where framing is strong, the transmitter explicitly regulates the distinguishing features of the interactional and locational principles, which constitute the communicative context. Where framing is weak, the acquirer has a greater degree of regulation over the distinguishing features of the interactional and locational principles that constitute the communicative context. This may be more apparent than real.

(Bernstein, 1990; p. 36)

In the present study, the 'transmitter' and 'acquirer' are both textual categories. It is certainly germane to discuss the nature of the relationship between these categories as constructed by the text. However, the acquirer as acquirer must always be objectified by the pedagogic text and thereby denied control in terms of pedagogic action. Consider, for example, a text which instructs the student to 'discuss' a mathematical pattern with a colleague. There is an apparent handing over of control to the student insofar as there is no direction over the nature or content of the ensuing discussion. Clearly, an empirical student might choose to discuss last night's football match or might discuss the aesthetic rather than the mathematical qualities of the pattern. The textual student, however, cannot make such choices. 'Discussion' must be included as a task by virtue of some presumed pedagogic (even therapeutic) value. The student is objectified as the site of this activity. The location of control with the textual acquirer is always 'more apparent than real'. Thus, the concept, 'framing', whilst clearly of analytic power in respect of classrooms, is backgrounded in my model which, as it has been thus far developed, excludes the interactional.

I have introduced concepts relating to the first of the three questions raised earlier. The private and the public in ideology can be distinguished in terms of the theoretical/practical logic described by Bourdieu. This distinction essentially differentiates between ideal types of ideologies. That is, between those exhibiting high and low discursive saturation respectively. This distinction is also relevant in differentiating within ideologies, as I shall argue in the next Chapter. The private and the public can also be distinguished in terms of the strength of classification of forms of expression and content. Again, this measure is capable of differentiating within and between ideologies. Its principal use in this thesis, however, will be in describing structure within ideology.

The second question concerns the distribution of ideology in the construction of included and excluded categories of subject. I have argued that the production of subjectivity is the achievement of pedagogic action. We can define the transmitter in the pedagogic relationship as already subjectified by the relevant ideology, that is, as being in possession of its principles (both in formal and practical terms). I have also referred to apprenticeship as the provision of metonymic links which enable the apprentice to 'rotate' between positions of subjectivity, or to participate within a proto-subjectivity. Metonymic chains may, for example, provide links between the everyday and the mathematical, or they may be confined within regions of mathematics which are either strongly or weakly classified with respect to the non-mathematical. Alternatively, links may be made metaphorically rather than metonymically. These different modes of chain will facilitate or deny access to the regulating principles of the ideology, which access is the condition for subjectivity. In either case, the pedagogic action will be subject to the regulating principles of the ideology. The latter will either be 'visible' or 'invisible'. Bernstein (1977, 1990) has introduced the term 'visible pedagogy' to refer to a form of pedagogic action which emphasises the evaluation of performance in terms of explicit principles. 'Invisible pedagogy' refers to pedagogic action which focuses on the elaboration and development of competence under circumstances in which the principles are implicit. Thus, from my perspective, we can define *visible pedagogic action* as generating metonymic chains which enable access to the principles of the ideology. *Invisible pedagogic action* can then be defined as not generating such chains.

Finally, in addressing the third question (how does one ideology view another) I want to draw again on the work of Bernstein's (1990). The 'recontextualising rules' of Bernstein's 'pedagogic device' constitute 'pedagogic discourse' which is:

... a principle which removes (delocates) a discourse from its substantive practice and context, and relocates that discourse according to its own principle of selective reordering and focusing. In this process of the delocation and the relocation of the original discourse the social basis of its practice, including its power relations, is removed. In the process of the de- and relocation the original discourse is subject to a transformation which transforms it from an actual practice to a virtual or imaginary practice. Pedagogic discourse creates imaginary subjects.

(Bernstein, 1990; p. 184)

Bernstein's construct is incorporated into a tightly defined theoretical framework. Thus, as was the case in respect of his work on speech codes, it is not possible to implant his concept into my theory. However, Bernstein's description of the action of pedagogic discourse has informed my own conception of the way in which one ideology 'views' another. Retaining the visual metaphor, I shall use the term *gaze* to refer to a mechanism which delocates and relocates, that is, which *recontextualises* ideological expression and content. The result of such recontextualising is to subordinate the recontextualised ideology to the regulating principles of the recontextualising ideology. In other words, the recontextualised ideology is constituted as an imaginary ideology and its subjects as imaginary subjects.

I have described a specific ideology as a particular articulation of a notional Global Semantic Universe. With respect to any given ideology, there will be regions for which the forms of expression and/or content must be comparatively weakly classified with respect to other ideologies; this was discussed above. Nevertheless, the realisation of these forms of expression and contents in texts must, to a greater or lesser extent, conform to the general principles of the ideology. To concretise: there are regions of school mathematics which involve forms of expression and content (signifiers and signifieds) relating to domestic ideology—say shopping. We can say that the *gaze* of school mathematics *recontextualises* shopping practices. In doing so, 'shopping' will be constituted as an imaginary set of practices. The principles governing texts which incorporate recontextualisations will first and foremost be those relating to school mathematics. The domestic principles regulating shopping practices must be subordinated, to a greater or lesser extent, to those of school mathematics. This is a derived generalisation. Nevertheless, it describes the 'mythologising' action of both utilitarianism and mathematical anthropology as these were represented in Chapter 2. In both of these epistemologies, the social bases of non-mathematical practices were elided by mathematical generalisations, generating imaginary or 'mythological' practices. The recontextualising action of the *gaze* of school mathematics will also be illustrated, empirically, in the analytic Chapters of this thesis.

This now concludes my discussion of the theoretical work which has been most influential in generating the Theoretical Propositions upon which my language of description is based. In the final Section of this Chapter, I shall present the Propositions.

3.4 Summary: Theoretical Propositions

The Theoretical Propositions given below constitute a summary of the key points in the preceding discussion. I have used the term 'ideology' throughout this Chapter, because of its familiarity. As I signalled at the start of the Chapter, however, I intend to use an alternative term in my own language. This is because my language establishes an analytic space and not an empirical one. Since the Theoretical Propositions are to be referred to in the next Chapter, I shall introduce the alternative term at this point: the term is 'activity'; its formal definition and some of its connotations will be discussed in Chapter 4.

Theoretical Proposition 1 (TP1)

Activity, in general, is the product of any division of labour. Activity, in particular, is the contextualising basis of all social practice. All activities are material. Activities vary internally and one-from-another according to the extent of the saturation of material practice by discourse, that is, their *discursive saturation* may be high (DS⁺) or low (DS⁻). Discursive saturation can never be total.

Theoretical Proposition 2 (TP2)

All activities are cultural arbitraries. They are to be understood as particular articulations of a notional Global Semantic Universe comprising forms of expression and contents (signifiers and signifieds). That is, an activity is a relational totality which exhibits principles, discursively and/or non-discursively (depending upon the level of discursive saturation). In discursive terms: activities exhibiting DS⁺ can give rise to relatively generalisable utterances; DS⁻ activities can generate only localised utterances.

Theoretical Proposition 3 (TP3)

Activities vary internally (and one-from-another) with respect to the strength of classification of their forms of expression and content.

Theoretical Proposition 4 (TP4)

A weakening of classification within a region of activity implies that forms of expression and/or content have been *recontextualised* from another activity via the *gaze* of the recontextualising activity. The principles of the recontextualising activity always effect a transformation upon the forms of expression and contents which are recontextualised.

Theoretical Proposition 5 (TP5)

Articulations between signifying elements of an activity may be metonymic or metaphoric. The distinction concerns the visibility or explicitness of the denotations and connotations which establish the articulation. Metonymic chains render visible, or explicit, metaphorical relations.

Theoretical Proposition 6 (TP6)

Activities are instantiated in individual subjectivities and in texts. Subjectivities and texts thereby (re)produce activities.

Theoretical Proposition 7 (TP7)

Human subjectivity is of necessity multiple.

Theoretical Proposition 8 (TP8)

Subjectivity in relation to specific activities are achieved by pedagogic action, which establishes metonymic links between the pedagogue and the to-be-

pedagogised. Pedagogic action may be visible or invisible depending upon the accessibility to the to-be-pedagogised of the regulating principles of the activity. Visible pedagogic action renders the principles available via metonymic rather than (or at least as well as) metaphoric links. This may be achieved ostensively or verbally, depending upon the degree of discursive saturation.

Theoretical Proposition 9 (TP9)

Successful apprenticeship to an activity is achieved (metaphorically) upon the completion of a one-hundred-and-eighty-degree rotation of the apprentice who thereby ‘moves’ from ‘outside’ to ‘inside’ the activity and becomes its Subject.

Theoretical Proposition 10 (TP10)

A text is an actional or discursive utterance in relation to a specific activity and constructs an author and a reader. A pedagogic text is an utterance within the context of a pedagogic relationship. A pedagogic text thus constructs one or more textual pedagogues or transmitters and one or more textual to-be-pedagogised or acquirers.

In the next Chapter, I shall introduce the main terms and structure of my language of description. The language will be further elaborated and extended in Chapters 5-10, which contain the analysis of the empirical texts.

Chapter 4

Introduction to a Language of Description

4.1 *The idea of a language of description*

The idea of a language of description is explicitly proposed by Bernstein in his 'Introduction' to *Class, Codes and Control, volume 5* (in press). However, the notion is implicit (at least) in all of his work and much of that with which he has been directly associated since the 1960s. Essentially, a language of description is concerned with the move from theory to the production of empirical data out of information, that is, with translating from one language (the observed world) to another (the theory). In his Introduction, Bernstein provides a *post hoc* summary of his methodology in a form which I have interpreted (with some minor alterations to terminology¹) in Figure 4.1. What I have referred to as 'theoretical referents' correspond to the theoretical work discussed in the previous Chapter. The ten 'Theoretical Propositions' (TP1-10) arose out of this discussion. The 'language of description' concerns, firstly, the production of a theoretical model which is to be derived from the Theoretical Propositions. Secondly, the language must specify what is to count as its empirical object—recognition rules. Thirdly, it must include 'realisation rules' which specify how information is to be read as data. The solid lines with single arrows, in Figure 4.1, indicate lines of deductive argument.

Since the language specifies what can count as its empirical object and how that object is to be interpreted, there is a danger that the model itself will not come under empirical scrutiny. Bernstein suggests that:

... the interface between the realisation rules of the model and the information the [something to which the model is applied] does or can produce is vital. There then must be a discursive gap between the rules specified by the model *and* the realisation rules for transforming the information produced by the something. This gap enables the integrity of the something to exist in its own right, it enables the something, so to speak, to announce itself, it enables the something to re-describe the descriptions of the model's own realisation rules and so change.

(Bernstein, in press, ms p. 69)

¹ The expression 'language of description' is Bernstein's. However, he tends more frequently to refer to 'principles of description' which, in the Introduction cited here, encompass the recognition and realisation rules derived from the model (and not the model itself). Bernstein's 'theory' would correspond to my 'theoretical propositions'. I have also included, in Figure 4.1, the theoretical referents (contextualising work) out of which these propositions arise.

Structure & Application of a Language of Description

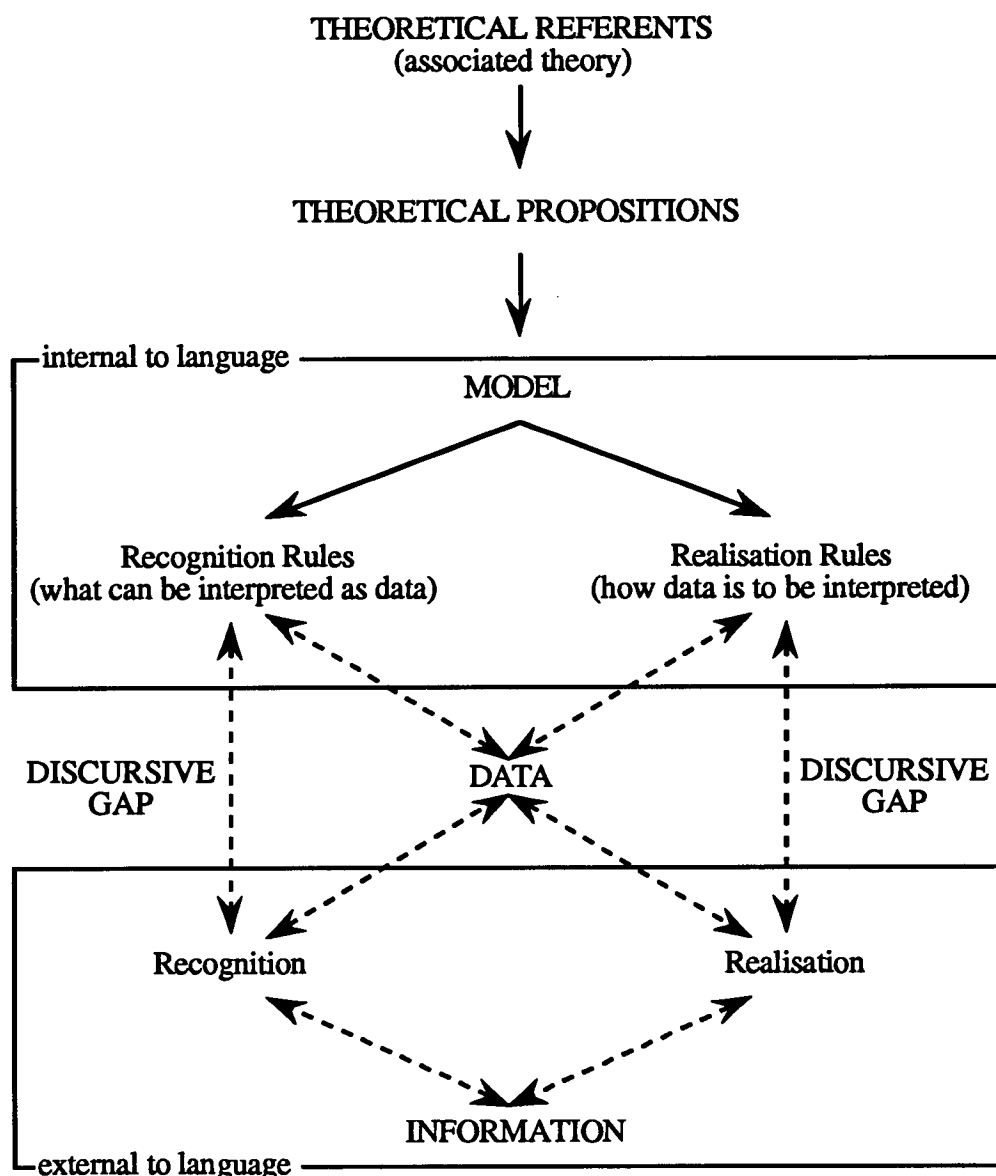


Figure 4.1

This is an important point if sociology is not to be reduced to the projection of armchair theorisings onto the world on the other side of the front room window. In Figure 4.1, the 'discursive gap' is 'between' that which is internal to the language of description and that which is external to it. Data is shown within this gap. Data can be understood as the product of the recognition and realisation rules of the language, but there will always be an excess in terms of possible interpretation. The 'discursive gap' is the region of the 'yet-to-be-described'. Figure 4.1 represents the 'discursive gap' as occupying a particular position within the methodological space. Given the impossibility of the closure of the discursive, discursive hiatuses are pervasive. It is,

therefore, not necessary to introduce a *formal* condition for reflexivity. Bernstein's point, however, concerns the need for an explicit recognition of the possibility of transformations in the language arising out of specific empirical engagements with the world.

This is certainly the spirit in which the present project has been carried out. The developing language of description has been under continual theoretical and empirical review. At the point of its final presentation, however, it necessarily acquires a patina of completeness. For this reason, as much as any other, there is a requirement that the language be rendered as visible as possible. At the end of his Introduction, Bernstein gives an appropriate fanfare for the work of this Chapter:

Finally it is relevant to point out that we all have models, some are more explicit than others; we all use principles of descriptions, again some are more explicit than others; we all set up criteria to enable us both to produce for ourselves, and to read the descriptions of others, again these criteria may vary in their explicitness. Some of our principles may be quantitative whilst others are qualitative. But the problem is fundamentally the same. In the end whose voice is speaking? My preference is to be as explicit as possible. Then at least my voice may be deconstructed.

(Bernstein, in press; ms pp. 69-70)

4.2 *An image of the social*

In this Section I shall introduce some of the basic concepts of the language. In doing so, I shall, where possible, index the relevant Theoretical Propositions (TP1-10) to which the exposition relates.

As I have already announced, I intend to avoid using the term 'ideology', because of its own ideological baggage. Instead, I shall use the term *activity*¹. It is important to specify that activity constitutes an analytic space. Activity is always ideologised in the empirical. That is, activities are, themselves, constituted by (and (re)productive of) the division of labour in society. I shall return, briefly, to this point at the end of this Chapter.

Activity is to be understood as the contextualising basis of social practice (TP1). Thus, firstly, any particular activity must specialise *practices*. This is achieved as a particular articulation of a notional global semantic universe (TP2). An activity thus

¹ This term is taken from the English translation of Leont'ev's work (1978, 1979). Its choice acknowledges a certain resonance between the hierarchical organisation in this model and in Leont'ev's 'activity theory'. Essentially, there is a certain similarity between the relationships between 'action' and 'operation' in Leont'ev's work and 'textual strategy' and 'textual resource' in my language. However, it is important to point out that the two theories differ in most other respects.

regulates what can be said or done or meant. Secondly, an activity establishes one or more positions which can be occupied by human individuals; these are specialised *subject positions*. These are not construed as 'roles' in G.H. Mead's sense (Turner, 1987) or as either 'roles' or 'social positions' in Giddens' use of these terms (Cohen, 1987). Subject positions are understood to be constitutive of human subjectivity rather than syndromes of expected behaviours, whether or not associated with a determinate physical locale. Concrete human *subjectivities* are to be interpreted as articulations of multiple subject positions (TP7). The relationship between the practices and subject positions associated with a particular activity is as follows: practices are distributed within the array (potentially hierarchical) of subject positions; an activity regulates who can say or do or mean what. Thus, practices and subject positions are realised in and by subjectivities. For example, school mathematics constructs a hierarchy of subject positions, such as teacher, students of different ages and abilities. This is achieved via the distribution of school mathematics practices (mathematical and pedagogic knowledge) within the hierarchy.

The practices and subject positions of an activity are also realised in *texts*. A text, in this project, is to be understood as closed. It is an utterance or set or sequence of utterances made within the context of a particular activity (TP10). The significance of the closure of the text is that its analysis is not concerned with the interactional; the ideal reader is effectively exhausted by the text¹ and is thus rendered passive. The empirical object of analysis of the present thesis is a school mathematics scheme. This scheme consists of a set of utterances of the activity school mathematics. The Chapters that follow are concerned with the analysis of these utterances and not with the interactions between them and what may occur in a classroom within which they are used nor, indeed, with what happens when an empirical reader takes up the texts. All texts construct authors and readers (Eco, 1979). All texts (re)produce, in part, the practices of the activity of which they are utterances. *Pedagogic texts*, which are the concern of this thesis, construct transmitters and acquirers. The transmitter is in possession of the regulative rules of the practices of the activity which the acquirer is to acquire. Transmitter and acquirer are textual constructions which are textual realisations of subject positions. The practices and subject positions of an activity are thus instantiated in pedagogic texts. The instantiation of *practices* will be referred to as *message*; the instantiation of *subject positions* will be referred to as *voices*.

I have now introduced two levels. At one level, activity constructs subject positions via the distribution of practices to a range of subject positions. At the

¹ More correctly, by the reading of the text.

second level, activity is (re)produced, firstly, by human subjectivities, which articulate multiple subject positions. Secondly, activities are reproduced by texts, specifically, pedagogic texts. Pedagogic texts distribute message over a range of voices and so (re)produce the practices and subject positions of activity. Activity is the structural level; human subjectivity and text are at the level of event. The relationship between these two levels is not one of simple reproduction or correspondence. Rather, the relationship between the structural and evental levels corresponds to that between *langue* and *parole*, where this relationship is conceived of as dialectical (see the discussion in Hassan, 1992a; see also Derrida's discussion of the *a* motif in his term *différance*¹). I have tried to establish this through the use of my term *(re)production*: subjectivities and texts (re)produce activities (TP6). I will discuss the potential for the productivity of subjectivity and of text later in this Chapter and in the following Chapters.

As is the case with linguistics, the structural level (activity or *langue*) is accessible only via the level of events (subjectivity/text or *parole*). In this thesis, the empirical and, therefore, the principal theoretical interest is in text rather than subjectivity. I will, therefore, refer to the level of events as the textual level of the language of description. Text as a material instance of activity is the empirical object of the study. Both activity and text are theoretical objects of the study: the language of description must structure the former and must provide recognition and realisation principles for the analysis of the latter.

My programme for the remainder of this Chapter is as follows. Firstly, I will discuss the composition of the structural level of the language: activity; its practices and their characterising; and subject positions. Secondly, I will introduce the language at the level of the text: the (re)production of practices and subjectivity via message and voice. This Section will be followed by the introduction of a third level of the model. Finally, I will pay some attention to the higher levels of structure, that is, to institutional level and subjectivity.

¹ 'The activity or productivity connoted by the *a* of *différance* refers to the generative movement in the play of differences. The latter are neither fallen from the sky nor inscribed once and for all in a closed system, a static structure that a synchronic and taxonomic operation could exhaust. Differences are the effects of transformations, and from this vantage the theme of *différance* is incompatible with the static, taxonomic, ahistoric motifs in the concept of *structure*. But it goes without saying that this motif is not the only one that defines structure, and that the production of differences, *différance*, is not astructural: it produces systematic and regulated transformations which are able, at a certain point, to leave room for a structural science. The concept of *différance* even develops the most legitimate principled exigencies of "structuralism"' (Derrida, 1981; pp. 27-8). See also the discussions in Sturrock (1979, 1986) and Bannet (1989).

4.3 The structural level: activity

4.3.1 Practices: domain & gaze

The global semantic universe comprises forms of expression and contents (signifiers and signifieds). These are articulated as relational totalities which constitute activities, that is, their practices and subject positions. The particular activity to be described in this thesis is school mathematics. It is asserted that school mathematics specialises its practices and subjectivities sufficiently to be referred to as an activity. However, it is of the nature of a relational totality that it resists all attempts at *a priori* definition¹. The positivity of school mathematics as an activity must emerge within the context of its description as an activity. In advance of this, however, it is also to be used as the principal reservoir of exemplary illustrations. Consider the following four extracts, which are all taken from the school mathematics scheme which is the empirical object of this thesis².

EXTRACT A

Solve	(a) $18x + 92 = 137$	(b) $0.7x + 3.2 = 4.88$
	(c) $2.9x - 3.5 = 19.7$	(d) $0.4x - 4.6 = -2.0$

(SMP 11-16 Book Y1, p. 66)

¹ See the discussion in Section 4.6.

² I have noted that the structural level of the model is empirically 'accessible' only via the level of events, that is, that activity is describable only in terms of its instances. There is a common tendency to disregard this on the part of grand theorists of social structure. Where social structure is described other than formally, theorists often make use of 'imaginary' texts, or 'texts' which have already undergone an implicit analysis. For example, Bowles & Gintis (1986; Gintis & Bowles, 1988) sketch out the structure of the social universe in terms of 'sites' of social practice. These sites are differentiated on the basis of their principle axes of power distribution. Bowles & Gintis offer 'government closes tax loopholes' as an illustration of a 'distributive' practice within the site of the 'liberal democratic state'. Just what makes such an action 'distributive' rather than 'political' or 'cultural' or 'appropriative' is not elaborated. Nor is the action described at the level of concrete practice, but at a sort of metatextual level. The actual texts which are associated with government actions are not introduced. Nor is it at all clear how Bowles & Gintis would be able to deal with them, given the highly general nature of their language: they have no language of description. A similar criticism can be made of Marx's analysis in *Capital* (1976). Bowles & Gintis' earlier work, *Schooling in Capitalist America* (1976) does not exhibit precisely the same problem, because empirical data is given. Again, however, there is no language of description. Their thesis is advanced on the basis of a 'correspondence principle' bolstered by a residual argument. (See also Bowles & Gintis, 1988).

The limiting of exposition on structure to imaginary or pre-digested texts seems, to me, to be an error. It demands a suspension of disbelief concerning both the genesis and application of the model. I shall, therefore, make use of empirical texts in the introduction of the structural level of my model, even though I have not yet described the tools which will enable their analysis. I shall begin with the characterising of practices. The extracts which follow were used illustratively in Dowling (1992b).

EXTRACT B

A4 What is the bill for each of these shopping lists?
Work out each bill in your head.

- | | | | | |
|-------------------|---------------|---------------|-----------------|------------------|
| (a) 1 kg potatoes | (b) 2 oranges | (c) 2kg spuds | (d) 1kg bananas | (e) 1 grapefruit |
| 1 grapefruit | 1 cauli | | 100g mushrooms | 1 orange |
| | | | | 1kg bananas |

(SMP 11-16 Book G1, p. 14; original shopping lists drawn as fragments of paper)

EXTRACT C

A café orders p white loaves and q brown loaves every day for r days.

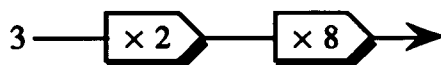
What does each of these expressions tell you?

- (a) $p + q$ (b) pr (c) qr (d) $(p + q)r$

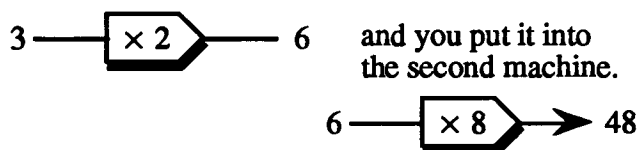
(SMP 11-16 Book Y1, p. 32)

EXTRACT D

Here is a machine chain.



You find the output from the first machine ...



(SMP 11-16 Book G1, p. 41; indexical borders and shading omitted, font standardised)

Extract A is relatively unambiguously a (school) mathematics text in terms of both its form of expression and its content. There are equations involving arithmetical symbols, positive and negative numbers, and unknowns (signified by 'x'); 'solve' is clearly to be interpreted as a mathematical process and not, for example, in the sense of solving a crime ('discover who was responsible for the following inscriptions'). This kind of text exhibits comparatively strong classification or specialisation of mathematical knowledge (TP3). Extract B, by contrast, clearly has some of the characteristics of a school mathematics text, but appears to index something else. The other that is indexed by this extract is the domestic practice of shopping. There is an absence of specialisation of either forms of expression or content. This extract signals weak classification of mathematical knowledge. Extract C is a hybrid. The content of the task is non-mathematical. That is, the context is an economic practice (running a café). However, the task incorporates specialised mathematical algebraic expressions.

The content of these expressions—their principal denotations—are non-mathematical: $p + q$, for example, signifies the total quantity of loaves ordered each day. The content thereby signals weakly classified mathematical knowledge. The form of expression of the algebra, on the other hand, refers to strongly classified mathematical knowledge. Finally, extract *D* is the other way around. Here, the context is comparatively unambiguously mathematical: all of the signifiers in the text principally denote mathematical objects. However, one of the signifiers, ‘machine’ is a non-mathematical expression; it introduces a metaphor, machine for arithmetic operation¹. In this case, a non-mathematical form of expression has been embedded within a mathematical context: the content represents strong classification, this expression (which has been foregrounded) refers to weak classification.

These four examples illustrate the characteristics of each of four *domains* of school mathematics practice. The four domains can, in fact, be derived directly from TP3 for any activity. Since the form of expression and content can be measured separately in terms of strength of classification, strong or weak, the product of these two scales generates the space illustrated in Figure 4.2.

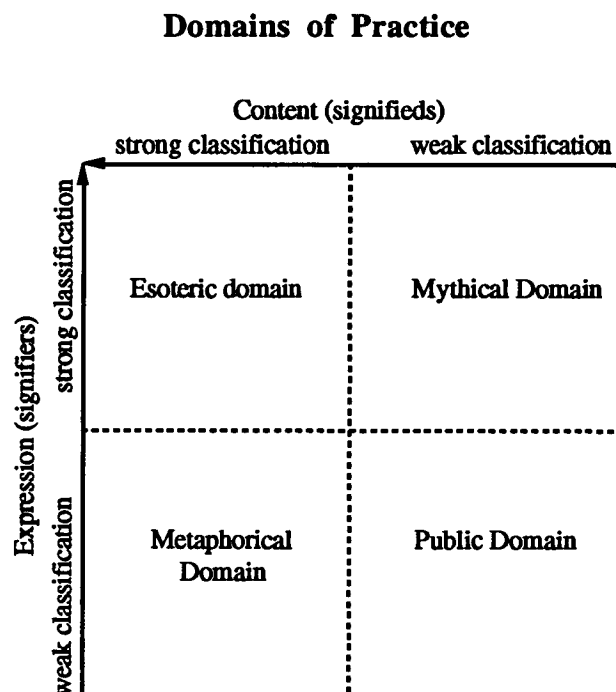


Figure 4.2

¹ See Chapter 6 for further discussion of this example.

The *esoteric domain* of practice refers to the region of an activity which is most strongly classified with respect to other activities. Both forms of expression and content are specialised. The esoteric domain may itself comprise sub-regions corresponding to Eco's 'sub-codes' (1976), these will be referred to as *topics*. School mathematics, for example, consists of topics such as algebra, statistics and probability, arithmetic, euclidean geometry, transformation geometry, and so on. Topics exhibit a certain degree of positivity, but are multiply interconnected (otherwise the activity itself could not exhibit positivity). Because ambiguity is minimised in the esoteric domain, specialised denotations and connotations are always prioritised. It is, therefore, only within this domain that the principles which regulate the activity can attain their full expression. The esoteric domain may be regarded as the regulating domain of an activity.

However, all activities must look beyond themselves for pedagogic if for no other reasons. Bernstein has made this point with great clarity, albeit in a different methodological framework:

If the culture of the teacher is to become part of the consciousness of the child, then the culture of the child must first be in the consciousness of the teacher.

(Bernstein, 1971b; p. 65)

If an activity were to make no references outside of itself, then it would be unable to create apprentices. The esoteric domain of an activity is therefore conceived as effecting a *gaze* beyond itself. The gaze lights upon external practices which are *recontextualised* by it (TP4). Recontextualising entails the subordination or partial subordination of the forms of expression and/or contents of practices of one activity to the regulatory principles of another. Extract *B*, above, provides a textual illustration of the effects of such a recontextualisation. Whilst the forms of expression and content derive from domestic activity, they have been referred, by the mathematical gaze, to mathematical interpretants, in this case, arithmetical interpretants.

The general effect of the recontextualising gaze is the production of a domain of practices which exhibits comparatively weak classification in terms of forms of expression and content. This is to be referred to as the *public domain*. The public domain has the appearance of non-specialised practice as is illustrated by extract *B*. However, it remains, to a greater or lesser extent, subject to the regulative principles of the esoteric domain. These principles, however, cannot be adequately expressed within this domain, because there can be no certainty of the prioritising of specialised denotations and connotations.

An alternative mode of recontextualisation occurs where the gaze combines non-specialised forms of expression with specialised content. This constitutes a metaphor, that is, referring to something as something else. In order to recognise, textually, the product of this mode of recontextualisation it is necessary to analyse a sufficiency of text. In extract *D*, for example, there is a sufficiency of context to enable its recognition as mathematical in terms of content, so that 'machine' appears as a metaphorical intruder at the level of expression. The domain of practice which is established by this mode of gaze is referred to as the *metaphorical domain*. Because the effect of the gaze is to recognise something else as substitutable for a specialised expression, this mode of gaze is referred to as *introjective*¹. Again, the regulative principles of the esoteric domain cannot be fully expressed within this domain.

Finally, the gaze may recontextualise a non-specialised context within which specialised forms of expression are projected onto non-specialised content, the latter being defined by the context. Thus, extract *C* projects algebraic expressions onto quantities of bread. This mode of gaze is referred to as the *projective gaze*. The combining of a specialised form of expression with a non-specialised content is again a metaphorical move. However, there is a difference in that the gaze is projective rather than introjective. That is, the specialised expression is being imposed upon the non-specialised content from the position of the esoteric domain. The esoteric domain is thus being implicated in a 'mythical' system. Figure 4.3 shows Roland Barthes' (1972) myth schema which shows the relationship between two semiological systems, language and myth.

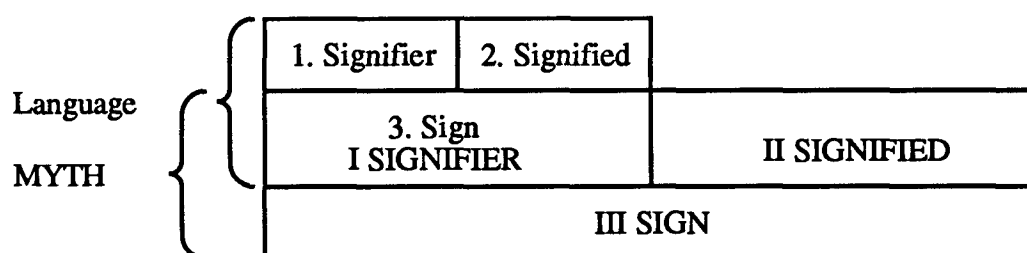
To refer to extract *C*: the signifier, *p*, signifies the quantity of white loaves ordered each day, which thereby corresponds to the signified within language in Barthes' diagram. This association produces the linguistic sign which now becomes a signifier within a mythical system: the signification of a non-mathematical signified by a mathematical signifier itself signifies the rationalising power of the esoteric domain of mathematics, which is the position from which the original signification is produced. This is a celebration of mathematics which is similar in form to the celebration of French imperialism in Barthes' example of the black soldier saluting the tricolour on the cover of *Paris-Match*. The domain in which there is strong classification of form of expression combined with weak classification of content is, therefore, to be referred to as the *mythical domain*. As will become apparent in the

¹ This term and the term *projective* are taken, metaphorically, from Klein (1975).

analysis in the following Chapters, there are two different kinds of *myth* which are implicated in school mathematics¹.

I have drawn primarily on TP2, TP3 and TP4 in describing the *domains* of practice within an activity. Essentially, the *gaze* is *projected* or *introjected* by the *esoteric domain* which is the regulating domain or an activity. The gaze generates three further domains, the *public domain*, the *metaphorical domain* and the *mythical domain*. These domains differ in terms of the strength of classification of forms of expression and content. I shall next consider the implications of TP1.

Barthes' Myth Schema



(Barthes, 1972; p. 115)

Figure 4.3

4.3.2 Practices: *discursive saturation*

In Chapter 3, I spent some time in a discussion of various conceptions of the modality of practice which led to the notion of *discursive saturation*. I argued that, whilst all practices are material, some practices minimise their dependency upon the material via the production of highly developed and articulated, that is, highly systematised discursive structures. Part of the value of taking mathematics as the case study in this thesis is that it often construed as exhibiting the highest form of systemativity. Foucault describes mathematics as:

... the only discursive practice to have crossed at one and the same time the thresholds of positivity, epistemologization, scientificity, and formalization. The very possibility of its existence implied that [that] which, in all other sciences, remains dispersed throughout history, should be given at the outset: its original positivity was to constitute an already formalized discursive practice (even if other formalizations were to be used later). Hence the fact that their establishment is both so enigmatic (so little

¹ 'Myth', in these terms, is also related to the mythologising of mathematics by utilitarianism and mathematical anthropology (Chapter 2). I will return to this in Chapter 11.

accessible to analysis, so confined within the form of the absolute beginning) and so valid (since it is valid both as an origin and as a foundation); hence the fact that in the first gesture of the first mathematician one saw the constitution of an ideality that has been deployed throughout history, and has been questioned only to be repeated and purified; hence the fact that the beginning of mathematics is questioned not so much as a historical event as for its validity as a principle of history; and hence the fact that, for all the other sciences the description of its historical genesis, its gropings and failures, its late emergence is related to the meta-historical model of a geometry emerging suddenly, once and for all, from the trivial practices of land-measuring.

(Foucault, 1972; pp. 188-9)

This is, perhaps, an undue mythologising of mathematics. It certainly dismisses the non-discursive in mathematics. Nevertheless, Foucault is probably correct in his assertion that '[m]athematics has certainly served as a model for most scientific discourses in their efforts to attain formal rigour and demonstrativity' (ibid)¹. It may be, then, that it is easier to demonstrate the high level of discursive saturation of mathematics² than of other school disciplines or of other activities in general. However, the discussion in Chapter 3 was intended to justify the assertion that discursive saturation is a quality which is generalisable beyond mathematics. Therefore, although school mathematics continues to be used as the principal exemplar, the contention is made that this aspect of the language as well as the others are of potential value in the analysis of pedagogic texts more generally.

The crucial distinction between practices exhibiting high and low discursive saturation is the extent to which their regulating principles are realisable within discourse. This entails that practices exhibiting high discursive saturation (DS⁺) are, at the level of discourse, highly complex and exhibit comparatively complete articulation. They are, furthermore, highly organised: discursive objects (signs) are always defined more or less formally and within discourse³. That is, any sign may be objectified within discourse, so that it is always possible to produce generalisations. It has already been established that regulating principles are only fully realisable within the esoteric domain of an activity. Therefore, the esoteric domain of a DS⁺ activity is characterised by a degree of discursive closure⁴.

¹ The text immediately following this extract reads: 'but for the historian who questions the actual development of the sciences, [mathematics] is a bad example, an example at least from which one cannot generalize' (ibid). My interest, of course, is not in the development of the sciences, so I shall take it that Foucault's caveat is not directed at me.

² There is an assumption, here, of at least a degree of continuity between school and academic mathematics.

³ In Vygotsky's (1986) terms, signs (more correctly, perhaps, signifieds) are generally constituted within such practices as 'scientific concepts'.

⁴ Although there is always a necessary openness at both the discursive and non-discursive levels since connotations can never be excluded. I am not, therefore, proposing 'closure' in the Piagetian/mathematical sense (Piaget, 1968).

The high degree of discursive organisation of the esoteric domain of a DS⁺ activity facilitates the generation, by such an activity, of languages of description having highly explicit realisation principles. This concerns the application of the gaze. The descriptive power of the esoteric domain preconceptualises practices which are recontextualised, so that these are easily subordinated to the grammar of the recontextualising esoteric domain. Indeed, such subordination is to a large extent necessary, because of the relative inflexibility of the grammar of the recontextualising esoteric domain. Such activities are, therefore, capable of producing highly generalisable descriptions both within and outside of the esoteric domain. School mathematics provides an obvious example of such an activity as the discussion on 'utilitarianism' and 'mathematical anthropology', in Chapter 2, illustrates.

The gaze itself must incorporate recognition principles. However, there is no logical necessity that these be discursively explicit, because the gaze is, first and foremost, concerned with pedagogy, that is, with the creation of subjectivity. It is probably more appropriate to understand the recognition principles of the gaze as deposited in the extension of a set of exemplars (cf Kuhn, 1970) rather than as being prefigured by intensional precepts. In the case of school mathematics, it is not at all clear that there are any limits on recognition. The arithmetic of the natural numbers, for example, seems to be universally applicable (although not, of course, universally useful). The prevalence of the view that 'mathematics is everywhere' (see Chapter 2) again attests to this.

Activities whose practices exhibit low discursive saturation (DS⁻) are characterised by implicit regulating principles. That is, specialisation is at the level of the non-discursive, but not, to any great extent, at the level of the discursive. These activities are characterised by what Bourdieu terms 'polythetic' thinking. Thus utterances within these activities are, of necessity, highly localised or context-dependent (Bernstein, 1990). This latter term requires a little elaboration. All utterances are context-specific, in the sense that they must be interpreted within the context of a particular activity. However, an utterance within a DS⁻ activity is also *context-dependent*, to the extent that it cannot be unambiguously interpreted outside of the context of its immediate production. Activities that are characterised by DS⁻ are those that are commonly (although not necessarily) referred to as 'manual' activities.

It is also the case that comparatively low levels of DS are represented by texts in more conventionally 'intellectual' activities. For example, insofar as it is concerned with student control, school mathematics can apparently lack discursive organisation.

The following two extracts are taken from the transcript of an interview with a mathematics teacher¹:

After having Jason who was on his own, working with Elaine, they seemed to get on OK, so I tried them together again and that seems to be working reasonably. Jason is quite capable but wanders all the time. Elaine struggles a bit and Robyn and Ann are quite strong so I put them altogether because they also seem quite considerate of the others' needs.

[...]

Over here I felt that Cyril he's reminded me of a lot of kids I see, particularly black boys, they come in and they are very capable but within a few months they start drifting off for some reason. There is something wrong there, so I've put him in with two very strong boys.

It would appear that the teacher, who was aware of the interviewer's expertise in education, has no recourse to a specialist language in describing his classroom organisation strategies. Such a language is clearly lacking in the first extract: 'they seemed to get on OK, so I tried them together again ...'. The second extract bespeaks a more or less explicit racism, but even this is not consistent as other black boys were described by the same teacher as exhibiting notable stability. The teacher is bricolaging his responses (cf Lévi-Strauss, 1972). It may be that the researcher will employ a well-developed language of description to organise the teacher's practices discursively. He does not appear, however, to be likely to achieve this himself. This is 'because the expertise of the secondary mathematics teacher resides in mathematical discourse and not in didactics' (Dowling, 1993a, in press).

I have now introduced the terms which enable me to talk about practices. These are to be described in terms of *domain* and *discursive saturation*. As I noted earlier, activities also construct *subject positions*. I shall now turn to this element in the final part of this Section.

4.3.3 *Subject positions*

Pedagogic activities, such as school mathematics, must logically construct a hierarchy of subject positions. That is, they must construct transmitting and acquiring positions. Since pedagogic action is extended in time, such activities must also construct a hierarchy within the category 'acquiring position' in terms of the degree of advancement. Empirically, not all acquirers become even potential transmitters.

¹ The interviews were conducted by Jayne Johnston as part of the empirical programme of her own doctoral research in the Department of Mathematics, Statistics and Computing of the Institute of Education, University of London. Students' names have been altered.

Pedagogic activities, therefore, construct hierarchies within the category 'acquiring position' which are not entirely defined by career. Such activities are, in other words, selective. Pedagogic activities may also construct hierarchies within the category 'transmitting position'. There are, thus relatively dominant and relatively subordinate subject positions. The most dominant subject position, we might say, exhausts the practices of the activity and is its subject. The most subordinate subject positions are more aptly described as being objectified by the activity. The teacher in the interview transcripts quoted above is clearly objectifying the students, even if not in any consistent way.

Thus dominant and subordinate subject positions are constructed via the distribution of practices. The activity, in effect, regulates 'who' can say or do or mean 'what'. Comparatively dominant acquirers may be described as 'to-be-apprenticed' subject positions, whilst comparatively subordinate acquirers are 'to-be-alienated'. In this respect, the most sensitive region of practice must be the esoteric domain and its gaze. In the case of a DS⁺ activity, such as school mathematics, the 'to-be-apprenticed' acquirer must be apprenticed into the regulative principles of the esoteric domain and the realisation principles of the gaze at the level of discourse.

It is thus possible to describe pedagogic action in relation to the 'to-be-apprenticed' (TP8 & TP9). The initial 'hailing', or 'interpellation' must take place within the public domain. Pedagogic action then proceeds via the construction of metonymic chains (TP5, TP8, TP9) which must enter the esoteric domain. In order to establish the semiotic complexity of the DS⁺ esoteric domain, pedagogic action must render accessible, again metonymically, the regulative principles and organisational structure of this domain, which must entail establishing explicit links within and between topics. In thus establishing the apprentice as a (limited) subject position with respect to any region of the esoteric domain, the apprentice will have undergone what can, metaphorically, be described as a one-hundred-and-eighty-degree rotation from the public to the esoteric domain (TP9). The recognition principles of the gaze may then be made accessible in the production of exemplars, via projection. The essential ingredient of apprenticeship is a visible pedagogic action.

The to-be-alienated must, correspondingly be denied access to at least the regulative principles of the esoteric domain. This may entail restriction to the public domain. Where esoteric domain practices are introduced, the subordinate subject position must be denied access to its semiotic complexity. The distribution of practices to the subordinate subject position, therefore, likely to be characterised by

an interrupted esoteric domain. There is an extent to which '*subject* position' is a misnomer in this case, because lack of access to the regulative principles of the activity must render the subordinate dependent upon the dominant.

This completes the discussion of the structural level of the language of description. I shall now present a brief introduction to the textual level. A more complete development of this level will be presented in the textual analysis which will be presented in the following Chapters. This is, firstly, for the purpose of avoiding redundancy, since the textual level is clearly most appropriately introduced via textual illustration. Secondly, since the production of the language was elaborated within the context of the analysis itself, it is inappropriate to present it in its entirety as an *a priori* deductive structure.

4.4 The textual level

Corresponding to practices and subject position, at the structural level of activity, are message and voice, at the textual level. The use of different terms for the two levels emphasises the difference in theoretical status. Message and voice are the direct products of the analysis of a particular text. The validity and reliability of the analysis are functions of the sampling of the text and of the extent to which the principles of the analysis have been made explicit. Practices and subject positions must be inferred from this analysis. The question, here, concerns generalisability and is a function of the standing of the analysed text within the archive of all extant texts. The practices and subject positions of an activity are structuring resources in the production of texts which, themselves, are produced as well as reproduced by texts; the relationship is dialectical. Activities are thus (re)produced by texts, specifically, by pedagogic texts. Pedagogic texts must, therefore, (re)produce the various features of activity described in the previous Section. I shall denote the various mechanisms by which they achieve this as *textual strategies*. Barthes (1974) describes a text as a 'weaving of voices' (his 'voices', not mine). In my language, a pedagogic text is a weaving of textual strategies which must be identified within the analysis. The principal categories of textual strategy are given below.

4.4.1 Message strategies

Texts in general and pedagogic texts in particular must (re)produce the domains and discursive saturation of the practices of an activity: this is achieved, principally, via *message strategies*. Clearly, pedagogic texts will always construct acquirers (TP10),

so that these message strategies cannot operate in isolation from strategies relating to voice: the strategies are 'woven' together. Nevertheless, there must be a range of practice (re)production with respect to any given subject position, so that it is appropriate to make an analytical distinction for the purposes of this thesis.

Message strategies must reproduce, firstly, the esoteric domain of practices. This must include the (re)production of the regulative principles of the esoteric domain which, for a DS⁺ activity such as school mathematics, are substantially discursive. The esoteric domain must, therefore, be (re)produced as *discourse*, which is a principal category of message strategy. As was noted above, however, the distribution of practices to subordinate subject positions (non-apprenticed voices) is such that there is no or limited access to these regulative principles in discursive form. The esoteric domain must, therefore, be (re)produced in a second form which obscures its regulative principles. The message strategy that achieves this is to be referred to as *procedure*. Examples of procedures will be given extensively in the Chapters that follow this one. It will be useful, however, to give a single illustration at this point. According to Pimm:

Too much algebra teaching is solely syntactic, in that much mathematical practice is coded into precepts which operate entirely on the symbols, rather than being combined with a meaning (and hence a purposeful goal), an interpretation in which the requisite transformations make some sense.

(Pimm, 1987; p. 174)

Pimm's language is inconsistent with that being used here: the notion of a meaningless symbol clearly does not fit in with the semantic structuring of school mathematics that I am employing. Nevertheless, the notion of the 'coding' of mathematical practice into a precept or procedure is instructive. Take, for example, the procedure which is commonly employed in the division of fractions: turn upsidedown and multiply. The effect of such 'coding' of mathematical discourse as an algorithm is to localise mathematical knowledge, to reduce its generalisability. The general quality which distinguishes discourse from procedures is that the latter tend to impoverish the complexity of the message, minimising rather than maximising connections and exchanging instructions for definitions.

Message strategies must also (re)produce the other domains of practice which have been established via the gaze. *Recontextualised message* may be *projection* or *introjection*, corresponding to the projective and introjective gaze, respectively. Projection strategies may be realised in mythical domain text, such as Extract C in Section 4.2. Introjection strategies may be realised in metaphorical domain text, such as Extract D. Public domain text (Extract B) may be instances of either projection or

introjection. The distinction is to be made in the context of how the text moves between domains. This must await further clarification until the more detailed analysis provided in Chapters 6-10.

I have argued that the regulative principles of an activity can be fully realised only within the esoteric domain. This being the case, there can be no discourse (in my sense) within the other domains. Empirically, however, strategies within these domains are not entirely restricted to procedures. The additional strategy, *quasi-discourse*, will be introduced in Chapter 7.

4.4.2 Distributing strategies

Message strategies concern the (re)production of the domains and discursive saturation of the practices of an activity. Insofar as particular message strategies are incorporated into texts which construct particular categories of voice (dominant/subordinate), they are also implicated in the (re)production of subject positions via the distribution of practices. For example, it will be shown in the analysis of the *SMP 11-16* materials that message associated with the subordinate voice is predominantly outside of the esoteric domain; where the message moves into the esoteric domain, it is characterised by procedures rather than discourse. In describing text in terms of message strategies, however, I intend to prioritise the (re)production rather than the distribution of practices (Chapter 8). In describing *distributing strategies*, on the other hand, I will be prioritising the (re)production of subject positions via the distribution of message over the range of voices.

As I have argued above, the most dominant subject position (no-longer-apprenticed voice) of any activity is that which exhausts its practices; this subject position is the Subject of the activity. Pedagogic action relating to the (re)production of the dominant subject position must, therefore, always tend to generalise. Esoteric domain message will be characterised by discourse. The full scope (recognition principles) and realisation principles of the gaze must be made accessible, so that the public domain is likely to be expanded. Successful apprenticeship to the dominant subject position entails the 'one-hundred-and-eighty-degree rotation' that constitutes the Subject of the activity (TP9). Pedagogic action must, therefore, minimise the local specificities of the acquirer and maximise the generalities of the esoteric domain. Textual strategies which construct a relatively dominant voice in this way are referred to as *generalising strategies*.

On the other hand, the subordinate subject position is, as has been argued above, objectified rather than subjectified by the activity. Textual strategies include the maximising of the individuality of the acquirer and the reduction of any tendency to generalise: message is made very much context-dependent, even where the activity is DS⁺. Public domain message will be predominant and the public domain is, itself, likely to be restricted in scope so that there is limited access to the breadth of the recognition principles of the gaze. Strategies which construct a relatively subordinate voice in this way are referred to as *localising strategies*.

4.4.3 Voice positioning strategies

The final category of strategies is, again, concerned with the (re)production of subject positions. Strategies in this category are to be referred to as *voice positioning strategies* and operate directly on voice rather than distribute message. There are two main forms of voice positioning strategy. *Intervoice strategies* construct relationships directly between voices. In the case of a school mathematics text, for example, it may produce explicit differentiation in terms of 'ability' and 'age'. Pedagogic texts may also differentially construct the relationship between the categories of transmitter and acquirer, apparently strengthening or weakening the classification between them.

Interactivity voice positioning strategies achieve relationships between voices by reference outside of the activity, that is, within the public domain. It will be argued in the analysis of *SMP 11-16* texts, for example, that these texts achieve a high degree of correlation between dominant and subordinate voices, on the one hand, and social class, on the other.

The strategies outlined in this Section will be brought into focus much more sharply in the textual analysis in the Chapters that follow this one. The schema of strategies will also be somewhat enlarged, although the three basic categories, message, distributing and voice positioning strategies will not be increased. I shall now move on to consider the analytically lowest level of the language.

4.5 Textual resources

4.5.1 The third level of the language of description

The form in which the language of description has been developed, so far, is essentially deductive. The ten Theoretical Propositions (TP1-10) arose out of a

discussion of a range of theoretical work in Chapter 3. From these Theoretical Propositions has been derived the notions of: activity, with its associated practices and subject positions; and pedagogic text, with its categories of message and voice which are realised by the three main categories of textual strategy. The language, thus far conceived, contains the recognition and realisation principles for the description of its empirical object—pedagogic texts. However, as I argued in the first Section of this Chapter, the language cannot exhaust the empirical. Thus, whilst the deductive structure advanced above entails the theoretical possibility of the categories of textual strategy, it cannot prefigure the empirical resources that will be implicated in these strategies. The category, message strategies, must include public domain text, but it cannot specify the external activities which are the objects of the recontextualising gaze. School mathematics must have a public domain, but what is represented in this domain: the domestic; professional activities of various kinds; other school disciplines? Where, as in the present study, the empirical object is a school textbook scheme, the theory can assert that there will be localising and generalising strategies. However, it cannot predict how these will be realised in terms of resources that are available for the production of textbooks: different kinds of verbal text; literary styles; pictures of various kinds; different kinds of binding; and so on.

Thus, textual strategies are realised in pedagogic texts via the implication of what will be referred to as *textual resources*. There is no *a priori* limitation on what can count as a textual resource. Nor is there a predetermination on how they are implicated into the various textual strategies, so that such implication may be close to what Lévi-Strauss (1972) has called *bricolage*, or it may consist of a more engineered approach. There is, then, a theoretical arbitrariness about textual resources which does not obtain with respect to textual strategies. The relationship between these two levels is thus similar in form to the relation between ‘action’ and ‘operation’ in Leont’ev’s ‘activity theory’ (1978, 1979). In Leont’ev’s conception, ‘actions’ are goal-oriented, whilst ‘operations’ are concerned with means. The latter are therefore comparatively contingent rather than necessary. This resonance between Leont’ev’s schema and my model is the principle reason for the choice of the term ‘activity’, although my use of it is clearly different from Leont’ev’s.

It should be emphasised that the arbitrariness of textual resources is purely a theoretical arbitrariness. Empirically, there must always be a selection from a notional reservoir of resources to constitute the repertoire(s) of resources which make up a particular text. The differential selection of resources in relation to different voices is crucial in the realising of all three main categories of textual strategy. For example, in

the analysis of the *SMP 11-16* texts, it will be demonstrated that the selection of different forms of binding of the textbooks for different categories of reader constitutes a voice positioning strategy.

Because this Chapter is concerned primarily with the theoretical derivation of the language of description, I shall not introduce, here, all of the empirical categories of textual resources. These will emerge from the textual analysis conducted in the following Chapters. I shall, however, discuss a single main category of resource which is to be implicated in a quantitative content analysis to be described in Chapter 9. The category to be introduced is to be referred to as *signifying mode*; the reason for placing this discussion here is that it of necessity involves some theoretical consideration.

4.5.2 *Signifying modes: icon, index, symbol*

Signifying mode describes a form of the relationship between expression and content that is implicated in sign production. That is, Saussure's contention that the general relationship between signifier and signified is one of arbitrariness is being rejected, here; rather, this relationship is understood as being motivated (Hodge & Kress, 1988, 1993). There are diverse ways in which such motivation can be realised. 'Signifying mode' refers to a particular repertoire of resources which is implicated in localising and generalising, which are important categories of textual strategy in my language. Furthermore, these particular resources can be described with sufficient precision to enable a content analysis of the empirical texts in this study. This repertoire thus offers an opportunity to provide, in a key area, a methodological triangulation by complementing the predominantly qualitative analysis used in this thesis with a quantitative component.

Plate 4.1 shows a page from *SMP 11-16* Book Y1 which incorporates all three of the signifying modes to be discussed here, that is, the *iconic*, the *indexical* and the *symbolic*. The picture at the bottom of the page portrays what looks like a sandy beach in front of a sizeable hotel; there are holidaymakers and vendors on the beach and swimmers—in various states of distress—a shark and an octopus in the water. Our view of the scene might be from an aeroplane coming in to land at a nearby airport: we can read the text by substituting for it a potential visual image and a physical viewpoint which fixes our own virtual presence as spectator of the scene. The image, in other words, is an articulation of a visual code and a physical

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 4.1
(Y1: page 18)

viewpoint; such a text is to be referred to as an *icon*¹. Crucially, an icon signifies the virtual, physical presence of the viewing reader and at least one of its readings is predicated solely upon this *visual code of presence*²—this is what it would look like if you were there³.

No assertion is being made that an icon presents an actual and uncoded resemblance or replication of a scene or of an object. We could match every posited resemblance between the beach scene in Plate 4.1 and a real beach (which, of course, may not actually exist) with a difference. Clearly, as Eco (1976) argues, iconic codes must be acquired and there are different kinds of iconic code: a line drawing may exhibit a greater or lesser degree of caricaturing, for example. Whether or not caricaturing is incorporated, there will be further classification of coding including, for example, the phallicising of masculine noses in many of the SMP cartoons. A pedagogic text must make the assumption that its reader has acquired certain codes⁴ and this is true of iconic codes. Again, it is the incorporation of the visual code of presence which characterises all iconic modes⁵.

The image adjacent to task D9, in Plate 4.1, is of a different kind, because, although producing it as a sign incorporates a visual code, the physical presence of a viewer is not signified, there is no code of presence. The image could be transformed into an icon by replacing the dots by drawings of the children: the reader would then be drawn into the unlikely position of flying above the children's heads, but it is the signification and not the plausibility of presence that is important. As it is produced

¹ The terms 'icon', 'index' and 'symbol' are used in marking out signifying modes and this clearly refers back to Peirce's semiology. However, my use of these terms does not coincide with any of Peirce's, in particular, the notion of 'index' used here has a considerable overlap with other uses of 'icon'. Furthermore, I am not ascribing essential qualities to the textual fragments which I am labelling icon, index or symbol, but am referring to conventions of signifying practice (cf Eco, 1976). Bishop's (1977) brief discussion in the professional journal *Mathematics Teaching* is illustrative of the cultural relativity of visual codes.

² 'Code' is being used, here, in a similar way to Eco's (1976). The reader must recognise her/his own spatial position as encoded within the icon (in terms of azimuthal and elevation angles, perspective, etc), thus producing (in Eco's sense, signs are not passive, but must be produced in their reading) the icon as signifying her/his presence.

³ A sound recording might be understood as the aural analogue of an icon; such resources are not, of course, available within the context of most school textbooks, although they do feature in some modern language courses.

⁴ Although not necessarily all: clearly, a text can address multiple readers as is instanced by the inclusion in the SMP texts of references to members of the authoring team—there is, probably, no assumption that the student reader will recognise these, although the alert teacher reader might, since the authors are listed in the Teacher's Guides.

⁵ Regester (1991) has developed a dual classification of 'visuals' in history textbooks. Visuals may be photographs, illustrations, tables, maps or graphs and they vary in their 'degree of visualisation' on a scale ranging from all words to no words. The motivation for Regester's study, however, is related to the apparent lack of knowledge concerning the influence of visuals on learning. This motivation has resulted in a form of classification which is inappropriate for the present purposes.

on the page, however, the D9 image is better described as a minimalist kind of map. The positions of the children are marked in such a way that mathematics can be performed using a pair of compasses or a piece of tracing paper. There are similar images at the top of the page in Plate 4.1: it is not possible, from the Plate, to say what the points are marking, because they refer to a task (D7) on the previous page; actually, they represent, simply, points—mathematical objects. This mode of signification is to be referred to as an *index*. An index incorporates visual or spatial codes, but does not assert the virtual, physical presence of the reader¹.

The final mode of signification in Plate 4.1 is provided by the alphanumeric text on the page which, primarily, is produced as signifying via an articulation of what might be described as a linear visual code (the numbers and letters must be written in a punctuated line which encodes the syntagmatic as well as the paradigmatic) with non-visual codes. This text is *symbolic*. A symbol is alphanumeric and is visual only in linear terms; it does not incorporate a code of presence². A rider must be added: it is clear that symbolic text may be laid out on the page so as to incorporate more complex visual codes; where this is the case and where it is intended to focus on the visual codes, it is appropriate to refer to the text as indexical or even iconic. For example, alphanumeric text may be tabulated, thus incorporating two linear dimensions: tables are generally referred to as indices. Plausibly, the text may be arranged so that, for example, the outline generated by the line beginnings and endings traces the profile of a human head; in such cases, it may be appropriate to distinguish between the symbolic and iconic modes incorporated by the text.

¹ It may be that certain non-representational forms of painting (some of the work of Joan Miró, for example) would thereby be described as indices rather than as icons within this language: this does not, of course, diminish them.

² It is clear that symbolic text may verbally signify a virtual presence; this is often the case in novels, where scenes may be described as if witnessed by the writer/reader. In the novel, the signified 'viewpoints' are often impossible 'positions', such as inside the mind of one (or more than one) of the characters or as an outsider, impossibly witnessing an intimate dialogue; they are viewpoints, nevertheless. Under such circumstances, however, it is generally the content that is signifying the 'viewpoint' and not the mode of signification which is signifying presence (onomatopoeia may be a partial exception, but the virtual presence is, in this case, aural rather than visual). Furthermore, there is clearly a sense in which any text encodes presence insofar as it presupposes a reader; in this sense, it is the work as text which is signifying presence. To define 'codes of presence' in such all-embracing terms would clearly be unhelpful; it is reserved, here, to indicate the signification of a virtual physical presence of the reader in addition to their presence *qua* reader; the distinguishing feature of the iconic mode is that it is the mode of signification itself which is signifying a virtual visual presence.

4.5.3 *Scaling icon, index & symbol*

The intrinsic nature of the visual code of presence in the iconic mode renders it particularly appropriate for incorporation into localising strategies. Thus, a comparison between the ratios of iconic to non-iconic modes in two textbooks has face validity as a measure of the relative extent of the use of localising strategies. Such a comparison was carried out on a sample of the G and Y series of *SMP 11-16*. This work will be described in Chapter 9. In anticipation of this, I shall, here, provide a little more delicacy to the categories of signifying mode which I have introduced.

Within the category 'icon', there are different degrees of localising in terms of what might be described as interruption of the code of presence. Compare, for example, the iconic significations in Plates 4.2 and 4.3. The drawings of Nadia and her classmates in Plate 4.3 is a simple, but 'straight' illustration with no exaggeration of features and without humour. John in Plate 4.2, on the other hand, would put Cyrano de Bergerac to shame. His brother, Charlie, has yet to reach puberty, but he seems to be able to grin almost from ear to ear and is sporting an ironic 'Big C' on his T-shirt. The caricature and humour in the drawings of the industrious John and Charlie have a tendency to interrupt the code of presence by asserting the impossibility of a viewpoint. There is no such interruption in the drawings of Nadia et al, even though they are clearly drawings and not 'the real thing'. Plate 4.4 incorporates a third iconic category in the photographs of grocery items. Photographic codes—and certainly the representational photographic codes implicated here¹—signify a real presence. This real presence is in the form of the camera which is a surrogate for the author/reader. In terms of the resources available within a textbook, the photograph, as a mode of signification, minimises the interruption of the code of presence. Thus three species of icon are defined which respectively specialise the definition of icon given above as: photographic; non-photographic but representational; representational, but incorporating exaggeration of features² and/or humour. These species are referred to as photographs, drawings and cartoons,

¹ It may well be appropriate to consider a wider range of photographic codes when considering certain other empirical texts, for example, in the context of astronomical photography or of the use of photography in art.

² Clearly there is commonly a certain exaggeration and certain cases may be difficult to call: the eyes of some of the characters in the drawings of Nadia et al (Plate 4.3), for example, are reduced to dots. However, given the scale of the drawings, this is interpreted as expedient; dots (or lines) for eyes in a much larger face would constitute exaggeration.

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Plate 4.2
(G2: pages 4-5)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 4.4
(G7: page 3)

Plate 4.3
(G2: page 19)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 4.5
(G2: pages 20-1)

respectively and, taken together, constitute an ordinal scaling of 'icon' in terms of interruption of the code of presence¹.

It has already been mentioned that the spatial arrangement of symbols into tabular form is interpreted as indexical, so that there are two species of index: tables and graphs, the latter being the residual category once tables are removed. Graphs include those 'mathematical' diagrams that are generally referred to as graphs (line graphs, histograms, pie charts, etc), also maps and the various forms of projection (the 'viewpoints' for which are formally defined and do not signify virtual visual presence). Also included within the category 'graph' are geometric diagrams, including, for example, drawings of polygons and polyhedra. This is because, in mathematical terms, what is being signified is not a visually realisable object, but a formally defined construct; it is not possible within this language to produce an icon which signifies such a mathematical object. On the other hand, it is possible to produce an iconic signification of an index as is illustrated in Plate 4.5. The inclusion of the hand, pencil, ruler and rubber in the images accompanying task A4 enable a virtual visual presence to be established: this is 'me' looking down on my hands whilst drawing a regular hexagon. Otherwise, an image might, for example, present a table showing prices on 'company headed notepaper', again evoking/being evoked by an iconic code of presence; Plates 4.6 and 4.7 show two contrasting examples, the 'Dumplings' price list in Plate 4.6 constituting a drawing, the 'length of day' text in Plate 4.7 is a table².

An additional visual code may be invoked in either indexical or symbolic signification where a deviation is made from the usual 'letterpress' form. Thus, in Plate 4.8, both symbolic³ and tabular significations are realised in a form which denotes handwriting. Elsewhere, especially in the G series, headings are produced in 'hand-drawn' outline form and graphs are obviously manually drawn, etc. This use of manuscript constitutes an 'iconicising' of the symbolic/indexical text, but it is,

¹ There is a resonance, here, between what I am describing as the degree of interruption of the visual code of presence and the notion of 'modality'; see the discussion in Hodge & Kress (1988).

² Instances such as the 'Dumplings' price list are, in fact, comparatively rare and there is rarely any difficulty in distinguishing between icons and indices.

³ Tasks D1-10 in Plate 4.8 are produced as if written by a student in their exercise book. Clearly there is an invoking of iconic code, here. However, it has been decided that it is the symbolic mode which is foregrounded, here, the exercise book, in this case and generally, being represented by little more than a border. In the context of the empirical analysis, an operational decision was taken to count such text as 'symbol' unless it is iconically embellished, eg by the inclusion of a hand and/or pencil or, as on the page preceding that shown in Plate 4.8, a burnt out calculator or some other object. A similar decision was taken with regard to indexical tables represented in this manner. Since this kind of text occurs far more frequently in the G than in the Y series, this coding decision will tend to result in an underestimation of the difference in iconic prevalence between the two series.

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Plate 4.7
(Y5: page 124)

Plate 4.6
(Y5: page 42)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 4.8
(G1: page 53)

nevertheless, the symbolic/indexical mode which is foregrounded—‘reading’ rather than ‘looking at’ clearly predominates. Nevertheless, in the analysis described in Chapter 9, it was decided to quantify manuscript and non-manuscript symbols and indices separately.

The modes of signification are summarised in Figure 4.4. This classification of this category of textual resources will be used in the quantitative analysis to be discussed in Chapter 9. Before concluding this Chapter, I want to give a very brief discussion of the possibility of extending the language of description to higher levels via the dimensions of *institutional level* and *subjectivity*. As I have already stated, however, these levels will not be implicated in the empirical work of the thesis: they adumbrate future work. Their importance here, however, is in placing the analysis within a broader conception of the social.

Signifying Modes

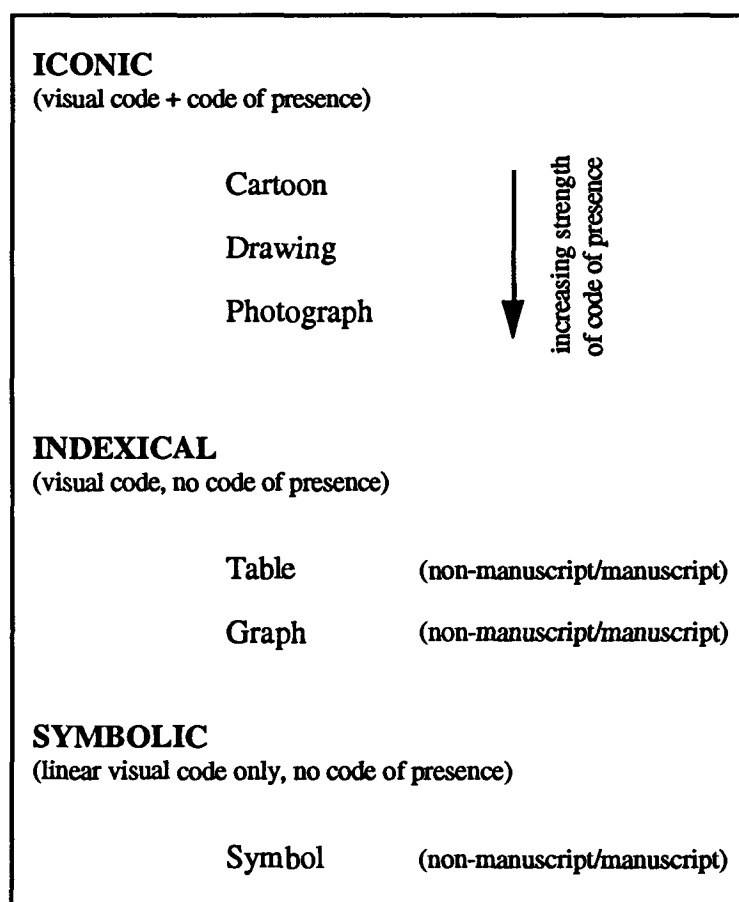


Figure 4.4

4.6 Institutional level & subjectivity

In introducing the term 'activity', I asserted that this constitutes an analytic space and that, activities are always ideologised in the empirical. An activity is defined as a specific articulation of a Global Semantic System which (re)produces (in part) a more general division of labour. Empirically, however, it is always a particular division of labour which is (re)produced: the content of an activity—its practices and subject positions—is always an expression of its position within a specific configuration. It is, therefore, appropriate to consider a higher level of analysis which addresses this positioning.

School mathematics has been described as an activity with its own rules of realisation and recognition. However, school mathematics must, in the empirical domain, exist within schooling as an institution. That is, school mathematics is positioned in relation to other school disciplines, thus (re)producing schooling¹. In conducting an analysis at this level, we might conceive of schooling as the structural level and school mathematics as the level of events. Schooling constructs practices (disciplines) and positions (teachers, students, parents) which are (re)produced by particular instances of activities (school mathematics in a particular school, and so on). Analysis at this level would (potentially) enable the consideration of the interactional.

Schooling is similarly positioned in relation to other institutions by higher order institutions which might be said to correspond to what Bowles & Gintis (1986; Gintis & Bowles, 1988) refer to as 'sites of social practice'. Analysis at this level would (potentially) enable the consideration of the historical. This methodology constructs the social as a kind of 'Russian doll' structure or alternatively, as a 'fractal' structure. The form that the analysis takes is independent of the level of the analysis². Thus the language of description might be conceived to operate, methodologically, as a lens of variable focal length, zooming in and out to produce descriptions at different levels of analysis. In this conception, there is no *a priori* highest or lowest level. Thus, whatever level is chosen, the language will always constitute an analytic space which will always be ideologised in the empirical. This constitutes the 'yet-to-be-described'

¹ In an earlier publication (Dowling, 1992a) I referred to this as 'framing'. I have decided to discard this term because of its potential confusion with Bernstein's use of the word. Such confusion would be particularly awkward in any extension of the language of description to include the interactional. In the context of such an extension, Bernstein's concept (or a recontextualisation of it) would undoubtedly be of importance.

² Fractal geometry exhibits this property which, it has been suggested, might also be applied to the description of coastlines, telegraphic transmission, and so on (Gleick, 1988).

in respect of an upward (in terms of level) orientation in the same way as the 'discursive gap' between the language and the empirical text constitutes a downwardly-orientated 'yet-to-be-described'.

Institutional level represents a variable relating to level of analysis. The dialectical relationship between institutional levels (implied by the term (re)production) raises the possibility of social transformation resulting from contradictions. Another variable is constituted by 'subjectivity'. It has already been asserted that human subjectivity is to be conceived as a multiple articulation of activities. That is, an individual human subject occupies and will have occupied many subject positions—occupational, domestic, religious, political, and so on—relating to the diverse activities in which s/he participates and in which s/he will have participated during her/his life history. Although subjectivity can occupy only one subject position at any one point in the social chronotope¹, there is no assertion, here, that subject positions are hermetically sealed one-from-another. Indeed, this cannot be the case given the existence of a public domain which is associated with each. Subjectivity is, therefore, a variable which concerns the possibility (although not the predictability) of transformation resulting from deliberate human action.

4.7 Summary

In this Chapter I have introduced the notion of a language of description and have provided an introduction to the structure and to the main terms of my particular language; the language of description will be further developed in the Chapters which follow. The main features of the language are illustrated in diagrammatic form in Figures 4.5 and 4.6. The language of description has been derived from the theoretical propositions, TP1-10, which themselves arose out of the theoretical discussion in Chapter 3. I have tried to indicate where a particular proposition is particularly relevant to a particular aspect of the language. The basic model consists of the dialectic of activity/text. Activity is the structural level of the language which constructs practices and subject positions: it regulates what can be said or done or meant by whom. Activity is (re)produced by text via three principal categories of textual strategies. These implicate repertoires of textual resources in the construction of message ((re)producing practices) and voice ((re)producing subject positions). I should emphasise again that, at the levels considered in this thesis, the language of description is designed for the sociological analysis of pedagogic texts which are

¹ Literally, 'time-space'. The term is borrowed from Bakhtin (1981) because it seems more appropriate than the English term with its connotations in physics and in science fiction.

Principal Features of the Language of Description

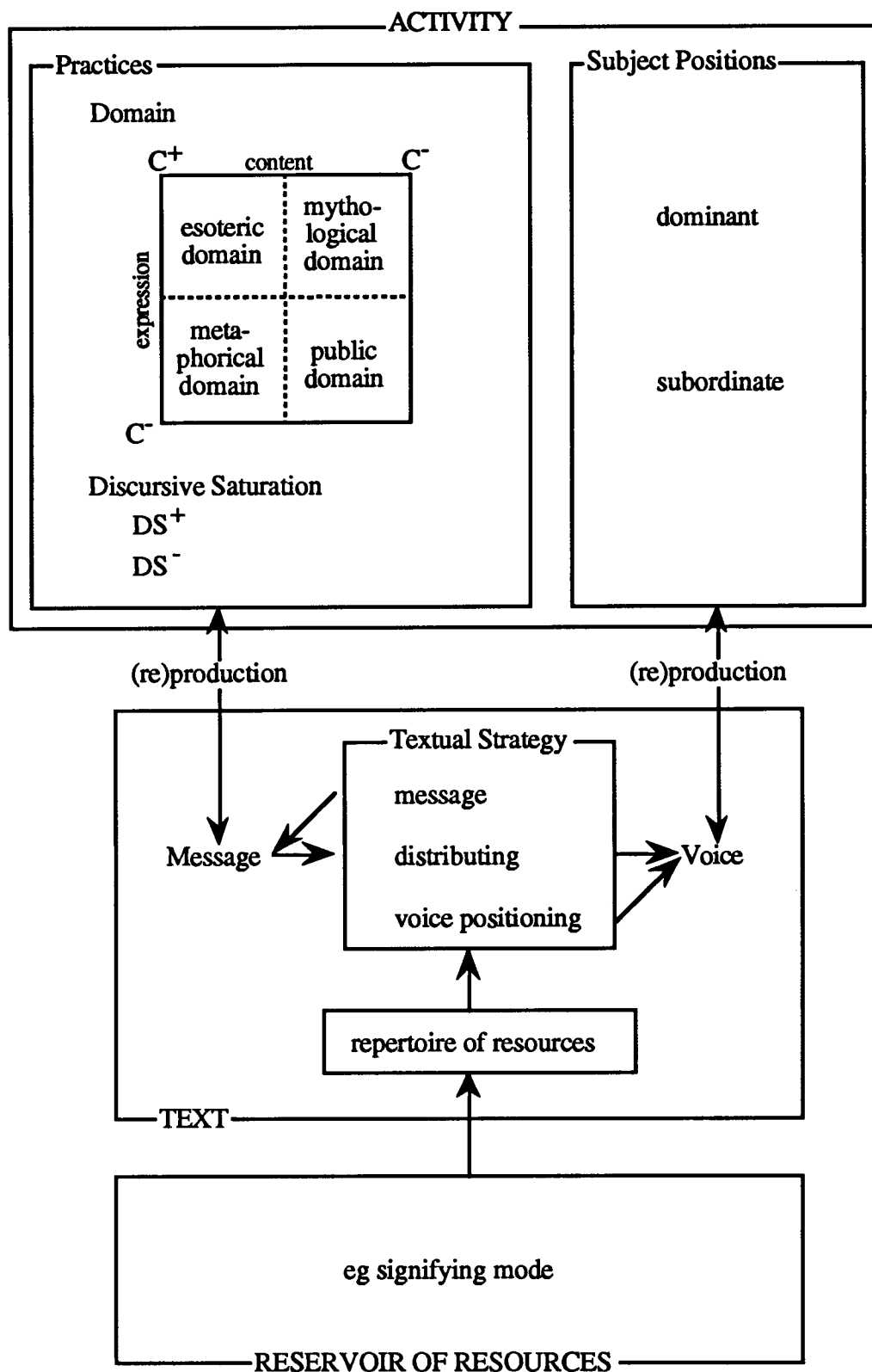


Figure 4.5

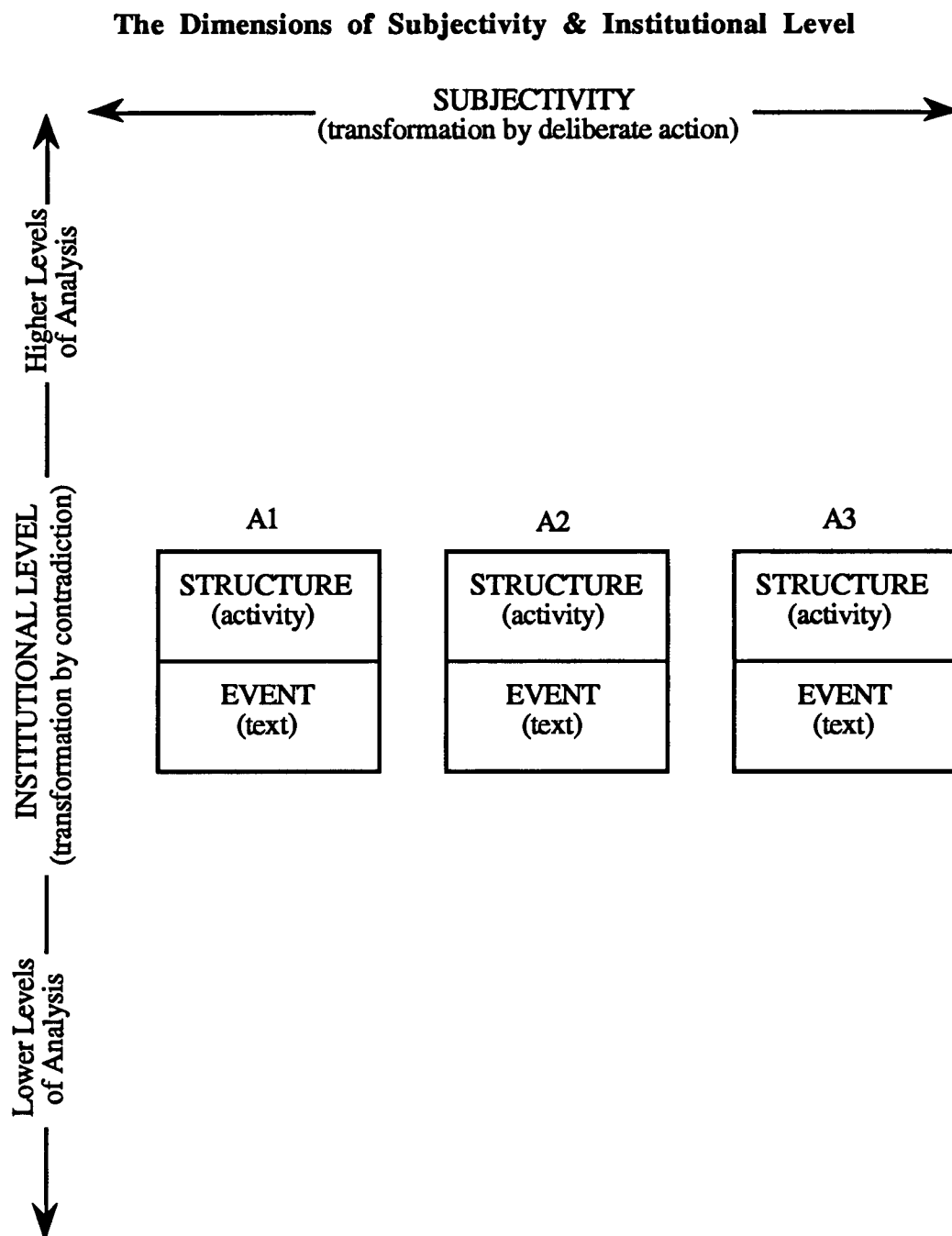


Figure 4.6

essentially closed. By the latter term I mean that the ideal readers of the texts are essentially passive in that they are exhausted by the text.

In Section 4.6, I considered, very briefly, the possibility of moving beyond the limitations the I have placed upon myself in producing an analysis of a school textbook scheme. Specifically, I considered the two variables of institutional level and subjectivity. These allow for the possibility of extending the application of the language to different levels of analysis. They also allow for the possibility of the introduction of interaction and transformation and for a historical dimension to the language. These extensions are undeveloped in this thesis. However, should these extensions be developed in a way which is consistent with the general methodology which is being employed here, it would remain the case that the language would constitute an analytic space only. Since the language of description must (within the context of academic sociology) be rendered in discursive form, it will recontextualise the social in the same way as 'programmes' might be understood to recontextualise 'technologies', in Foucault's work, by excluding the non-discursive (see Chapter 3). Institutional level and human subject, concrete as they may sound, are, nevertheless, merely analytic arenas.

In the following Chapters, I shall move to the empirical analysis itself. In Chapter 5, I shall provide some background information on the School Mathematics Project (SMP) and on its scheme, *SMP 11-16*, which is the empirical object of this thesis. I shall also report on a preliminary, high level analysis of the scheme which gave direction to the sampling of the texts. Chapters 6 and 7 comprise readings of the texts as they relate to two mathematical topics. In these Chapters, the intention has been to produce comparisons between two tracks of the SMP scheme. Thus the ordering of the readings has, to a very large extent, been dictated by the ordering of the empirical texts. My intention is also that this should give some indication of the importance of inductive processes in the genesis of the language; in this sense, Chapters 6 and 7 are intended to balance the present Chapter and Chapter 3. Chapters 8, 9 and 10, by contrast, serve as a practical and systematic introduction to the language of description as a whole and also mark out the major substantive 'findings' of the analysis. Finally, in Chapter 11, I shall conclude the thesis by considering the possibilities and limitations of the language of description and of the readings which have been produced and also directions for further work. In the final part of Chapter 11, I shall also discuss a possible form of the relationship between sociology and pedagogic practice.

Chapter 5

SMP 11-16: Introductions

In Chapter 4, I introduced the general structure of a language for the sociological description of pedagogic texts. The context for most of the discussion in Chapter 4 was the *activity* of school mathematics and, within this context, a number of direct references were made to the UK secondary school mathematics scheme, *SMP 11-16*. In the present Chapter and in the five Chapters which follow it I shall focus attention directly on *SMP 11-16*. My intention is both to illuminate the delicacy of the language and its gaze, and to produce a sociological reading of school mathematics as instanced in *SMP 11-16*. In this Chapter I shall first present a brief introduction to SMP and to the *SMP 11-16* scheme. Secondly, I shall give an introductory analysis of the SMP G and Y series of books; these books will be the focus of my analysis in Chapters 6-10. This introductory analysis will enable me to index two mathematical topics which will be discussed in Chapters 6 and 7.

5.1 *SMP 11-16: an introduction*¹

5.1.1 *The School Mathematics Project: the 'numbered' & 'lettered' books*

SMP 11-16 is the current SMP text for the eleven to sixteen age range. It is the product of the School Mathematics Project which was, according to its principal founder, conceived in the garden of Culver Lodge in Winchester at 11 a.m. on Monday 18th September 1961, at a meeting of the Heads of Mathematics of Charterhouse, Marlborough, Sherborne and Winchester and Professor Bryan Thwaites, Professor of Theoretical Mechanics at the University of Southampton (Thwaites, 1987). The meeting followed a conference, organised at Southampton by Thwaites, which was 'aimed specifically at producing an "ideal" school mathematics syllabus' (Thwaites, quoted by Cooper, 1985). The project was one of a large number of initiatives arising out of a series of conferences in Europe and in the US in the late 1950s and early 1960s. These initiatives fall under the broad heading of 'modern mathematics' or, in the US, 'new math'. In the US, the School Mathematics

¹ For further description of the development of the School Mathematics Project and of 'modern mathematics', see: Cooper (1983, 1985); Griffiths & Howson (1974); Howson (ed) (1987); Howson, Keitel & Kilpatrick (1981); Moon (1986). More generally, regarding curriculum innovation in the 1960s in Britain and in the US, see MacDonald & Walker (1976).

Study Group (SMAG) was sponsored by the American Mathematical Society, the Mathematical Association of America, and the National Council of Teachers of Mathematics (Moon, 1986). The materials developed by this group were produced largely by mathematicians and emphasised an axiomatic and highly rigorous approach (Ling, 1987; Tammadge, 1987). Events in Europe and, particularly, in the UK followed closely those in the US. Barry Cooper has described SMP as the result of negotiation:

... it required interested actors, utilizing the climate of 'crisis' resulting from the campaign on teacher supply as a major resource, to enter various arenas in order to persuade others of the 'need' for change.

[...] such actors, especially the university pure mathematicians, with their followers in the [Association for Teaching Aids in Mathematics (ATAM)], and the applied mathematicians, with their industrial allies, did have considerable success in convincing school teachers of the 'need' for change in school practice, while fending off attacks 'from below' on their own curricular practices. In both cases, the success of the university sub-disciplinary segments can be seen as having partly resulted from their having found allies outside of their own organizations. The supporters of modern algebra within the ATAM, who had come into contact with the pure mathematicians in various European-wide meetings [...] gradually made 'modern mathematics' more acceptable to many school teachers by combining it with those elements of their original mission concerned with Piagetian ideas, and proposing 'intuitive' and 'experiential' approaches to its study. The applied mathematicians found themselves able to claim that 'industry' also approved of 'modelling', numerical methods, and so on, as well as being able to derive resources of personnel and money from companies with which to further the promotion of their version of 'mathematics'.

(Cooper, 1985; pp. 230-1)

Funding was obviously very important, indeed, the initial and continuing success of SMP is, according to Howson (1987), due to its financial autonomy which was originally established through 'generous grants from Industry and the Leverhulme Foundation' (p. 7). In the US, federal funding for SMAG followed the successful launch of the Soviet Sputnik in October 1957 (Moon, 1986; Tammadge, 1987). Europe had only recently emerged from war and work by Vig (reported in Cooper, 1985) suggests that politicians and commentators in Britain were increasingly assuming that economic success depended upon the application of scientific research to industrial processes. Cooper also notes that the rapidly increasing number of mathematics graduates being employed in industry was 'the likely structural basis of the increased interest in mathematics education expressed by industrial employers in the late 1950s' (1985; p. 93).

The first SMP materials to be published were *SMP Books T* and *T4*, which focused on the final two years of the GCE (General School Certificate) O-level course. These were succeeded by *SMP Books 1-5* (often referred to as the 'numbered' books), which covered the whole of the secondary age range up to O-level. John Ling (the *SMP 11-16* team leader) notes that the most obvious difference

between the SMP course and 'traditional' courses was in the content, which was a 'distinctive blend of pure and applied mathematics' (unsurprising, in the light of Cooper's work).

On the pure side, there was a highly unified development in which the concept of function/mapping/transformation played a key role. The Euclidean approach to geometry was replaced by the study of transformations, the latter being linked with vectors and matrices. The concept of function dominated the presentation of algebra, and algebra itself was enlarged to include a concrete approach to abstract algebra. On the applied side, topics such as statistics, probability, linear programming, networks and computing were introduced.

(Ling, 1987; p. 34)

However, Ling points out that advocates of SMP also laid great stress on the experimental 'discovery' approach. Alan Tammadge, one of the three principal authors of *SMP Books 1-5*, has described his view of their task:

... we arrived at the idea of introducing new work by means of a problem whenever possible. We tried to find problems associated with children themselves, so that they would really want to solve them. Such problems are fairly easy to find, but posing them and helping with their solution is sometimes a lengthy process. We had decided to address our books to the pupil, to talk to him or her as a person and to expect interest and willingness to participate in exploration of ideas. [...]

The theory behind our writing was that pupils would work through the introductory problems on their own, perhaps for homework. The teacher would then pull their ideas together, correct misconceptions, go through difficult parts again and then leave them to practice their new understanding.

(Tammadge, 1987; pp. 25-6)

Although Tammadge recognised that some teachers were bypassing the introductory material and 'simply telling their classes how to get on with the ensuing exercises' (ibid), the authors of the materials were clearly constructing an eager and essentially independent student. This, perhaps, is hardly surprising, given that SMP was directed exclusively at 'pupils in grammar schools, independent schools, and that growing number in secondary modern schools who aspired to an O-level' (Howson, 1987; p. 3), that is to 'the top 25 per cent or so of pupils' (ibid). In fact, four of the original eight pilot schools were precisely those independent schools which were represented in Thwaites' garden meeting, referred to earlier. Indeed, it was these four heads of mathematics at prestigious independent schools, together with Professor Thwaites, who had received his own education at Dulwich, Winchester and Cambridge (Cooper, 1985), who wrote the first materials (later to become Books T and T4) for the first year pupils in these schools (Tammadge, 1987). The fact that these independent schools recruited at age thirteen explains the emphasis on the final two years of the O-level course (Howson, 1987). A breakdown of the occupational locations of participants at the Southampton Conference is given in Tables 5.1 and 5.2. Comprehensive schools, in 1961, were few and far between. However, those

schools attended by the majority of secondary students, that is, secondary modern schools, were not represented at all and girls schools were in a clear minority. The social class as well as the gender and 'ability' bias in the origins of SMP are clear.

Occupational Locations of Participants at the Southampton Conference

Occupational location	Number
University	34
College of technology	1
College of education	1
Private industry	15
Government research	3
HMI	1
Schools	73
Uncoded	2
Total	130

Adapted from Cooper (1985)

Table 5.1

Categories & Gender of Students of Schools Represented at the Southampton Conference

Category of School	Student Gender			
	Boys	Mixed	Girls	Total
Independent	26	0	1	27
Direct Grant	7	0	3	10
Grammar	12	8	8	28
Technical Grammar	1	1	0	2
Bilateral	0	2	0	2
Comprehensive	3	1	0	4
Total	49	12	12	73

Adapted from Cooper (1985)

Table 5.2

SMP Book 1 was published in 1965, the year of the first CSE (Certificate of Secondary Education) examinations. SMP responded by producing a new series, *SMP Books A-H* (the 'lettered' books), the first volume of which was published in 1968. Geoffrey Howson, the editor of *SMP Books 1-3*, notes that:

Significantly [the authors of *SMP Books A-H*] came from comprehensive schools, or from grammar or independent schools which wished to enter their average ability pupils or low attainers for the new examination. Rather than being a new course designed from

first principles for those of 'average ability', the lettered books tended to present a subset of what was found in *SMP Books 1-5*.

(Howson, 1987; p. 9)

Again, there is no inclusion of secondary modern schools. Nevertheless, Howson (writing with H.B. Griffiths) does seem to suggest, in an earlier statement, the possibility of a relationship between 'ability', as measured by the distinction between the 'numbered' and the 'lettered' series, on the one hand, and social class, on the other:

With all types of pupil, the final teaching language may have to take account of their *social* language: it is no good using the language of mandarins to the children of factory workers, as studies by teachers of English have shown. For example, the early SMP texts T and T4 were written in the language of mathematical specialists intent on getting the mathematics right. These were rewritten in the language of grammar school boys, and the resulting books 1-5 were again rewritten (with modifications) in the language of 'CSE' children, as books A-H.

(Griffiths & Howson, 1974; pp. 340-1)

In this extract, girls are, again, invisible and the identification between "CSE children" and 'the children of factory workers' is almost irresistible. Also encoded in the statement is a relationship between 'the language of the mandarins' and that of 'mathematical specialists', that is, the language of Head Masters Conference schools teachers (who wrote the materials) and, presumably, their male students.

As well as the 'numbered' and 'lettered' series, SMP material for the 11-16 age range included *SMP Cards I and II*, five booklets of supplementary questions and calculator-based supplements to the O-level material and *SMP Books X, Y and Z*, which provided a route to O-level from *SMP Book G*. The project also produced a scheme aimed at middle schools, *SMP 7-13*, an Advanced Level scheme (and associated publications on specific mathematical topics) and, against original intentions, a scheme for A-Level Further Mathematics. SMP also produced and collaborated on the production of materials for use in other countries.

I want now to consider the second main development in the 11-16 age range, which resulted in the series which is the principal focus of my analysis, *SMP 11-16*.

5.1.2 *The School Mathematics Project: SMP 11-16*

The lettered series was highly successful, becoming, for a time, the most widely used textbook series in England (Ling, 1987). Despite the production of Books X, Y and Z, however, the scheme was not considered suitable for the 'upper ability' range. Nor had it been intended for use with 'lower ability' students. The corpus of

materials for the 11-16 age group had all been derived from the original O-level course in rather an *ad hoc* fashion and, as John Ling comments, 'the whole collection was not organised in a way to make it satisfactory for comprehensive schools' (Ling, 1987; p. 39). SMP held a conference in Bristol in July 1976 to discuss the next development; Ling notes that:

Two major concerns emerged at the conference. One was a concern with the mathematical needs of the lower ability group, hitherto not catered for by the SMP material, the other was a concern with the organisation of course materials. It was felt strongly that material written for comprehensive schools should be flexible in use, allowing teachers to select and adapt to suit the needs of different groups of pupils.

(Ling, 1987; p. 39)

In September 1977, John Ling was appointed team leader, responsible for the production of the new materials. Writing teams, initially comprising between fifty and sixty people, were set up in January 1978. According to Ling, the members of the team (which diminished in size as the work progressed) were mainly teachers in comprehensive schools. They were initially divided into five writing groups, each working on a specific topic, the list of topics comprising number, algebra, graphs, space, and statistics. The team was particularly concerned with the issue of 'learning difficulties' and received input from the Concepts in Secondary Mathematics and Science (CSMS) project based at Chelsea College¹. Ling notes that the team's

... concern with learning difficulties became part of a much wider concern that mathematics should be *meaningful* to pupils. The considerable use of concrete materials and games derives from this concern, as does the extensive use of real life contexts which are authentic and which do not give rise to unnecessary 'noise' but facilitate a grasp of the underlying mathematical idea.

(Ling, 1987; p. 40)

Ling does not provide any examples of 'real life contexts', but his brief description is redolent of Tammadge's description of the principle underlying his production of the 'numbered' texts. Ling's 'real life contexts', like Tammadge's problems, which were 'associated with the children themselves', were there to facilitate mathematical knowledge. The team decided upon an organised collection of booklets for the first two years of the scheme. Illustrators and writers met to work together on the booklets and the team received specialist help in respect of readability.

The booklet scheme comprises levels 1, 2, 3 and 4, each divided into sub-levels, (a) and (b), the latter level being at a marginally higher level of difficulty, but this is

¹ The relevant report of this project is available in Hart (1981). This has been a highly influential study, boasting, on its cover, a representative sample of some 10 000 children. In fact, this claim is, arguably, disingenuous, given that the figure represents the aggregate of a large number of very much smaller samples. The study is also seriously flawed in a number of other respects, see O'Reilly (1991).

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Plate 5.1
(Level 4(b) booklet *Rotation*: front cover)

‘not so great as to prevent pupils doing some (b) before (a)’, although an order is specified in some cases (publisher’s publicity materials). Levels 2-4 each have an additional (e) or ‘extension’ level which is described in the publicity materials as intended:

... for pupils who would find little difficulty with the corresponding (a) and (b) booklets. They are not designed to be worked through as a block, but as challenging material to be used as appropriate.

Numbers & Length of SMP 11-16 Booklets

Level	8-page booklets	16-page booklets	Total pages	Total booklets
1(a)	3	5	104	8
1(b)	2	6	112	8
1	5	11	216	16
2(a)	2	9	160	11
2(b)	1	11	184	12
2(e)	3	4	88	7
2	6	24	432	30
3(a)	3	7	136	10
3(b)	2	7	128	9
3(e)	6	9	202	15
3	11	23	466	34
4(a)	6	3	96	9
4(b)	5	3	88	8
4(e)	11	7	200	18
4	22	13	384	35
Total	44	71	1498	115

Table 5.3

The booklets all have the same basic cover design (shown in Plate 5.1) and are colour-coded according to the five topics: red = number; green = algebra¹; blue = graphs; grey = space; mauve = statistics. Each booklet has either eight or sixteen pages; the numbers of booklets in each level and sub-level is given in Table 5.3. The shift in emphasis (in terms of number of booklets and number of pages) from the (a) and (b) sub-levels, in levels 1 and 2, to the (e) booklets, in levels 3 and 4 suggest that there is a widening of differentiation of students. Ling also notes that there is a

¹ It might be noted, in passing, that the colour denoting algebra, perhaps the most ‘esoteric’ of the topics, is also the colour used to denote the G series for ‘low ability’ pupils in the third and subsequent years. By and large, the covers of the G books are not, in fact, green.

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Plate 5.2
Overall Structure of the Scheme (publicity materials)

‘widening gap between the mathematical levels of the “main” and the “extension” material’ (p. 41) which militated against the production of a unitary ‘level 5’. Thus:

A split into differentiated courses was inevitable. In theory these differentiated courses could have taken the form of parallel booklet schemes rather than parallel series of textbooks. The argument in favour of textbooks was strongest in the case of the upper ability group. To be economical, a booklet scheme must contain units of work which can be done in many different orders, otherwise queuing will occur in the classroom and can only be avoided by having more copies of each unit. But as the subject progresses, topics become increasingly inter-related, and this restricts the number of possible routes through them. Also, the writers who worked on the level 4 extension booklets on algebra felt that they were reaching the limit of what could be taught effectively in algebra, to all but the ablest of pupils, through the medium of the printed page.

(Ling, 1987; p. 41)

Although the argument in favour of more conventional books rather than booklets was weaker in respect of the ‘middle and lower levels of ability’, there was some concern about overburdening schools with organisational complexity. Accordingly, it was decided to produce three parallel series of books, with the middle series branching into two after the first two books. The structure of the whole course is shown in Plate 5.2. The G series is clearly the most complex. There are eight books in the main G series compared with five in each of the B, B/R or Y tracks. The G series also includes Supplementary Booklets, Resource Packs, G Booklets, and Topic Booklets. In addition to the five main books, the B series shares the Topic Books with G, and the Y series has two ‘extension’ books (YE1 and YE2); there are no additional R materials¹.

These materials were produced by two sub-teams of writers, one working on the G books, the other on the B, R and Y books. The Teacher’s Guides list nineteen G authors and twenty-one B/R/Y authors; six individuals are on both lists. John Ling, who was a B/R/Y author as well as being overall team leader, describes some of the differences between *SMP 11-16* and its predecessors:

There are important differences in mathematical content between SMP 11-16 and the earlier SMP courses. The syllabus of the G series, which has no counterpart in earlier SMP materials, conforms closely to the ‘Foundation list’ given in the Cockcroft Report. Compared with *SMP Books A-H*, *SMP Books 1-5*, etc., the syllabus content of the other SMP 11-16 series is reduced, this being more true at the middle and lower levels of ability. Algebra is confined to the ‘traditional’ algebra of generalised arithmetic; there is no mention, except in the extension materials for the Y series, of other ‘algebras’, and no work on sets or operations on sets. The explicit concept of function is introduced much later in SMP 11-16, and then only in the Y series. The concepts of ratio and proportionality are given greater weight. In geometry, the topic of transformations does not have the central importance it had in the original course (but there is no ‘return to Euclid’). Spatial work in three dimensions is more prominent at

¹ There are also worksheets associated with each series (the number of worksheets decreasing in the sequence G, B/R, Y) and more recent publications on specific topics, for example, including work on mathematics and microcomputers.

all levels of ability. Matrices were included in the draft version of the Y series, but were later omitted. Some other innovations of the original course remain, such as work on probability and statistics, the latter being taken further to include practical work on sampling.

(Ling, 1987; p. 43)

Two of the members of the G team (Afzal Ahmed and John Hersee) were also members of the Cockcroft Committee of Inquiry, the report of which was published in 1982, three years before the publication of Book G1. The 'Foundation list', referred to in the above extract was intended by the Cockcroft Committee to form a part of the curriculum for all students and 'by far the greater part of the syllabus for those pupils for whom CSE is not intended, that is those pupils in about the lowest 40 per cent of the range of attainment in mathematics' (Cockcroft, 1982; p. 134). This is the group targeted by the G series. The Committee also noted that:

... as has been pointed out in official publications of various kinds over many years, formal algebra is not appropriate for lower-attaining pupils.

(Cockcroft, 1982; p. 141)

Although it was believed 'that efforts should be made to discuss some algebraic ideas with all pupils' (ibid). Algebra is marked out in two of the extracts from Ling's paper which are quoted above. Ling argues that algebra 'is not needed in everyday life' (1987; p. 42) and that the 'return', in terms of wider applicability, on effort in algebraic manipulation is not apparent until one has mastered a number of techniques, so that the applicability of these techniques is unlikely to be immediately apparent. The principle that was employed in respect of algebra in the production of the *SMP 11-16* books is described in the following terms:

As it is not known for certain which pupils will need algebra and which will not, and as it appears that algebraic skill needs to be built up over a period of time, the only safe policy seems to be to include algebra in the course to the extent that pupils can succeed at it, and to accept that it is likely to be unnecessary for many of them.

(Ling, 1987; p. 42)

The singling out of algebra by Ling and by the Cockcroft Committee suggests that its treatment in *SMP 11-16* is worthy of some consideration in the present study. This is further supported by the analysis in the next section of this Chapter.

The G series was targeted at a group of students who were not expected to take either GCE O-level or CSE examinations. Ling states that:

Working outside the framework of the public examination system gave the G series team the freedom to design a graduated assessment scheme for its target group.

(Ling, 1987; p. 43)

The resulting assessment scheme looks very different from a conventional CSE or O-level examination and includes practical, oral and mental tests as well as written papers. The scheme is certificated by the Oxford and Cambridge Examinations Board. The summative assessment for students following the B, R and Y series was within the mainstream public examination system, and a joint GCE/CSE 16+ syllabus was set up for first examination in June 1985. This scheme also differed from other mathematics examinations at this level in that it included a thirty per cent coursework element. Most recently, two National Curriculum GCSEs have been approved by the Schools Examination and Assessment Council (SEAC): SMP 11-16 and SMP Graded Assessment.

Draft *SMP 11-16* materials were trialed in approximately thirty pilot schools from September 1980 following pre-draft trialing in authors' own schools. Following revision, the scheme proper began being used in schools in September 1983. The first 16+ awards were in 1985 and the first GCSE awards in 1989. Currently, *SMP 11-16* is by far the most popular secondary mathematics text in the United Kingdom, being used, according to the publishers (Cambridge University Press, private communication) by more than fifty per cent of schools. A National Curriculum Council Report (NCC, 1991) found that 48.5% of schools sampled used the scheme and that the next most popular scheme was used in only 8% of the sample schools. There is also a Welsh language version of the scheme and a Dutch scheme which is based on the English materials. There is some considerable diversity in the form of current secondary mathematics texts, ranging from the workcard-based SMILE¹ scheme to the more 'traditional' *ST(P) Mathematics*, published by Stanley Thorne. Nevertheless, the popularity of *SMP 11-16* clearly lends credibility to the assertion that it is representative of a dominant voice in school mathematics.

Having given a brief discussion of the background and structure of the *SMP 11-16* materials, I shall move on to an introduction to their analysis.

5.2 The analysis of the texts: sampling the materials

The language of description which was introduced in Chapter 4 is designed to reveal the nature and distribution of practices with respect to subject positions within the context of an activity, in this case school mathematics. This structure of an activity is (re)produced through its texts in the (re)production and distribution of message and

¹ Secondary Mathematics Individualised Learning Experience: an Inner London Education Authority (ILEA) scheme which first appeared in the early 1970s.

the positioning of voices. A decision has been taken to analyse *SMP 11-16* because of its singular popularity within UK secondary school mathematics. The principal method of analysis which is to be employed is, as has already been announced, semiotic. This form of analysis entails the elaboration of denotations and connotations incorporated into the text via a detailed reading. The total number of pages in the Levels 1-4 booklets and the four series of books (G, B, R and Y including the Teacher's Guides and the two extension books), YE1 and YE2 is 4918. In addition to this, there are ancillary materials and Guides associated with the booklets and the additional G and B materials shown in Plate 5.2. Clearly, a detailed semiotic analysis of such a large body of text is not possible within the constraints imposed upon a study of this kind. It is therefore necessary to sample the material.

I have decided to focus attention on the G and Y series of books, because it is in the division between these two series that differences in the structuring of message in relation to voice will be most explicit. Whereas the booklet scheme acknowledges differences between students in terms of pacing, many individual booklets (all those in Levels 1 and 2 with the exception of 2(e)) are intended to be attempted by all students and all of the (a) and (b) booklets in Levels 1 to 3 are intended to be attempted by the majority of students. At the start of the third year of the course, however, the scheme effectively divides students into G-students, B-students and Y students. The greatest differentiation within the mainstream should therefore be visible through a comparison of the G and Y series¹. Even with this restriction, however, there remains a need to sample. I have decided, firstly, to concentrate my attention on the main series of books in each case, that is, books G1-8 and Y1-5, together with the relevant Teacher's Guides. I have decided to organise the analysis such that it both illustrates the language of description and reveals the 'findings' of the reading of the SMP texts that the language produces. My intention is to begin with a general overview of the two series in terms of the domains (esoteric, public, mythical, metaphorical) within which the text is located. This analysis will be discussed below. The findings will index two areas of the texts that will be the focus of Chapters 6 and 7.

¹ It might be argued that it would be useful to make a comparison between the Y and B books, since these were produced by the same team of authors, whereas only a subset of this team worked on the G books. However, the intention of this analysis is not to retrieve the intentions of the authors. It is the activity of school mathematics generally which gives authority to *SMP 11-16* within which scheme this authority is to be revealed.

5.2.1 Isolating domains

I shall use the expression *textual time* to refer simply to a measurement of quantity of text or page space. The term 'time' is preferred to 'space' because it suggests a one-dimensionality and a uni-directionality, connoting passage through a text. This kind of passage is proposed by the G and Y books both within and between their respective chapters via, for example, the numbering of chapters and sections. The intention of the analysis in the present Chapter is to obtain a relatively coarse picture of the amount of textual time spent within the four domains, esoteric, public, mythical and metaphorical. This will give an initial comparison of the two series and between mathematical topics. The comments made by John Ling in relation to algebra, for example, suggest that we might expect to find that this topic is predominantly within the esoteric domain. The analysis in this Chapter will give some indication as to whether or not this is the case in respect of each series of books.

The books were analysed by ascribing domains to sections of text. The recognition rules associated with each domain are as follows.

i) *Esoteric domain (E)*

The significations carried by a particular section of text (ie excluding connectives etc) are mathematically specialised in terms of both expression and content, that is, they are strongly classified with respect to the non-mathematical.

ii) *Public domain (P)*

Significations are not mathematically specialised in terms of either expression or content, they are weakly classified with respect to the non-mathematical. The significations may relate to other specialised discourses (such as other school or professional activities) or to popular discourses (for example, relating to domestic settings).

iii) *Mythical domain (My)*

The text incorporates mathematically specialised expressions within predominantly public domain settings. That is, these expressions are strongly classified with respect to the non-mathematical, whilst the associated content is, contextually, less strongly classified.

iv) *Metaphorical domain (Me)*

The text incorporates expressions which are not mathematically specialised within settings which are predominantly esoteric domain. That is, the expressions are weakly classified with respect to the non-mathematical, whilst their content is more strongly classified.

Each chapter in each G and Y book was analysed in terms of the movement of the text through the four domains. I decided to omit the tasks that appear between the G chapters (and, very occasionally, between the Y chapters) and the reviews as these are often not easily associated with a single mathematical topic. To illustrate the procedure, I will describe the analysis of section A of chapter 2 in Book Y2 (Y2.02), 'Accuracy'. This section is reproduced in Plates 5.3-5. The first section of the chapter opens with a discussion about human populations, a public domain setting, but incorporates number lines and the terms 'interval' and 'interval approximation' in their description. The text is, therefore, mythical domain. The text moves on (task A5, Plate 5.4) to a new public domain setting concerning manufacturing tolerances, but retains the mathematical expressions and so remains within the mythical domain.

As the text moves from task A6 to A7 (Plate 5.5) it moves out of the mythical domain. The sub-heading, 'tolerances', remains, to a degree, present in the content of these tasks by virtue of their inclusion under it. However, there is no explicit reference to machines or to manufacturing in task A7, and the 'intervals' in this task are not dimensioned. In other words, A7 and the subsequent task, A8 are concerned with numbers rather than with quantities. There is, therefore, a shift in task A7 away from a local public domain setting. It is appropriate, therefore, to refer to tasks A7 and A8 as esoteric domain text.

Finally, the section moves back into the mythical with tasks A9 and A10, which concern metal blocks, which are not mathematical objects. We can describe the *trajectory* of this section by the following expression:

$$\text{My} \rightarrow \text{E} \rightarrow \text{My}$$

The analysis proceeds for the rest of the chapter, yielding the results shown in Table 5.4, in which the trajectories for each of the sections, A-D, in the chapter have been delineated on separate rows. This was repeated for each chapter, generating 75 tables for the Y series and 59 for the G series; the Tables are included as Appendix 2. These tables together describe, very roughly, the domain topography of the two

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Plate 5.4
(Y2: page 13)

Plate 5.3
(Y2: page 12)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 5.5
(Y2: page 14)

series. It may be noted at this stage, that the incidence of metaphorical domain text is comparatively rare in both series, occurring in less than one-third of chapters in each series and only occasionally in each of these chapters.

Domain Topography in Y2.02, 'Accuracy'

Chapter Y2.02: 'Accuracy'							
Section	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
A	My	E	My				
B	My	E	My	E			
C	My	E	My	E	My	E	My
D	E	My	E	My	E	My	

Table 5.4

Esoteric Domain Time in SMP 11-16 Y Books

Book	Total number of chapter pages	Total number of esoteric dom. pp.	% of text in esoteric domain
Y1	129	47.25	36.6
Y2	142	53.5	37.7
Y3	136	52.0	38.2
Y4	143	73.5	51.4
Y5	125	68.0	54.4
Y series overall	675	294.25	43.4

Table 5.5

Esoteric Domain Time in SMP 11-16 G Books

Book	Total number of chapter pages	Total number of esoteric dom. pp.	% of text in esoteric domain
G1	44	6.25	14.2
G2	49	9.0	18.4
G3	47	7.75	16.5
G4	49	2.0	4.1
G5	50	5.5	11.0
G6	52	0.5	1.0
G7	51	1.75	3.4
G8	48	2.5	5.2
G series overall	390	35.25	9.0

Table 5.6

The topographies illustrated by the 134 tables mentioned above are clearly incomplete in that they give no quantitative indication of the textual time spent within each domain. A problem arises over the operational distinction between the mythical and metaphorical domains. This distinction rests on a judgement as to the predominance of the esoteric domain or the public domain in relation to the setting¹. However, a principal purpose of the analysis in this Chapter is heuristic. The particular heuristic value concerns the distinction between esoteric domain and non-esoteric domain text and is less dependent upon whether non-esoteric domain text is mythical, metaphorical or public. The loss of reliability relating to this difficulty is, therefore, not considered to be crucial. The decision was thus taken to 'quantify' textual time in terms of the proportion of textual time within the esoteric domain. A rough measure of the proportion of textual time spent inside the esoteric domain was achieved by estimating the amount of page space (in units of one quarter of a page) in the esoteric domain. This measurement was carried out with respect to each of the 134 tables, that is, with respect to each individual chapter in the two series. The results are collated for each book in Tables 5.5-6.

Before discussing the findings of this analysis, I shall give some consideration to two problems relating to validity and reliability.

5.2.2 Problems of validity and reliability

There are clearly some difficulties relating to the validity and reliability of these tables because of the complexity of the embeddings of expressions and contents. Firstly, this form of reading of the text constitutes an implicit, that is, untheorised punctuation of the text in terms of where a particular section of text begins and ends. There is, in other words, a problem in deciding upon what constitutes a unit of text. This problem arises because, in the form of analysis being adopted in this Chapter, it is necessary to assign sections of text to single domains. In the forms of analysis presented in Chapters 6-10, there is generally no such requirement and so the problem does not arise. I will illustrate the operational rules which have been applied here by considering a specific example. Thus, consider the domain assignment of the following extract:

For example, suppose you are paid £24 for 7 hours' work. You want to know how much you are being paid for each hour, so you divide £24 by 7.

(Y1, p.1)

¹ Very occasionally, the Tables in Appendix 2 include the coding My/Me. This indicates a balance between esoteric and public domain in terms of setting.

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 5.6
(Y1: page 1)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 5.8
(G3: page 47)

Plate 5.7
(G3: page 46)

Up to the final clause ('so you divide £24 by 7') the text seems to be public domain. This final clause, however, introduces a syntactical precision that is consistent with the esoteric domain, but not with this public domain setting. In other words, the final clause in this extract introduces a relatively strongly classified form of expression into a relatively weakly classified setting, so that it is mythical domain. The decision, then, is whether to register the above extract as public domain, up to the final clause, and then mythical domain, or whether to assign the whole extract to the mythical domain.

The SMP texts are composed of (apart from titles and subheadings etc) sections of exposition (which generally constitute the reader as passive) and tasks (which activate the reader). Both expositions and tasks may incorporate pictures and diagrams etc. The general rule that has been adopted is that a unit of text is taken to be a complete task or a complete section of exposition. However, exceptions to this rule are made where an exposition or task incorporates a clearly marked semantic break. The above extract is embedded in the exposition shown in Plate 5.6 (up to task A1). The paragraphing and the use of (red) borders emphasise semantic breaks which constitute domain shifts between esoteric and mythical domain (after the first paragraph) and between mythical and esoteric (at the first bordered section). The first paragraph of the exposition is a general esoteric domain statement about the use of calculators. The second paragraph constitutes a move from the general to the particular by inaugurating a public domain setting. The bordered section moves back into the esoteric by dropping the dimensions (£) and thus abstracting a number from a quantity. However, it is the emphasis that is placed on these shifts through the use of indices (paragraphing, bordering) that justifies the decision to break the exposition for the purposes of trajectory analysis.

A second problem arises because the esoteric domain is accessed only through its realisations in the text. That is, the esoteric domain can neither be operationally defined *a priori*, nor, because of its extensiveness, can its possibilities be listed exhaustively. The esoteric domain must be elaborated in the reading of the text. However, this part of the analysis of the SMP texts is intended to assist in providing a general picture of an extensive text. This would be impossible using a closer reading such as that included above in the exposition on section A of Y2.02. Since the analysis in Chapters 6-10 will incorporate much closer reading of the text, it is felt that some play can be allowed in terms of validity and reliability, here. The results of this Chapter are, in other words, to a certain extent triangulated by the finer grained

analysis in the Chapters which follow. Nevertheless, two points of clarification will be made in relation to difficulties that arose during the course of the analysis.

The first difficulty concerns the distinction between number and quantity. The section of exposition reproduced in Plate 5.6 incorporates, as has been mentioned, an abstraction of a number from a quantity, 3.4285714 from £3.4285714. The distinction between number and quantity (or between number as noun and number as adjective) is an important point of splitting between the esoteric and the non-esoteric. Thus, for the purposes of trajectory analysis, the general rule is applied that the inclusion or exclusion of dimensions will effect a shift from esoteric to non-esoteric or vice versa, respectively. Clearly, however, there are occasions when quantities are incorporated into esoteric domain text, specifically, in the topic of geometry, where number is used to quantify lengths and angles, etc. Thus, degrees and centimetres occur within esoteric domain text. However, other units of length are generally taken to indicate a non-esoteric domain referent¹.

A second difficulty arose over the labelling of figures. The labelling of a length using arrows is read as a specialised mathematical expression (although this may be somewhat weakened where the units are not centimetres), but it is a specialism which is shared with certain other specialised messages within, for example, engineering. The general rule is that the inclusion of such dimensioning shifts an otherwise public domain text into the mythical (unless it is consistent with the non-mathematical setting). Thus, the text in Plate 5.7 is public domain, that in Plate 5.8 is mythical domain. The mathematical labelling of an otherwise public domain object with letters (to denote vertices etc) also constitutes a shift into the mythical.

I shall now proceed to a discussion of the findings of this analysis.

5.2.3 Findings

From the analysis we can describe a coarse measure of the relative *visibility* of the esoteric domain in each of the Y and G series. That is, more than 40% (in terms of chapter time) of the Y text, but less than 10% of the G text is within the esoteric domain. Furthermore, the Y series increases its proportion of esoteric domain text in that the figure for the first three books in the series is a little below 40%, whilst books

¹ Geometrical generalisations are independent of absolute dimensions and centimetres are conventionally used, presumably because of the size of the textbook page. The G series sometimes uses centimetres and millimetres rather than the decimal form of centimetres.

Y4 and Y5 are both above 50%. The G series, on the other hand, decreases its esoteric domain representation: the average for the first five books in the series is 12.8% and that for the final three books is 3.1%. The suggestion is, then, that the esoteric domain is relatively visible in the Y series and that it becomes more visible in the later books, whilst it is relatively invisible in the G series, becoming even more invisible as the series progresses. These progressions towards and away from the esoteric domain are consistent with the proposition that the Y series constitutes an apprenticing text, whilst the G series is alienating, the apprenticing and alienation increasing with progression through the course of books.

The amount of esoteric domain time is not evenly distributed between the chapters within each series. The distribution for the Y books is shown in Table 5.7. This information suggests a polarising between two sets of chapters. 24 Y chapters comprise 10 per cent or less esoteric domain. This is less than or approximately equal to the mean esoteric domain text per G series book (9.0 per cent). 19 Y chapters comprise more than 80 per cent esoteric domain text. This amount is more than the approximate minimum of non-esoteric domain text in a G book (81.6 per cent in G2). The Y chapters having minimum esoteric domain time are listed in Table 5.8 and those having maximum esoteric domain time are listed in Table 5.9.

Esoteric Domain Time in Y Chapters

% of text in esoteric domain	No. of chapters (n = 75)
0-10	24
11-20	3
21-30	8
31-40	3
41-50	3
51-60	5
61-70	5
71-80	5
81-90	9
91-100	10

Table 5.7

Y Series Chapters with Minimum Esoteric Domain Time

Chapter	Chapter title	% of text in esoteric domain
Y1.02	Loci	9
Y1.08	Investigations	0
Y1.10	Percentage	6
Y1.12	Probability	3
Y2.04	Rates	0
Y2.07	Investigations (1)	0
Y2.08	Distributions	10
Y2.10	Points, lines and planes	7
Y2.15	Investigations (2)	0
Y2.16	Periodic graphs	0
Y2.17	Probability	3
Y3.04	Percentage (1)	0
Y3.06	Investigations (1)	0
Y3.07	TV programmes survey	0
Y3.11	Percentage (2)	0
Y3.12	Investigations (2)	0
Y3.15	Problems in planning	0
Y3.17	Distributions	0
Y4.01	Selection and arrangements	0
Y4.08	Types of proportionality	8
Y4.13	Optimisations	0
Y4.14	Sampling	8
Y5.02	Optimisations	0
Y5.06	The Earth	0

Table 5.8

Of the 19 chapters represented in Table 5.9, 11 explicitly concern algebra¹ (and, of these 11, only 2 enter the metaphorical domain). On the other hand, there are only two chapters in the entire G series which relate explicitly to this topic (chapters G1.03 and G6.01, both titled 'Formulas'). The G chapters remain almost entirely outside of the esoteric domain apart from a section of less than a half of a page in G6.01. This section relates to the equivalence of different forms of expression of the arithmetic operation 'division' and not specifically to algebra. The high representation of the topic 'algebra' in Table 5.9 marks it out as a key topic of the esoteric domain. This is consistent with the singling out of this topic by John Ling which was mentioned earlier in this Chapter. The status of this topic is also confirmed by its virtual exclusion from the G series other than in the context of non-esoteric domain text. There are a number of other chapters in the Y series which signal algebra in their titles, but which are less confined within the esoteric domain. These chapters, together with the 11 algebra chapters from Table 5.9, are listed in Table 5.10. As is

¹ These chapters are: Y2.05, Y2.09, Y2.11, Y4.03, Y4.05, Y4.09, Y4.15, Y4.17, Y5.03, Y5.05, Y5.07.

illustrated by the information in this Table, the algebra chapters exhibit a wide spread of esoteric domain content, one (but only one, chapter Y4.08) also being included in Table 5.8 which lists the Y chapters having minimum esoteric domain text. We might infer from this (on the basis of titles only, at this point) that, in the Y series, not only is the esoteric domain in relation to algebra highly visible, but so is the gaze projected from or introjected to this topic. This is crucial. Algebra may be glossed as the topic in school mathematics concerned with the structure of systems which is explored and described through the introduction of variables. Algebra, in other words, is concerned with objectification, it comprises what might be described as a language of description (and is described as a 'language' in Y1.03) which facilitates a degree of context independence¹. The other topics represented in Table 5.9 are: trigonometry (2 chapters); vectors (2 chapters); other geometry (2 chapters); and sequences (2 chapters).

Y Series Chapters with Maximum Esoteric Domain Time

Chapter	Chapter title	% of text in esoteric domain
Y2.05	Algebraic expressions	93
Y2.09	Re-arranging formulas (1)	83
Y2.11	Re-arranging formulas (2)	100
Y2.13	Area	88
Y3.13	Right-angled triangles	87
Y4.03	Linear equations	100
Y4.05	Algebraic fractions	86
Y4.06	Vectors	94
Y4.07	Sequences (1)	92
Y4.09	Manipulating formulas	88
Y4.11	Sequences (2)	100
Y4.15	Functions	89
Y4.16	Three dimensions	81
Y4.17	Quadratic functions and equations	85
Y5.03	Algebraic fractions	100
Y5.05	The sine and cosine functions	100
Y5.07	Equations and graphs	89
Y5.09	Iterations	98
Y5.12	Vector geometry	97

Table 5.9

In contrast to algebra, the chapters signalling (by their titles) the compound topic 'probability and statistics' are almost all listed in Table 5.8, which comprises those chapters having not more than 10 per cent of esoteric domain text². These 7 chapters

¹ Arithmetic, of course, also does this, but in a much more limited way.

² These chapters are: Y1.12, Y2.08, Y2.17, Y3.07, Y3.17, Y4.01, Y4.14.

are supplemented in the Y series by just one more, chapter Y4.10 which falls only just outside of the criterion for inclusion in Table 5.8, having only 11 per cent of esoteric domain time. However, the whole of the esoteric domain text in Y4.10 concerns the addition of fractions, and is not explicitly related to probability. Thus, all 8 chapters concerned with statistics and probability effectively fall within the category of minimum (not more than 10 per cent) esoteric domain time. Statistics and probability are, it seems, presented without theory. These topics represent, in a sense, the limits of visible pedagogy in the Y series. The mathematical objects that are projected onto (or introjected from) the empirical world are never elaborated so that the text may appear to be concerned with essential properties of that world, but not with mathematics.

Also included in Table 5.8 are all 5 chapters headed 'Investigations' and all 3 headed 'Percentages'. These 8 chapters are in the first three books in the Y series. There are 2 chapters concerning rates and proportion, 2 concerning geometry and 1 concerning graphs, all of which topics appear in other chapters having greater esoteric domain visibility. There are 4 chapters, 'Problems in planning', 'Optimisations' (two chapters) and 'The Earth', the titles of which index the public domain.

Tables 5.7-10 illustrate the heuristic aspect of this part of the analysis by pointing to specific areas of the scheme for closer reading. Algebra appears in the scheme to be of particularly high status and is prevalent in those Y chapters which are substantially esoteric domain. Statistics and probability, investigations and percentages are reproduced substantially outside of the esoteric domain.

The distribution of esoteric domain time in the G books is shown in Table 5.11. Of the 59 G series chapters, only 23 enter the esoteric domain. The amount of textual time in the esoteric domain, in these 23 chapters, ranges from 2.3 per cent (G1.01) to 66.7 per cent (G1.06 and G2.04). These 23 chapters are listed in Table 5.12. As is clear from Table 5.12, the chapter titles in the G series are sometimes rather more ambiguous than the Y chapter titles in terms of their indexing of esoteric domain topics. In deciding upon which topics are represented, it was therefore necessary to refer to the chapter contents. The topics represented in the 23 G chapters are listed in Table 5.13.

The text represented in Table 5.13 constitutes only 9 per cent of the G series chapter time, 35.25 pages in all. Nevertheless, more than one-third of this time is concerned with arithmetic operations and more than one-sixth with the representation

Y Series Algebra Chapters

Chapter	Chapter title	% of chapter in esoteric domain
Y1.03	The language of algebra	43
Y1.09	Brackets	80
Y2.01	Relationships	36
Y2.05	Algebraic expressions	93
Y2.09	Re-arranging formulas (1)	83
Y2.11	Re-arranging formulas (2)	100
Y2.12	Proportionality	38
Y2.14	Linear equations and inequalities	35
Y3.02	Linear relationships	67
Y3.08	Direct and inverse proportionality	25
Y3.16	Linear equations	55
Y4.03	Linear equations	100
Y4.05	Algebraic fractions	86
Y4.08	Types of proportionality	8
Y4.09	Manipulating formulas	88
Y4.15	Functions	89
Y4.17	Quadratic functions and equations	85
Y5.03	Algebraic fractions	100
Y5.05	The sine and cosine functions	100
Y5.07	Equations and graphs	89
Y5.11	Inequalities	67

Table 5.10

Esoteric Domain Time in G Chapters

% of text in esoteric domain	No. of chapters (n = 59)
0-10	43
11-20	5
21-30	5
31-40	3
41-50	1
51-60	0
61-70	2
71-80	0
81-90	0
91-100	0

Table 5.11

G Chapters Incorporating Esoteric Domain Time

Chapter	Chapter title	% of text in esoteric domain
G1.01	Estimating and scales	2.3
G1.05	Chains	28.6
G1.06	Calculate ...	66.7
G2.03	Working backwards	37.5
G2.04	Polygon patterns	66.7
G2.05	Further beyond the point	16.7
G2.06	Button pressing	3.1
G2.07	Percentages	11.1
G3.02	Percentage scales	41.7
G3.03	Area of rectangles	36.4
G3.05	Carrying on	21.4
G4.02	Thousandths	29.2
G4.05	Long and short numbers	5.0
G5.02	Times 10, times 100	28.1
G5.04	Dividing by 10 and 100	35.0
G5.06	Percentages and your calculator	25.0
G6.01	Formulas	4.2
G6.08	Averages (2)	8.3
G7.02	Sixteenths and all that!	5.0
G7.03	Circles	18.8
G8.01	Calculating well	20.0
G8.03	Large numbers	16.7
G8.08	Cones, cylinders and spheres	10.0

Table 5.12

G Series Topics Involving Esoteric Domain Time

Topic	No. of chapters	No. of pages	% of chapter time
arithmetic operations	8	13.0	3.3
representation of number	6	6.75	1.7
geometry	3	6.0	1.5
percentages	3	5.0	1.3
area	1	4.0	1.0
statistics	1	0.25	0.1
linear measurement	1	0.25	0.1
Total	23	35.25	9.0

Table 5.13

of number (mainly decimal representation). Percentages and statistics are also included here: these topics are amongst those with minimum esoteric domain time in the Y series, although 2 of the Y chapters on statistics and probability have comparable esoteric domain time in the respective chapters (10 per cent of Y2.08 and

8 per cent of Y4.14 compared with 8.3 per cent of G6.08). Algebra is completely absent from Table 5.13 as are vectors, sequences and trigonometry: all of these are represented in Table 5.9 as amongst those topics with maximum esoteric domain space in the Y books. There is evidence, then, that a number of those topics which involve the most esoteric domain time in the Y books do not occupy esoteric domain time in the G series. Correspondingly, percentages and statistics, which occupy very little esoteric domain time in the Y series, are amongst those maximising esoteric domain time in the G books. This chiastic prioritising in the Y and G schemes is another consistency with the description of the former scheme as apprenticing and the latter as alienating with respect to esoteric mathematics: where the Y books apprentice, the G books alienate; where the Y books alienate, the G books apprentice.

5.2.4 Summary

This quantitative analysis has limitations in terms of validity and reliability, as has been noted earlier. In addition, the distance of the analysis from the substance of the text must also place limitations on the delicacy of the description that it produces. Nevertheless, it does reveal a considerable difference between the Y and G series in terms of the textual time spent in the esoteric domain. More than 40 per cent in the Y books, but less than 10 per cent in the G series is esoteric domain. In other words, 90% of the G series concerns or is expressed in terms of something other than mathematics. The esoteric domain is all but invisible in the G series. Furthermore, the proportion of textual time in the esoteric domain increases as the Y series progresses, whilst it decreases in the G series, so that there is what might be described as a general textual career towards the esoteric domain in Y and towards the public domain in G. Finally, certain topics are marked in terms of their high degree of representation within the esoteric domain in the Y books; algebra, in particular, stands out in this respect. Other topics, notably, probability and statistics and percentages, minimise esoteric domain time within the Y books. In terms of their representation in the G books, however, there is a degree of reversal of the relationship between topic and domain. Whilst it is the case that the absolute amount of esoteric domain time is very small, statistics and, particularly, percentages occupy rare esoteric domain time within the G series, whilst algebra is entirely absent from this domain. Certain topics occupying maximum esoteric domain time within the Y series, notably, vectors, sequences and trigonometry, are excluded from the G series altogether.

This comparison of the two series of books has revealed evidence of a reversal of priorities in terms of domain, both in aggregate time and in respect of career, and in

terms of particular topics. These distinctions between the series are consistent with the claim that where the Y series is apprenticing, the G series is alienating.

5.3 *SMP 11-16: concluding introductions*

The purpose of this Chapter has been to provide some background information on the School Mathematics Project and, in particular, to its current main secondary school mathematics text, *SMP 11-16*. In this introduction, the middle class and masculine base of at least the early activities of the project are made quite apparent. The comments by John Ling on the *SMP 11-16* materials also mark out algebra as mathematically important but, in some sense, 'difficult'.

I have also introduced an initial coarse grained analysis of the Y and G materials which has begun to characterise these texts in relation to the language introduced in Chapter 4. This analysis has suggested that certain topics are worthy of closer analytic scrutiny; these include algebra, the topic highlighted by Ling. There remains a need for some limited selection and I have decided to look at the introductory chapters in Algebra, in Chapter 6, and Probability, in Chapter 7, these topics being at opposite ends of the range of esoteric domain priority within the Y text, which clearly relates to the dominant voice in school mathematics. The other topics which are highlighted in this Chapter will be touched upon in the analysis in Chapters 8-10.

Chapters 6 and 7 will comprise close and comparative readings of the whole of the first chapter in each series of books which explicitly concerns the relevant topic. Since the results of the analysis are dependent upon the particular contents of the selected text, it is to be expected that there will be a certain amount of repetition in the two Chapters which follow. However, I have felt that it is important to include both Chapters, for three reasons. Firstly, they are concerned with topics which are treated differently by the Y text, as has been indexed in this Chapter. Secondly, because they also explore rather different features of the language of description and of the SMP texts. Thirdly, the analysis in these Chapters takes advantage of a comparatively rare opportunity to compare the same topic treated differently by the Y and G series.

Algebra in *SMP 11-16*

This Chapter and the one which follows it are concerned with close and comparative readings of sections of the Y and G texts which were identified in Chapter 5. My intention is, firstly to explore the ways in which the text is constituted by a 'weaving together' of the textual strategies introduced in Chapter 4. I intend, secondly, that this form of exposition will begin to 'socialise' the reader into the language of description. There is a sense, in other words, in which the language is being presented as 'emerging' from the text. This is also intended to represent the importance played by analytic induction in the production of the language, thus balancing the heavily deductive form of Chapters 3 and 4. Finally, the topics which are discussed in this and the following Chapter—Algebra and Probability—have been selected as critical cases, in the sense outlined in Chapter 5. Therefore, these Chapters are also intended to reveal something about the text being analysed, the *SMP 11-16* scheme, as a whole.

It may be useful to provide a brief reminder of the essential structure of the language. School mathematics constitutes an *activity* which specialises *practices* and *subject positions*. The activity is (re)produced in *subjectivities* and *texts*, the interest, here, being in the latter. Practices and subject positions are textually (re)produced by *textual strategies* which establish *message* and *voices* (dominant and subordinate) via the recruiting of *textual resources*. There are three basic categories of textual strategy. *Message strategies* (re)produce the domains of practice (esoteric, mythical, metaphorical and public) as either *discourse* or *procedure*, and also (re)produce the *gaze* (projective, introjective). *Distributing strategies* (generalising, localising) distribute message across the range of voices. Essentially: the more dominant voice is to be associated with the esoteric domain, a visible gaze, and discourse (*generalising strategies*); the more subordinate voice is to be associated with the public domain, an invisible gaze, and procedure (*localising strategies*). *Voice positioning* strategies act directly on the voice structure either by reference within the activity (for example, by constructing an 'ability' hierarchy) or by reference outside of the activity (for example, by reference to social class).

The topic of this Chapter is algebra, which was identified by the analysis in Chapter 5 as of particular importance in discriminating between the Y and G series of books. I noted that algebra is prevalent amongst those chapters in the Y series which have the highest proportion of esoteric domain time. 'Algebra' was glossed, in Chapter 5, as 'that topic in school mathematics which is concerned with the structure of systems which is explored and described through the introduction of variables'. School algebra is often particularly concerned with the manipulation of formulas (and other variable expressions) and the solution of equations. It is difficult to find a single chapter, in the Y series, which does not involve the use of symbolically expressed variables to highlight a general structure (although a few, such as Y1.02, do not take this very far). By contrast, the G series is almost completely free of the use of such variables¹ and only two G chapters (G1.03 and G6.01, both titled 'Formulas') explicitly involve algebra in their titles. Empirically, therefore, the presence and absence of algebra signal, respectively, dominant and subordinate voices and the topic is thereby implicated in positioning strategies. However, the description of systems in terms of the principles which regulate them is precisely the meaning of discourse, so that algebra is, by virtue of the definition given above, theoretically associated with the dominant voice, because we would expect it to be centrally involved in generalising strategies. This theoretical and empirical importance of the topic, algebra, has motivated the focus of this Chapter which will constitute a close, comparative reading of the first explicit instances of algebra in each series, that is, chapters Y1.03 and G1.03.

I should note that there is some tension between the demands that the reading be 'close' and that it be 'comparative'. By 'close' reading, I mean, simply, that the reading follows the order of the empirical text and that a substantial amount of the empirical text be explicitly referred to in the reading. However, there are two textual components (the G and Y books) and I intend that the reading should highlight the differences between them, that the reading should be 'comparative'. Clearly, comparisons are to be made in terms of my language of description. This entails that I shall need, upon occasion, to prioritise the categories of the language over the strict sequencing of the text. The difficulty in reconciling the demands relating to close and comparative readings is, to a certain extent, exacerbated by the fact that the Y chapter is rather more extensive than the G chapter with which it is being compared.

¹ All significations are variables in that they are essentially repeatable, that is, they can stand for any instance of any of their connotations or denotations. The use here, however, is restricted to the use of specifically mathematical significations, normally expressed non-verbally (eg using letters).

The first two Sections of this Chapter describe the introductory sections in the Y and G chapters, respectively. The third Section contrasts the subsequent development in the Y chapter with the continued rehearsal in the G chapter. Section 6.4 compares the use of a particular symbol used by both texts. Here, it is necessary to consider a second chapter in G1 in order to sustain an optimum comparison between the texts. Section 6.5 contrasts the insistence upon mathematical convention, by the Y text, with the idiolectical description of mathematics in the G chapter. Section 6.6 describes the introduction of an exposition on calculators in the Y1 chapter and compares this with the treatment of calculators in the final book in the G series. Section 6.7 returns to a comparison of Y1.03 and G1.03, this time in respect of the development of the gaze. Finally, Section 6.8 compares the endings of the Y and G chapters.

6.1 *'The language of algebra'*

Y1.03 is titled 'The language of algebra'. The first section (Plate 6.1) is called 'A review of some shorthand' and involves at least five textual strategies. Firstly, the section is exclusively typeset, foregrounding the symbolic mode of signification. As was discussed in Chapter 4, the symbolic mode is associated with generalising.

Secondly, the section is a celebration of esoteric domain competence. The section is labelled as a 'review', but the initial exposition is not a straightforward repetition of something which has gone before, but an extension of the currency of that which has already been acquired:

The language of mathematics is international.
You may not know what language this problem is written in,
but the algebra is the same as you would find in an English book.

I.82. Sa se arate ca expresia

$$E = 4n^2 - 2n + 13$$

nu se imparte exact la 289, pentru nici un numar n intreg.

(Y1, p. 21; accents omitted)

The reader must be able to recognise as algebra the specifically mathematical expressions included in this extract in order to be able to make sense of the exposition. S/he is produced as being able to do this because the extract is apparently taken from a school textbook and because algebraic strings have arisen before in the

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Plate 6.1
(Y1: page 21)

scheme (hence 'review' in the title of the section)¹. The extract given above is followed by an extract in Japanese and another in Arabic, the latter (but, unexpectedly, not the former) requiring some transliteration in the English text. The text thus refers the reader's already and formally acquired, esoteric domain knowledge to an international community of students in a similar position. The section, therefore, employs a generalising strategy. In this case, the strategy identifies the reader with mathematics students in other countries and so diminishes local linguistic differences.

Thirdly, the section carries an internationalising of mathematics itself, specifically, esoteric domain mathematics. Algebra has the status of a language which is, furthermore, international and here it is, appearing in an unidentified, European language, in Japanese and in Arabic. 'Language', in the chapter heading, might signify simply a code, like a computer language; however, the play on the word language in the first two lines of the above extract and the extracts in Japanese and Arabic emphasise 'language' and 'international' as foregrounded significations. To the extent that the foreign language extracts are expressed in non-mathematical terms (the foreign language words and symbols are not unambiguously mathematical since they cannot be recognised as such within an English language context) and signify a non-mathematical content (foreignness) they are located within the public domain². However, the extracts also implicate specifically and recognisably mathematical signifiers (the letters 'x' and 'y' are conventionally italicised) which are examples of (ie they are metonyms for) algebra, to which attention is also drawn in the text: in this respect, the extracts are within the esoteric domain. The foreign language extracts thus carry contradictory significations.

Within the textual context, there is no elaboration of the public domain element other than the note that 'Arabic is written from right to left' and the associated irony in a 'translation' of the Arabic algebra as mirror writing. This ironic 'translation' effects a projection out of the public domain context (the foreign language) thus diminishing its priority. The irony also emphasises the foreign connotation of the extract and so enhances the signification of mathematics as international. Mathematics extends even

¹ The text does not state that the 'problem' is taken from a school textbook, but it appears to concern the solution of a quadratic equation which is compatible with school mathematics at a slightly more advanced level than that signified by Y1; the item reference, 'I.82.', is also associated with textbooks.

² Public domain refers to the domain of weak classification of mathematical message in terms of expression and content and so does not necessarily imply general familiarity: in most instances, public domain message does have a wider semantic currency than esoteric domain message, but this is clearly not the case in these instances.

to the most foreign parts, unreached, we might suppose, by other, less international disciplines. The esoteric domain significations, on the other hand, are elaborated within the English text. Furthermore, the final quarter of the page (after the mirror writing) is more or less exclusively esoteric domain and so confirms the priority of the esoteric over the public domain. This is, again, a generalising strategy, which lends esoteric domain mathematics a universal currency: it doesn't matter what the *words* in the extracts mean, the extracts themselves are all metonyms for the language of algebra.

Fourthly, the section carries an imperative invitation into the esoteric domain practices of an international community of mathematicians through the indexical emphasising (in red print) of its agreed rules of expression:

It would be confusing if everyone used their own shorthand, so mathematicians have agreed that

$3a$ means $3 \times a$	a^2 means $a \times a$
ab means $a \times b$	a^3 means $a \times a \times a$
$\frac{a}{b}$ means $a \div b$	$2ab$ means $2 \times a \times b$

(Y1, p. 21; rules of expression emphasised by red print in original)

This exposition is followed by task A1 which constitutes an imperative action in which these rules of expression are rehearsed by the reader, confirming her/him as a neophyte mathematician. This involves an intervoice positioning strategy which renders visible the authority of the discourse: this is what mathematicians (the most dominant voice of the activity) have agreed; now you do it.

The rules employ different variable labels from the preceding exposition (which implicate n and x and y , but not a or b). This marks them out in the text as does their emphasis in red print. Both of these contribute to their exemplary status, as does their subsequent rehearsal in specific realisations (ie tasks). The rules are thus produced as general principles, rather than local statements and hence the text involves discourse, which is thereby distributed to the Y reader (generalisation). This is the fifth textual strategy implicated in the introductory section of Y1.03.

6.2 'Formulas'

Chapter 3 of Book G1—'Formulas'—is quite different. To begin with, half of the first page of the G chapter (Plate 6.2) is taken up by cartoons and drawings. This page thereby contrasts with the Y page by its foregrounding of the iconic and local

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Plate 6.2
(G1: page 23)

rather than the symbolic and general. This localising is supported by the content of the first cartoon and by the opening exposition which constitute a closed narrative, specifying the work setting in some detail. Via the cartoon, the reader is shown the front of the company building and a man who appears to be the proprietor. The exposition introduces Peter (the main character of this part of the chapter) and gives some details regarding the nature of the company business and of Peter's role; Peter is also represented in the cartoon at the bottom of the page.

The cartoon of the 'Blagdon Hire Company'—open for business—is at the head of the chapter, almost on a level with and much more prominent than the chapter title. This is a scene setter, announcing the public domain setting of the chapter far more forcefully than the title announces its esoteric domain topic (six of the seven pages of the chapter concern the Blagdon Hire Company). As is the case with the mirror writing in the Y text, the cartoon incorporates irony. The display window of the Blagdon Hire Company denotes a manager wearing a suit, what might be a watch and chain and a bow tie—unlikely garb in a contemporary establishment of this kind. The display also includes a broom amongst the items for hire. It is not at all clear who might want to hire a broom; the manager appears to be looking at the broom with raised eyebrows, perhaps he wonders what it is doing there as well. These ironies effect a projection out of the public domain setting. However, whereas the irony in the Y text is resolved via a prioritising of the esoteric over the public domain, the ironies in the G text can be resolved only within the public domain, because there is no esoteric domain text. The irony signifies 'not this', allowing a connotative signification to come to the foreground. This connotative signification is a mapping of the G text onto comic literature, an interactivity positioning strategy: the setting is transformed into a joke. Thus the work setting which apparently motivates almost the entire G chapter is satirised at the very instant of its announcement.

A drawing of a 'card', incorporating an algorithm used in the company and, alongside it, what is presumably a paint sprayer neatly bisect the page and bisect the content, exposition/tasks. The card incorporates mythical domain significations through the use of the 'flowchart'¹ symbols referred to as 'machines'. The strong classification (with respect to the non-mathematical) of the flowchart formula is muted by its appearance on a public domain object. Both the formula itself (incorporating non-mathematical expressions) and its signifying mode (as manuscript on a drawing) are markedly less mathematical than the formulae in the Y text, discussed above. The

¹ These do not appear to be standard flowchart symbols, but have been pedagogically modified to incorporate directionality.

G text thus draws away from the mythical domain significations (the flowchart symbols), again foregrounding the public domain. The text moves on from the card (which is further localised by the drawing of the paint sprayer) to an instruction to the reader to copy it. Accompanying this instruction is a drawing denoting the appearance of the current page of the reader's exercise book when they are part way through copying the algorithm. A pencil (not a pen) point is poised in mid-trace. The second part of this task asks how much it costs to hire the paint sprayer for 3 days and includes a cartoon showing Peter thinking about the problem. The final two tasks on the page require the reader to rehearse the same algorithm for different numbers of days.

Mathematically, we might describe the formula as linear in one variable and might generalise it as $C = An + B$. This is the mathematical form of all of the formulae in the chapter, yet there is no exposition concerning either its esoteric domain structure or (more pertinently) its public domain origin. It is not clear, from the text, why this particular formula should be used by the Blagdon Hire Company to price the hire of its paint sprayer (and, indeed for all of its hirings). There is no discourse, because the public domain setting (which is central to this page and to most of the rest of the chapter) is a procedural domain; the esoteric domain is absent.

Peter appears as a junior in the company (Peter works 'for' the company, the proprietor is shown in the cartoon) and so stands in a possible career relationship with the reader: *you* might be doing this 'manual' job when you leave school. Metonyms that are thereby attached to Peter thus ionise the character with respect to the reader. This identification is sustained in the rest of the chapter, where the other characters are at a greater semantic distance. The manager and supervisor are Peter's superiors in age and rank and all of the hirers all have titles (Mr, Mrs, Miss) rather than first names, Mr Evans appears to be the school caretaker¹. The surrogate relationship between Peter and the reader (and the weaker relationship between Peter and the hirers) projects the subjectivity of the reader into the public domain narrative which is a possible realisation of the reader's own future. The text thereby objectifies

¹ The reader identification with Peter is somewhat confused by the form of address used in the above extract: '*You* can hire them for a number of days ... When *you* bring them back *Peter* has to work out how much *you* pay'. However, the strong career association between Peter and the reader, the fact that the reader is to do Peter's job (ie calculate the cost of hiring), the clear semantic distance between all of the hirers and the reader certainly outweigh the apparent addressing of the reader as hirer. In fact, an alternative to '*you*' would possibly render the exposition too specific for an exemplar (an individual's name might be used which would also involve an introduction) or, perhaps, too abstract (by using, 'someone'). Thus '*you*' is being used as a sort of generalised other which is, nevertheless, not too alien: the reader might, after all, become a hirer in due course; all of the hirings do connote manual labour.

the reader. Additionally, the public domain context is satirised, as has been noted, so that the reader's subjectivity is, at least partially, shifted within the public domain to that of reader of comic literature, which is an infantilising of the reader, whose construction resonates with a reader of comics.

Both the Y and the G pages incorporate exemplars. However, whilst the algebraic recognition rules of the Y text have general significance within the esoteric domain, the G exemplar (the algorithm which appears on the 'card', in the 'think bubble' and, partially, on the exercise book fragment) is highly localised. The algorithm is confined not only to the public domain setting, but to a specific item of equipment.

The opening pages of the two chapters exhibit both distributing and positioning strategies. The G text positions its reader by projecting its reader onto junior clerical work and onto infantile behaviour (comic reading). These projections are established denotatively and connotatively, respectively. That is, Peter, the junior clerk, is denoted in the text, whilst comics are connoted. These are categories of interactivity positioning strategy which will be referred to as *denotative mapping* and *connotative mapping*, respectively. Thus, the Y text positions its reader by producing a denotative mapping onto mathematical work through the invitation to join the community of mathematicians.

The G text localises mathematical message via the foregrounding of the iconic, via its detailed and closed narrative relating to the public domain setting, and via its reliance upon a highly local algorithm (procedure). The Y text generalises mathematical message in a number of ways: via the exclusion of all but the symbolic mode of signification; via the internationalising of esoteric domain message and the despecialising of language; and via the introduction of discourse. Crucially, the G reader is to recognise her/himself in the text and is thus objectified by it. The Y reader is to recognise mathematics in the text, so that her/his individuality remains exterior and irrelevant to it. The Y reader is, thereby, an apprentice subject of mathematics.

6.3 Developing the language of algebra: rehearsing formulas

Section B of Y1.03 (Plate 6.3) extends the discourse introduced in the previous section by addressing a possible ambiguity arising out of a combination of two of the rules that were given before. The initial approach is empirical: $3a^2$, when $a = 5$, might be calculated as 75 or as 225, depending upon whether a is squared first or multiplied by 3 first. The empirical approach is, of course, highly local and generalisation

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 6.3
(Y1: pages 22-3)

depends upon making the move from the particular cases (numbers) used in the example to numbers in general. This move is made in the rule which is printed, in red, below the example and in the subsequent exposition. Thus far, the rule has simply been asserted and barely extends beyond procedure. However, the text moves on to provide an exposition which is more recognisably discursive. The two diagrams (indices) in the middle of page 22 (Plate 6.3) firstly map the two algebraic expressions, $3a^2$ and $(3a)^2$, respectively, onto two geometric figures which can be compared in terms of area. Although the figures have specific shapes, that they have different areas is not dependent upon the absolute value of a , so that the comparison is a form of proof that $3a^2$ and $(3a)^2$ have different values. The proof is achieved via a mapping between topics within the esoteric domain, that is, between algebra and geometry. This not only confirms the general status of the need for the disambiguating rule which is printed in red on the page, but it also increases the connectivity of the esoteric domain: the text is thus doubly discursive.

Below the diagrams in Plate 6.3 are a worked example and a number of tasks which relate to the syntactical rules given in the current and in the previous section. However, the tasks are not simply rehearsals of the rules, but involve the extension of their application. For example, B1(f) includes a fractional coefficient and B2 introduces a number of variations, including exponents in the denominator of an algebraic fraction. Having been invited into the community of mathematicians, on the previous page, and having been introduced to some of the principles of the esoteric domain, the reader now begins to take the position of the dominant voice of school mathematics, the subject of esoteric domain discourse, in further elaboration of its message.

Section C of Y1.03 covers page 23 (Plate 6.3) and half of page 24 (Plate 6.4) of Y1. This section follows the same pattern as section B: an exposition of esoteric domain principles; worked example; tasks (rather more, this time) which also extend the application of the principles. The tasks are, additionally, punctuated by the provision of an additional rule concerning the omission of the multiplication sign in algebra. In terms of the definitions given Chapter 4, the whole of this section foregrounds the symbolic mode of signification. However, task C6 can be read as iconically signifying part of a page of an exercise book with mathematical expressions in manuscript. The task instructs the reader to ‘... put brackets in the correct places to make these statements true’. This device is distinguished from the exercise book fragment appearing on page 23 of the G text (Plate 6.2) because it cannot be identified with the reader’s own exercise book which is to show the same mathematical

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 6.4
(Y1: pages 24-5)

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Plate 6.5
(G1: pages 24-5)

expressions, but with correcting brackets. Rather, the manuscript and iconic mode facilitate a shift towards the public domain. The use of symbolic mode is muted by the intrusion of the iconic, which thus localises the mathematically false statement. A similar device is used elsewhere in Y1 (and also in the G series), for example, in Y1.04, as illustrated in Plate 9.7. In this case, the iconic mode is introduced in the two cartoon sequences and in the two exercise book fragments, localising mathematically false statements¹.

The second and third pages of the G text (Plate 6.5) are again dominated by the iconic mode and incorporate three prominent cartoons, thus extending the connotation of comic literature introduced on the first page of the chapter. The tasks on these pages involve a rehearsal of the basic algorithm introduced on the first page; each task is accompanied by the 'card' relating to the relevant item of equipment. In the case of the first task, the card is produced as part of a drawing which also shows part of a hand holding a scrap of paper on which are written the details of the hire. The hand is a left hand: this could be Peter's hand, offering the paper to the reader whilst standing at her/his shoulder. Alternatively, it might be the reader's own hand. The reader is placed either in Peter's position, or in a position in which Peter is supervising the reader who is to calculate the hire charge.

In the lower lefthand corner of the same page is a cartoon showing Mr Evans using the floor cleaner that he has hired from the Blagdon Hire Company². Behind him is the Blagdon High School noticeboard on which three notices and two graffiti are legible. 'Do More Maths', advises one of the notices, 'Yes Please!' reads a graffiti. The second graffiti, 'Wow! Maths is Fun!', is just above one of the other notices, 'Extra Maths Club: Tues. Evening'. These two notices are, ironically, identifying school mathematics with a leisure activity through their out-of-school timing (do *more* maths, *extra* maths club) and through their juxtaposition with a Disco notice. This is a leisure activity, furthermore, about which the students are thoroughly enthusiastic—'Wow! Maths is Fun!' The fun in this chapter, however, is all in the ironies contained in the cartoons: the broom and the overdressed manager on the previous page; Mrs Jones wobbling dangerously on her hired ladder on the next page; manual road workers being violently shaken and deafened by a pneumatic drill at the bottom of page 27 (Plate 6.6); a hired road roller and a hired mechanical digger

¹ In the case of the first cartoon sequence in Plate 9.7, the mathematically *true* statements that are incorporated within it are generalised by the caveat, signified symbolically and immediately preceding it.

² It is not entirely clear why a school caretaker might need to do this: it seems odd that the school would not possess its own cleaning equipment.

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 6.6
(G1: pages 26-7)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 6.7
(G1: pages 28-9)

sporting bandages and band-aids (Plate 6.7); Mr Ling trying to sell a 'new 24-volume pocket edition' of mathematics exercises (Plate 6.7). Mathematics is fun, because mathematics textbooks contain comic cartoons¹. The subjectivity of the reader is projected into the public domain, but once more, the public domain setting that apparently motivates the tasks is satirised by these ironies. It is the scene-setting cartoons that carry the real fun of mathematics which is comic literature. The third legible notice on the board behind Mr Evans is an advertisement for a 'Disco' for which the main attraction is 'The Idiots'—presumably, a live band, what might be a guitarist also appears on the notice. This notice signifies, literally, student leisure activities; the name, 'The Idiots', satirises the reader her/himself.

The second and third pages of each of these two chapters (Plates 6.3 & 6.5 respectively) extend the positioning and distribution of the opening pages. The Y text is more fully discursive in its increasing articulation of message within the topic of algebra and between algebra and geometry. The text is almost exclusively esoteric domain, so that the local, public domain conditions of the reader are excluded. The Y reader is to be apprenticed to mathematical discourse and this involves subjugation, in the suppression of individuality, and subjectification, in constituting the dominant voice of school mathematics.

The G text, in contrast, sustains the projection of its reader onto infantilism via the continued emphasis on cartoons (connotative mapping). The satirising of mathematics in the ironic notices on the school board produces a spurious collusion between the pedagogic voice and that of the reader, rendering invisible the authority of the former. The pedagogy in the form of the mathematical principles which have structured the text, the esoteric domain discourse, is also invisible, because the text remains outside of the esoteric domain. Peter, on the other hand, is highly visible as is the public domain setting within which he is placed. It is Peter who, as surrogate for the teacher, apprentices the G reader into his own public domain setting. However, the public domain always recontextualises the non-mathematical resources which are appropriated by the gaze. In the case of the Blagdon Hire Company, we might imagine that the cost of hiring the paint sprayer would be more easily expressed as £5 plus £4 per day; the tendency of the firm's customers to return the equipment an hour before the deadline is also somewhat suspicious². The apprenticeship of the G

¹ And this is a construction of the reader in relation to what they consider to be 'fun'.

² The Teacher's Guide notes that 'We have not included examples of the type 'Monday 10 o'clock to Tuesday 11 o'clock for the sake of simplicity'.

reader, unlike that of the Y reader, is a spurious apprenticeship as the collusion of the G teacher is a spurious collusion.

6.4 *Flowcharts & machines*

Section D of Y1.03 is titled 'Chips' and represents the first substantial move of the Y text outside of the esoteric domain (Plates 6.4 & 6.8). The section opens in the public domain, but employing a relatively open narrative. 'Chips' are integrated circuits which can be bought and which add, multiply, etc voltages. However, it is not certain who will be buying the chips, a school science department, perhaps, although task D3 refers to a 'micro-electronics kit' which connotes a hobby; the ascription of the pronoun, 'you', in the first line of the section may or may not refer to the reader. The electronics setting is sustained throughout the whole of the section, but mathematical expressions are first introduced to the setting and are ultimately prioritised over it. The text moves into the mythical domain with the first icon which superimposes a flowchart symbol on a drawing of a part of a circuit. The subsequent icons move further into the mythical domain through the introduction of algebraic notation. At task D3 (and for the remainder of the section), the chips are signified by flowchart symbols only, the drawings being replaced by indices, and the algebraic expressions are now foregrounded. The section constitutes a textual trajectory from the public domain into the mythical and, by foregrounding the mathematical expressions, points towards the esoteric which is where it returns in the subsequent section.

The flowchart symbol which is incorporated into section D is basically the same symbol as is used in G1.03, although, in the latter case, it always appears in manuscript. In G1, the symbol is referred to as a 'machine' which is glossed, in the Teacher's Guide, as 'a box containing a single operation' (G1TG, p. 25). This metaphor occurs again in G1.05 and also in G2.03¹. G1.05, in particular, opens with a cartoon (Plate 6.9) which, through an ironic reification of the metaphor, satirises the flowcharts as superfluous pedagogic props, to be scrapped, like their metaphors. The flowcharts make brief reappearances on the following page and in G2.03 (still referred to as 'machine chains'), but their use in the G books ends there. The G reader is provided with imaginary machines. The cartoon in Plate 6.9 connotes some form of heavy machinery, insofar as the objects look as if they are made of steel and are being dragged, with some effort, by heavy chains (a pun on the reference to

¹ This chapter is called 'Working backwards' and is a representation of inverse arithmetic operations as 'number puzzles', thereby contextualising algebra within the public domain.

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 6.8
(Y1: pages 26-7)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 6.9
(G1: page 41)

'chains' in section A of G1.05). It is not at all clear what the non-mathematical purpose of such machines might be.

The reader of Y1.03, on the other hand, is provided with real, albeit recontextualised, machines in terms of the integrated circuitry. In the Y chapter, the relationship between the mathematical signs (flowcharts, arithmetic operations and algebraic expressions) and the 'machines' is not metaphorical, because the former are not identified with the latter, but, rather, they describe them. The flowchart is, initially, superimposed on a drawing of the 'machine' and is constituted as a mathematical description of the operation of the chips which 'instead of numbers [...] add and multiply voltages'. The relationship between mathematics and machinery is made more explicit, so that they stand as metonyms for one another. In other words, the gaze is made comparatively visible and this is enhanced by the textual trajectory referred to above. Furthermore, the flowchart symbols are not, as in the G series, a dispensable pedagogic prop, but are used, albeit fairly infrequently, throughout the Y series¹.

As is the case with the public domain setting in G1.03, the electronics setting in Y1.03 has been recontextualised. Quite apart from the mythical domain significations, a certain amount of violence has been done to the setting. Although the manufacturing of integrated circuits such as those described in the section is entirely possible, it is not clear that they are actually available. Furthermore, the voltages mentioned are very high, for this kind of circuitry, and the representation of voltage in the icons and indices seems to be closer to current, since a voltage needs to be measured with respect to an earth line, which is absent. In other words, although the public domain setting is electronics, the text is clearly not about electronics, just as the G1.03 text is not about hiring equipment; both are about formulae. As has been argued, however, the G text is presented as entirely motivated by the public domain setting and so constitutes a spurious apprenticeship. The Y text, by contrast, through its textual trajectory and through its use of a comparatively open narrative, backgrounds the public domain in favour of the mythical algebraic expressions which are clear metonyms for the esoteric domain algebra in the sections before and after section D.

¹ More conventional flowcharts are also introduced in the Y series (in Y4 and Y5) and also appear in B1.

6.5 *Mathematical vs idiolectical shorthand*

Section E of Y1.03 (Plate 6.8) returns to the esoteric domain and extends the algebraic conventions introduced in sections A and B. Section B of G1.03 also extends mathematical extension, but, again, without entering the esoteric domain. The first page of this section (Plate 6.6) is even more heavily dominated by the iconic than the previous pages in this chapter, yet, mathematically, it is making the transition between flowchart and algebraic conventions. The algebra is initially referred to as a 'shorthand', which is the term used in the first section of Y1.03. The Y text makes it quite clear that idiolects are unacceptable: 'It would be confusing if everyone used their own shorthand, so mathematicians have agreed that ...'. By contrast, the G narrative positions Peter as the author of the shorthand and the public domain setting as the motivation for its production: 'I can use a shorthand for this', Peter thinks, on being told that he will have to copy out all of the cards, not 'I remember what we used to do in maths when I was at school'. Even the introduction of the term 'formula' is expressed in public domain terms, it is 'this way of writing the card'. A right hand holding a pencil and copying out the new card appears at the bottom of the page; there are three of these in this chapter and one idle left hand. The hand icons emphasise the manual nature of the task and balance the 'think bubbles' in two of the cartoons. The mathematics, which is mental, optimises the public domain, which is manual. In this case, of course, the optimising works only because a rather long-winded form of mathematical expression was adopted in the first place. Furthermore, it is Peter, within the public domain, and not the dominant voice of school mathematics which appears as the subject of this optimising.

The mythical expressions, such as $n \times 3 + 2 = c$, in Plates 6.6 and 6.7 are certainly mathematical. However, these expressions represent a restructuring of esoteric domain syntax, because, firstly, the dependent variable, c , is placed last, rather than its more conventional place at the beginning of a formula. Secondly, the dependent variable, n , is positioned before rather than after its coefficient ($n \times 3$ rather than $3 \times n$). In addition, the multiplication sign has been retained, contrary to the rule on the first page of the Y chapter. The restructured syntax has the advantage of being in accord with that of the 'machine chains' and this localises the algebra more effectively than would be the case if a more conventional, and therefore more abstract, syntax were imposed.

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Plate 6.10
(Y1: pages 28-9)

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Plate 6.11
(Y1: pages 30-1)

6.6 Calculators

Section F of Y1.03 (Plates 6.8, 6.10 & 6.11) is titled 'Using a calculator' and opens with a classification of calculators in terms of their logic of operations, generating four categories, LTR (left to right) with or without bracket keys and MDF (multiplication and division first) with or without bracket keys. There follows an exposition on bracket keys and the classification described above together with a worked example corresponding to each type of calculator. The final utterance of the initial exposition which is emphasised through the use of red type:

If you are ever in doubt whether a key sequence is correct,
try it out on some simple numbers where you know what
the answer should be.

(Y1, p. 28)

This is followed by task F2:

The following calculations are simple ones. First work out
each answer without using a calculator. Then experiment
with your calculator until you find the key sequence
which agrees with the answer. Write down the key sequence ...

(Y1, p. 29)

Having described the principles of the classification of calculators the task of classifying specific calculators and so generating specific calculating sequences is delegated to the reader. Between tasks F4 and F5 is another fragment of exposition which gives a generally applicable procedure:

In very long calculations such as this one:

$$\text{Calculate } \frac{a^2 - 2ab}{3b^2 + 4} \text{ when } a = 6.3 \text{ and } b = 0.75$$

it is risky to try to find a single key sequence. It is safer first to write
the calculation with the numbers in, putting in brackets where necessary,
and then use the calculator to work out each separate part, like this ...

(ibid)

Again, however, it is left to the reader's discretion in assessing the level of 'risk' and so determining whether to apply this or the previous procedure. The issue of calculator logic does not appear in G1.03, but is introduced towards the end of the G series, in G8.01 (Plate 6.12). The opening exposition and task describe the syntax for the mental computation of a 'bill' and contrast this with the operation of 'most calculators'. The text introduces brackets to 'tell you to do the multiplying first' and provides an algorithm for using calculators: '... work out any multiplying first, and then write down the answers. Then add up your answers at the end.' The subsequent tasks rehearse this algorithm, first with reference to a 'bill' and then using brackets in

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Plate 6.12
(G8: page 3)

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Plate 6.13
(G8: pages 4-5)

the esoteric domain. The following section (Plate 6.13) provides an alternative algorithm (in the form of a simplified, esoteric domain flowchart) for use with calculators which have a memory. This algorithm is subsequently to be rehearsed in a series of public domain tasks.

The G8 text is a rare example, in the G series, of a textual trajectory which moves between the esoteric and public domains. However, the esoteric domain text is entirely procedural and the chapter ends where it began, in the public domain with comparatively closed narratives (Plate 6.13). The public domain is constituted so as to motivate the chapter, including its esoteric domain sojourn which is presented as a subroutine for the facilitating of public domain tasks. The algorithms which are provided will work with any calculator, the choice between them being contingent upon whether or not the machine has a memory. There is thus no need to discuss the variations in calculator logic¹. Whereas the G8 reader has no options within the esoteric domain, the Y1 reader is provided with an esoteric domain schema which will help to enable them to optimise their own use of the calculator. The calculator is a highly limited tool for the G8 reader, but an object of discursive analysis for the Y1 reader.

6.7 Extending & restricting the gaze

The mathematical object which acts as the esoteric domain interpretant for all of the algorithms in G1.03 remains, as has been noted, unchanged throughout and, until the final page of the chapter, is projected, mythically, onto a single public domain setting, which is described in some detail, generating a very closed narrative. Furthermore, only small natural numbers are involved in the tasks and, as was mentioned above, there has been a restructuring of esoteric domain syntax in the algebraic expressions. This high degree of localising is only minimally disturbed in the final section of G1.03 (Plate 6.7), where two additional public domain settings are introduced, again using closed narrative. In these cases, a single alteration has been made to the formula, p is substituted for n ; the restructured syntax is retained to the end of the chapter. Algebra, within the G text, is localised in both form and setting.

The Y text, by contrast, has approached algebra discursively, introducing principles (rules of expression) and constituting articulations between algebra and another esoteric domain topic, geometry. This being the case, any formula for which

¹ This is not, however, a generalising strategy, because it in fact limits the use of the calculator, which is untheorised.

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 6.14
(Y1: pages 32-3)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 6.15
(Y1: pages 34-5)

the interpretation lies within the scope of the rules becomes comprehensible, regardless of setting. Apart from section D ('Chips'), the Y1.03 text discussed so far has remained within the esoteric domain. Following the exposition and first five tasks in section F, however, the chapter moves out of the esoteric into the mythical domain, where it more or less remains until its conclusion. These final tasks in section F and those in section G (Plates 6.10, 6.11, 6.14 & 6.15) minimise detail of the settings, thus employing more or less open narratives. In most cases, the open narratives connote, rather than denote, a range of settings. There is also a wide variation of predominantly public domain settings as the text moves between tasks. Thus, mathematics is represented as itself constituting a powerful language of description. I will discuss a number of these tasks, although there are no appropriate G tasks with which to compare them.

F8 (Plate 6.11) begins, 'The weight in kg that can be supported at the middle of an oak beam is given by the formula ...'. The formula for the weight is simply stated without explanation. Most of the chapter has been esoteric domain text and has been concerned with the evaluation of formulae. In F8, however, the reader is provided with a public domain instance of the descriptive power of mathematics. Whilst there is a certain ambiguity in the setting (the narrative is fairly open) the advantage of being able to calculate the load that can be supported by a beam without actually having to test it is clear.

F9 is a little different, because the narrative is more closed, particularly via the provision of a marginal cartoon. The formula, which concerns the stopping distance when driving a car, is again stated rather than derived, but this time there is a weak hint as to where it comes from: it must incorporate thinking time and braking time. The task involves the completion of a table and drawing a graph, similar tasks were introduced in chapter Y1.01, so that there is a reference back to another topic. This implicit reference to another topic, the similarity between the formula and esoteric domain formulae which appeared earlier in the chapter, and the hint about the origin of the formula are beginning to make visible the action of the gaze of school mathematics. In this case, the result of this action is to produce a graph which describes an aspect of motoring. The graph is drawn in the Teacher's Guide. Although the narrative in this task is closed, the task, unlike those in G1.03, does not point to participation in the public domain setting: the appearance of a child in the road when driving a car would suggest braking rather than drawing a graph. Rather than denoting participation, the task constitutes a rationalising of the setting which, of necessity, involves its objectification.

The final task in section F illustrates the effectivity of the recontextualising gaze of school mathematics. The narrative is again fairly closed and a marginal cartoon is again included. The text provides a formula which is used by nurses in calculating children's doses of drugs. The formula is not fictitious, its mathematical equivalent being cited by Pirie (1981), who was investigating the 'uses' of mathematics in medical practice. Pirie cites the formula as follows:

$$\text{Childs dose} = \frac{\text{Age} \times \text{Adult Dose}}{\text{Age} + 12}$$

Although this is mathematically the same formula, the variables are, in Pirie's version, expressed in words rather than letters. Furthermore, Pirie's formula breaks with mathematical convention because the variables are quantities rather than numbers. The formula itself does not specify the units in which 'Age' and 'dose' are to be measured and would clearly produce different results if the age of a child were to be measured in months rather than in years¹: a child's dose of one-thirteenth of the adult dose if an age of 1 year is used as opposed to a child's dose of one-half of the adult dose if 12 months is used instead. The formula in the Y1 text, by contrast, is presented as a numerical relationship insofar as *n* represents the *number* of years in the child's age. This convention is, to an extent, also broken by the Y1 text in that no allusion to units is made with reference to *C* and *A* (it is clearly important only that they refer to the same unit of measurement). We can describe the formula in task F10, therefore, as constituting a shift into the mythical domain.

A second point concerns part (b) of the Y1 task. 'At what age does a child receive half the adult dose?' Essentially, it is difficult to conceive of circumstances in nursing practice when such a computation would be necessary: the nursing setting cannot motivate the task. School mathematics, on the other hand, can, because the task involves solving a linear equation (formally or informally) and 'the solution of linear equations' is a sub-topic of algebra (although this sub-topic has yet to appear explicitly in Y1). This particular equation, furthermore², is somewhat more complex than those which have appeared explicitly in the course as a whole, for example in the level 4e booklet 'Equations'. The action of the gaze has thus transformed the mode of expression of the formulae so that it conforms more or less with mathematical, not nursing, convention and it has incorporated the formula into a strategy which is

¹ Pirie does provide this information which would certainly be available to and/or known by nurses, the point here, however, concerns the mode of expression of the formula itself.

² The equation may be formalised as $\frac{A}{2} = \frac{An}{n + 12}$ or $\frac{n}{n + 12} = \frac{1}{2}$.

(albeit implicitly) extending esoteric domain message. In this latter respect, the gaze has reduced the level of the formula from that of textual strategy (it is an algorithm within nursing) to that of textual resource. This reduction in level is a general effect of recontextualisation.

Both of these final tasks in section F begin with a scene-setting introduction which includes a marginal cartoon. The first cartoon (F9) signifies a child chasing a ball out into the road in front of a car and being urgently hailed by another. There is no irony within the cartoon, indeed, this particular cartoon is very close to being a drawing in terms of the criteria introduced in Chapter 4. Nor does the Y cartoon satirise the public domain activity as do many of the G chapter cartoons. Indeed, there is a literal relationship between the verbal scene setting introduction and the cartoon: both signify an inherent danger in the situation. However, as the elaboration of the task progresses to its specific imperatives—‘Copy and complete this table ...’, ‘Draw a graph ...’—both the verbal introduction and the marginal cartoon are revealed as ironic. The problem is not, after all, how to make sure we can stop the car (or stop children from running out into the road), but to complete a table and to draw a graph, neither of which would be very likely to be of assistance to the driver who, were they to be looking at their speedometer at the time, would probably hit the child anyway.

Precisely the same trajectory is rehearsed in the following task. Again, the marginal cartoon does not, alone, contain its irony (although the nurse’s uniform is, perhaps, a decade or so out of date), but stands in literal relationship with the verbal scene-setter: the nurse is shown pouring out a spoonful of medicine. As noted above, however, the nursing activity cannot motivate the task as a whole, so that there is a textual trajectory out of the public domain and into the mythical domain which is thereby prioritised.

Section G, the final section in the Y1 chapter, includes, very near the beginning, a cartoon which is somewhat different from those in section F in that it is a rare instance in the Y series of hyperbole in the caricaturing of an (anonymous) individual. The section also contains five other cartoons which implicate puns or elements of the ironic or ludic. These cartoons are not simply scene setters, but humorous elaborations of an aspect of the verbal text; they carry their own humour. Task G4 (Plate 6.14—the actual task number has been omitted from the text) is accompanied by a cartoon showing a cat sitting on a box of tins of ‘moggy food’ and clutching the end of a ball of string which seems to have rolled across the page underneath the verbal text of the task. The cat and box are weakly associated with the task. The latter

does not specify that the ' n tins in a box' are catfood and part (b) refers to tins which each weigh 3kg and which are, therefore, certainly not catfood (at least, not the catfood signified by the cartoon). The playfulness of the cat, with its ball of string which intrudes in the serious business of doing mathematics, and the ironic labelling of the box lighten the page of otherwise symbolic text and diagram.

The setting for G10 is ludicrous: why are the children in separate rooms; why do we not, initially, know how many there are or how much the trifle weighs; whoever heard of a trifle weighing 60kg; how are the children going to eat 3kg of trifle each (the same weight as is to be eaten by the virtual cat in G4 part (b))? The verbal text is accompanied by a cartoon which visually balances the cat in G4 (they are in the top righthand and top lefthand corners of the double page spread respectively). This cartoon signifies two doorways out of each of which children (apparently standing on top of each other or, alternatively, of conveniently graduated heights) are staring at a huge trifle in the foreground. This task contrasts with the section F tasks, F9 and F10. The latter fix their public domain settings with little ambiguity via the use of closed narratives, even though they implicate an ironic trajectory out of the public domain. G10, on the other hand, certainly connotes a recontextualised domestic setting, but implicates irony right from the start, in the public domain, to an even greater degree than the introduction to the G1 chapter, because the verbal text as well as the cartoon is ironic. The effect of the irony is to alienate the public domain significations. However, and in contrast to the G chapter, there remain the mythical domain significations which themselves connote the esoteric domain expressions in previous sections of the chapter. The public domain setting is almost entirely driven by the esoteric and the mythical domain denotations and their esoteric connotations allow this to be seen. The trajectory is from public to mythical domain and, connotatively, to esoteric domain. The ludic nature of the setting simply ensures that priority is given to the esoteric domain.

Section G is specifically referred to in the generally laconic Y1 Teacher's Guide:

Section G is particularly important, and if it is felt that the chapter is taking a long time, or if the pupils find the first few questions in section G very difficult, then the section can be postponed. However, whenever the work in section G is introduced it is likely to be difficult for many pupils.

(Y1TG, p. 18)

Concessions may be made for the many pupils who will find this section difficult, but only by postponing the work, that is, by reducing the pacing. The section cannot be postponed indefinitely, however, because the work is 'particularly important' and, in any event, postponement will not necessarily make the tasks any easier. This

Teacher's Guide statement signifies a possible lack in competence in the reader, and this lack in competence is signalled in the student's book by the introduction of the ludic in the cartoons and the trifle task in this section.

The mathematical distinction between section G and the previous section is that, now, the algebraic expression, the formula, is not given, but is to be derived by the reader. There is, however, a further distinction in the plausibility of the settings. The section F tasks that involve public domain settings (F6, F8, F9 & F10) vary in terms of the openness or closedness of the narratives that are employed. However, it is clear that they all concern plausible mathematical descriptions, that is, a plausible casting of the gaze onto something plausible—the setting. This is the case even where the setting cannot itself motivate the mathematical task as in F10. The application of the gaze effects a recontextualising which appropriates strategies within non-mathematical activities (braking within motoring, Young's Rule within medicine) and reduces them to resources within school mathematics.

Plausibility breaks down almost entirely, however, in some tasks in section G. G10 has already been described as implicating a ludicrous setting. Elsewhere, it is the effect of the gaze itself which is ludicrous. Consider the final task, G19, for example (Plate 6.15). The setting is signified by a comparatively closed narrative and is certainly grave in comparison with the earlier trifle. However, the specificity of each of the three possibilities for the will¹ is starkly contrasted with the indeterminacy in the numerosity of the woman's offspring². The opening words of the text also seem to specify the woman ('A woman has ...') and to place her death in the past ('When she dies, she leaves ...'): her identity and her death are fixed, how many sons and daughters she has is not.

Most of the other tasks in the section avoid the ludic (although G17 apparently mathematise a strawberry picking expedition). However, the mathematical gaze in most of the public/mythical domain tasks remains very distant from any possible recontextualised activity. In G6, for instance (Plate 6.14) the answer to (a)—the total weight of the train—might plausibly be of interest to railway managers/engineers in relation to the power of the engine (although algebraic generalising would be unlikely to be of procedural value). However, there is no attempt to motivate the task in this or in any other way, which contrasts strongly with the tasks in the G chapter. What the

¹ The political validity of constructing a will on the basis of gender cannot, of course, be questioned, because this would be to engage the setting at the level of strategies rather than resources (see, also, Brown & Dowling, 1989).

² A contrast which is reproduced elsewhere in the *SMP 11-16* scheme; see Dowling, 1991c, p. 3.

task is achieving, on the other hand (and this applies equally to the ludic tasks discussed above), is an apprenticing into the action and validity of the mathematical gaze. The question, ‘What does *nb* tell you?’ invites the reader to describe a public-mythical domain trajectory by giving the answer in public domain terms: ‘*nb* tells you the total weight of the train’ (Y1TG, p. 21). The question also celebrates, through the use of the word ‘tell’, the descriptive power of mathematics: mathematical expressions can *tell* us about non-mathematical things. This resonates with the utilitarian mythologising, discussed in Chapter 2. But the reader must beware of profligacy. The gaze is to be applied with discrimination: $a + b$, for example, has no meaning, in this case, because ‘It makes no sense to add the weight of a coach to the total length of the train’ (ibid). The general nature of this answer, given in the Teacher’s Guide, again creates a distance from any possible setting. The fact that very few of the tasks in this section seem to make any sense (outside of school mathematics) is left unaddressed.

6.8 Concluding the chapters

Both Y1.03 and G1.03 end in irony, the Y chapter with the testate woman and her family, and the G chapter with the second of two new settings introduced in its final section (Plate 6.7). The G chapter is finally completed with a cartoon drawing of Mr Ling posting a notice—‘Buy them today!!’—on a copy of his ‘New 24 volume pocket edition’ of ‘Ling’s Exercises’. As with the other settings, motivation for the task is located within the public domain: Mr Ling can do the necessary calculation himself and so structure his texts appropriately. Again, mathematics is optimising the public domain in terms of participation. The cartoon, however, carries its own ironic significations: a 24 volume *pocket* edition (each volume appears to be of encyclopaedic proportions); who, particularly a ‘less able’ reader of the G text, would want to buy such a publication? By juxtaposition with the symbolic text, the cartoon emphasises the otherwise weak irony in the task, part (c) in particular. Again, there is no esoteric domain to resolve the irony, so that the comic cartoon comes to the foreground. Furthermore, it is the institution of school mathematics itself which is being satirised, here, as is the case with the noticeboard behind Mr Evans. Mr Ling also connotes John Ling, the *SMP 11-16* team leader (see Chapter 5), so that the irony also infects the *SMP 11-16* project itself, by signifying the futility of producing mathematical texts for such readers¹.

¹ Clearly, this latter connotation is likely to be missed by most readers, especially since John Ling’s name does not appear in the student texts (other than ironically). The analysis here, however, is concerned with the construction of the reader by the text as an exemplar of the activity of school

The page opposite the final page of Y1.03 (Plate 6.15) is the first of three individual pages entitled 'Mathematics of Yesteryear', which implicate extracts from old school mathematics textbooks, this one apparently from *Rural Arithmetic*, published in 1916. In terms of content, this text constitutes a return to the kind of task at the end of section F, in which a formula relating to a public domain setting is presented. Following an initial exposition, there follows a worked example and a list of wind velocity ranges corresponding to verbal descriptors. Underneath the extract appears a drawing of a windmill and a task. It is not made clear whether or not the task is also taken from *Rural Arithmetic*¹. The extract, task and drawing are on a black background which is bordered by Corinthian order columns and indeterminate (but classical-looking) stylobate and entablature; the title, 'Mathematics of yesteryear' appears on a scroll which is mounted on the entablature. The mathematical content of the extract connotes the formulae in section F, thus linking this page with the preceding chapter and effecting its punctuation. The chapter thus opens with a synchronic universalisation of mathematics—the language of mathematics is international—and ends with a European diachronic universalising extending from ancient Greece through early twentieth century Britain to contemporary times.

In general, the final part of the Y chapter which concerns 'formulas' (from F6 onwards) extends the contrast with the G chapter which is entitled 'formulas' in terms of the differential textual strategies that are implicated in the two texts. Firstly, the Y text continues the subjectification of the reader with respect to the esoteric domain through the denotative and connotative prioritising of the latter by the domain trajectories which it incorporates, in particular, by the resolution of irony which projects out of the public domain and into the mythical domain and, connotatively, into the esoteric domain. The G text, by contrast and as has been argued earlier, prioritises the public domain by locating task motivation within recontextualised activities. Because the G text never enters the esoteric domain and only weakly enters the mythical, irony can only be resolved within the public domain, by emphasising comic literature which, through its association with children (and with working class literature, see Dowling, 1990b, 1991a, 1991c), infantilises the G reader relative to the Y reader. Thus, a hierarchy measured explicitly in terms of mathematical 'ability' (in

mathematics; the reader's response is not under consideration, nor should it be in the absence of appropriate empirical text.

¹ There is a windmill in Saxtead Green in Suffolk.

the Teacher's Guides) is brought into correspondence with a public domain hierarchy measured in terms of maturity (and class); this is a connotative mapping.

Secondly, the Y text apprentices the reader into the action of the mathematical gaze: this has been partially achieved in the opening sections of the chapter through the introduction of esoteric domain discourse; in the later sections, the application of the gaze is practically demonstrated in worked examples and in tasks. Such apprenticing is not achievable by the G text which never enters the esoteric domain wherein lies the subjectivity of the gaze. Furthermore, the G text constructs mathematics as idiolectical in contrast to the Y text which celebrates the power of mathematics to generate valid description within the mythical domain.

Thirdly, the Y text implicates a diverse public domain, thus stressing the generalisability of mathematics in comparison with the very narrow public domain in the G text which confirms the context-specificity of the procedures which are implicated. Fourthly, it is only in the Y text that there is any linking between esoteric domain topics (which constitutes discourse); the G text, as has been stated, does not enter the esoteric domain at all. Finally, whilst the G chapter ends with a satirising of school mathematics itself, the Y chapter—through its extension into 'Mathematics of yesteryear'—ends as it began, with a celebration of the universal validity of mathematics.

6.9 Conclusion

The textual strategies which are implicated in these two chapters (re)produce quite fundamental differences in voice and message. In the Y chapter, there is a consistent foregrounding of esoteric domain discourse via generalising strategies. Where the text does involve public domain settings, there is always a movement away from the public domain which prioritises the esoteric. This movement is sometimes achieved via the use of irony, as is the case in the ironic 'translation' of Arabic as mirror writing and in the bizarre and ludic settings at the end of the chapter. Then, the text might 'peel off' the public domain significations, foregrounding the mythical, as occurs in the 'chips' section (cf Walkerdine's (1982) 'stripping away the metaphors', see Chapter 3). Alternatively, the text may constitute a mathematical rationalising of the setting, objectifying it as a field to be described rather than one which invites participation; this is the strategy involved in the tasks on braking distances and children's doses. Finally, the text may incorporate many diverse settings which are weakly fixed by open narratives, thus backgrounding the specificities of the public

domain setting and rendering more visible the structured action of the recontextualising gaze; this explains the variation in settings, particularly in sections F and G of the Y chapter. Crucially, it is always the mathematics that must be recognised in the text and which must subjectify the apprentice dominant voice. There is no room for the individuality of the reader in this chapter. The chapter opens with an invitation to the reader to join an international community of mathematicians, we might call this a recruitment strategy. The chapter ends with historical and, particularly, classical references which celebrate the timelessness (and the class basis) of mathematical authority.

The G chapter is entirely different. Throughout the chapter, the text never enters the esoteric domain and never transcends the procedural. Localising strategies include the foregrounding of the iconic and, particularly, the cartoon mode, which also positions the reader as infantile. The chapter is almost entirely confined to a single public domain setting, which is defined via a closed narrative. Through this narrative and, especially, via the introduction of Peter, the reader is objectified by the text, so that s/he must recognise her/himself as potential participants in the public domain setting. There is to be no recognition of mathematics. The algebraic content (defined in terms of the esoteric domain) is highly limited, that is, to a single expression. Esoteric domain syntax, in respect of the rules of expression given in the Y chapter, are, in the G chapter, negated in simplifying the tasks. Furthermore, mathematical authority is replaced by mathematics as idiolect, as Peter invents his own shorthand. Finally, the public domain referents in the G chapter are predominantly low in terms of socioeconomic class. Only Mr Ling stands out as a professional and he is clearly dominant with respect to the reader. The satirising of this particular teacher as altogether too earnest is consistent with a tendency, in G1, for the authorial voice of the text to collude with the reader in opposition to mathematical authority.

I will present a systematic discussion of textual strategies in Chapters 8, 9 and 10. Before that, however, I will give a second comparative close reading of two chapters which introduce the topic 'probability'.

Chapter 7

Probability in *SMP 11-16*

This Chapter is a companion to Chapter 6 and is produced in a similar form and with essentially the same intentions. Of necessity, there will be some repetition. However, I have decided that it is important to include the analysis on ‘probability’ for the three reasons noted at the end of Chapter 5. Firstly, ‘probability’ was one of those topics indexed in Chapter 5 as contrasting with ‘algebra’. Its exclusion would, therefore, unduly bias the description of the SMP texts. Secondly, the analysis enables me to introduce an additional category of textual strategy—*quasi-discourse*—which has not yet been discussed. Thirdly, as was the case with ‘algebra’, this Chapter enables me to compare apparently the same mathematical topic treated in different ways by the two texts. This comparison differs from that in Chapter 6 because the principal texts being analysed are from Y1 and G8, that is, from the beginning of the Y series and from the end of the G series.

Whereas algebra is the topic which is most consistently realised within the esoteric domain in the Y texts, statistics and probability has been identified, in Chapter 5, as the topic which minimises esoteric domain time within the books which are associated with the dominant voice. Thus, the three Y series chapters which are titled ‘Probability’ spend 3% (Y1.12), 3% (Y2.12) and 11% (Y4.10) of their respective textual time in the esoteric domain, whilst Y1.03 (‘The language of algebra’) spends 43% of its time within the esoteric. This latter figure is approximately equal to the mean for the Y series as a whole and is considerably exceeded by the other algebra chapters.

Eight chapters in the Y series deal with statistics and probability, but there are only four G chapters concerning this topic¹. I decided that the most appropriate comparison could be made between two chapters having the same title, that is, Y1.12 and G8.05, both titled ‘Probability’. As noted above, this also constitutes a comparison between an early chapter in the Y series and a late chapter in the G series, which contrasts with the comparison in Chapter 6 (between chapters in the first book

¹ These chapters are: Y1.12, Y2.08, Y2.17, Y3.07, Y3.17, Y4.01, Y4.10, Y4.14; G3.04, G6.04, G6.08, G8.05. There are also four intercalary tasks in the G series which are concerned with probability: ‘Escape’, ‘Dice’ and ‘Detective dice’, in G1, and ‘Flowers’, in G3. I shall make some reference to ‘Detective dice’ in this chapter.

in each series). As with the analysis of the algebra texts, which was presented in the previous Chapter, the ordering of the SMP texts will generally dictate the order of the discussion. Again, however, the decision to produce a comparative analysis will necessitate some deviation from this principle. Thus, in the first Section, I will compare the introduction of the topic in the two chapters. In Section 7.2, however, I will contrast a task in the Y chapter with an apparently similar task in Book G1. Finally, I will consider the development of 'probability' in the Y1 and G8 chapters.

7.1 Introducing probability in Y1 & G8

7.1.1 A close comparative reading of the introduction of probability

I shall begin with a fairly detailed description of the introductory sections of each of the chapters being analysed. Following this description, I shall summarise the similarities and differences between the two texts that are revealed by the language of description.

Both Y1.12 and G8.05 open with a prominent icon. The G chapter (Plate 7.1) shows a black woman TV weather forecaster whose performance is being attended by a white female viewer. Y1.12 (Plate 7.2) shows a white female interviewer who is making notes using a clipboard while a white female mechanic is up to her elbows under a car bonnet and a white man looks on, somewhat bemused. The expressive styles in the two pictures are very different. In common with almost all of the non-photographic icons in G8, this one is a drawing and is conventionally representational and humourless. The public domain is, by the end of the G course, to be taken very seriously. The Y1 cartoon displays mild caricaturing and mild humour. The prim, middle aged interviewer contrasts with the young and scruffy mechanic, the femininity of the latter being confirmed by the shape of the nose. The man is doubly castrated. He is in a subordinate position in the scene and, unlike many of the other caricatures of males in Y1 (and in the cartoons in the early G books), his nose is a mere vestige of a phallus. Both icons foreground the feminine in their denoted actions by backgrounding the masculine (in the Y image) or excluding it (the G drawing). This illustrates an overt 'political correctness' with respect to gender. This is a general feature of the books which might be contrasted with their more stereotypical images of social class (see Chapter 6 and 10).

Both of these images are in the public domain, but, as has been mentioned, there is a degree of satire in the Y cartoon which is not present in the G drawing.

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Plate 7.1
(G8: pages 30-1)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 7.2
(Y1: page 135)

Furthermore, whilst both the chapter and section titles of the Y chapter ('Probability' and 'Relative frequency') are mathematically specialised terms, the section heading in the G chapter—'Is it likely'—is more like an everyday translation of the chapter title. The Y chapter, therefore, asserts the authority of the esoteric domain and diminishes that of the public, whilst the G text moves immediately and literally into the public domain.

The image in the G chapter positions the reader as a participant in the scene, sitting next to the other viewer. The weather forecaster is the authoritative voice in the scene, although this authority is placed within the public domain. The exposition below the drawing indexes terms which are or which might be used within the setting and associates these with 'predict' and 'how likely'. The drawing is recalling an everyday, domestic setting and placing the reader as close as possible to the position that they might adopt within that setting. The terms which are indexed by the exposition must be familiar and must be recognised by the reader. The setting is 'real' and the reader must recognise themselves within it, just as they must recognise themselves in the character of Peter in the Blagdon Hire Company setting (Chapter 6). The text, itself, is generated by an introjective gaze, because the familiar terms can stand in a metaphorical relationship to measures of probability (this contrasts with the projective gaze illustrated in Chapter 6). Within the exposition, however, there is no movement out of the public domain setting; the gaze is entirely invisible.

The Y text is rather different. The setting is, like the G setting, public domain. However, although the setting is associated with an everyday practice—motoring—it directly concerns research into or commentary upon that practice. In the text beneath the table, the reader is introduced to the rationality of the research and, rather than simply pointing to familiar, everyday words, the text introduces specialist mathematical terms, 'proportion', 'relative frequency'. These terms shift the text into the mythical domain at the point at which they are introduced ('To compare models we need ...'), but the public domain setting follows the text into the mythical, so that the naive reader cannot be completely sure whether 'proportion' and 'relative frequency' are specialist mathematical terms or whether they relate specifically to surveys like the one in the setting¹.

¹ Moving into the esoteric domain would involve (at least) the abstraction of 'variable', for example, 'the relative frequency of a specified value of a discrete variable is the ratio of the frequency associated with that value to the sum of frequencies across the variable domain'.

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 7.3
(Y1: pages 136-7)

The icon, in contrast with the G image, does not position the reader within the setting, but as a non-participating observer. Indeed, the reader is not identified with any of the characters in the drawing, neither the interviewer, nor the mechanic, nor the passive man. On the contrary, the reader is associated with the analysis of the information collected by the interviewer. The exposition supplies the relevant information and the rationale for the analysis which the reader is to perform in the subsequent tasks and which is discussed in a second section of exposition (Plate 7.3). The reader is engaging in a rationalising of motoring, so that there is an objective distancing from it. However, this objectification is not achieved by moving into the esoteric domain, but by the creation of an objective space within the public/mythical. This space might be indexed by the term 'survey' and has been supplied with apparently specialist signs such as the mathematical terms, 'proportion' and 'relative frequency' and the diagram in Plate 7.3. This space might be described as a surrogate for esoteric domain discourse and is an example of *quasi-discourse*.

The final paragraph of the exposition introduces a splitting of mathematical expression.

The relative frequency is a way of 'measuring' the likelihood of getting a major fault. In the diagram below, the relative frequencies are marked on a scale which goes from 0 to 1. It is usual to give relative frequencies as decimals or fractions rather than percentages.

(Y1, p. 136)

In the reports of surveys such as this one it is quite usual to speak in terms of percentages (of new cars having major faults etc). In mathematics, however, 'it is usual to give relative frequencies as decimals or fractions rather than percentages'. Percentages, whilst basically mathematical, are generally referred to public domain interpretants, whilst mathematical discourse prefers decimals or fractions. This is a splitting of the esoteric domain topic 'representation of number' which is consistent with the finding, in Chapter 5, that none of the three chapters in the Y series which are headed 'Percentages' spend more than 10% of their textual time within the esoteric domain. However, the splitting, like the projective gaze which has produced the exposition and tasks, is not entirely visible, here, precisely because the setting is sustained throughout so that there is no entry into the esoteric.

The final task in this section introduces a second setting, this time a work setting, rather than a domestic setting, but the abstraction of 'relative frequency' remains implicit. The introduction of the second setting points to something which is generalisable, but this something is not realised within its mathematical, that is,

esoteric domain context. Nevertheless, the use of two settings hints at a certain arbitrariness in each with respect to that which is invariant, 'relative frequency'. This arbitrariness is also emphasised by the treatment of the two settings, particularly the first one. As has been mentioned, the opening picture is mildly caricaturing the characters, but the use of mythological (not 'mythical') characters as the names of car models makes it quite clear that we are not dealing with a real survey. This is compounded by the playful correspondence between the likelihood of major faults and the malignancy of the respective creatures: Goblins, and especially Demons, just have to be more faulty than Pixies, Gnomes and Elfin: the setting is satyrised. The second setting is illustrated by a second picture of the front of the light bulb manufacturer's office or factory: an illuminated sign above the building probably reads 'HI-GLO BULBS', but we can't be sure, because some of the bulbs in the sign have blown; again, a satirising of the setting. The variation of setting and the satirical style, both directed at the public domain, enable the foregrounding of the quasi-discursive space and is thus a generalising textual strategy.

The exposition opening G8.05 is followed by a sequence of four tasks culminating in the production of a diagram (at the bottom of page 31 of the G text, Plate 7.1) which is, on the face of it, not unlike the Y1.12 diagram in Plate 7.3. The two diagrams are, in fact, different in a number of ways. Firstly, only the G diagram is embedded within a task; it signifies the student's work¹. Since the student is to 'copy' the diagram, its length is given as is the parenthetic and exclamatory advice to 'use your own percentages!' Here, as throughout the G series, it is important that progress through the tasks is facilitated with a minimum of ambiguity. This was apparent in the restructuring of algebraic syntax described in Chapter 6. Secondly, the diagram in the G text, but not the Y diagram, is labelled in manuscript. This introduces the iconic and, again, references the fact that the student is to produce a diagram like it. However, the manuscript also weakens the authority of the diagram: students' work is, after all, not definitive². Thirdly, the diagram is labelled in percentages rather than decimals. In contrast with the Y chapter, percentages are retained throughout the chapter (although fractions are introduced as an alternative mode of representing probabilities), indeed, section B is titled 'Percentages'.

The diagram within the G text, like the table which precedes it in Plate 7.1, is produced as a part of a 'simple game' rather than as a commentary on it. The reader is

¹ Although the diagram is reintroduced later in the chapter as a 'probability scale': a spurious mathematical signification.

² The use of this device was also noted in the discussion on the Y chapter on algebra in Chapter 6.

to complete a table about the possible scores in advance. In task A1, the student is to rehearse the kind of everyday terms introduced in the opening exposition. In the subsequent task, for which 'You need a partner and two dice', the game is to be played. In fact, the 'game' seems to involve each person throwing the dice fifty times whilst the other records the scores as tallies and, subsequently, computes the frequency and percentage for each possible result, before constructing the diagram in task A4. There would appear to be no competition, no winning or losing, no goals or strategies in this 'game'. Indeed, the instructions are given apparently as unambiguously as possible. The tables and diagrams on which results are to be recorded are shown in blank form in the text and one of the tasks (A3) incorporates an indexical arrow to emphasise the correct location (shown in a fragment of a table) of a result. The following sequence from task A2 illustrates the level of explicitness of the instruction:

Both of you make a copy of this table.

[...]

One person throws the dice, the other person
puts tally marks in their copy of the table.

Throw the dice 50 times.

The result is the difference between the scores on the dice.

When you have thrown the dice 50 times
the person filling in the tally marks fills in the frequency column.

Then swap over.

Throw the dice 50 times and fill the other person's table in.

(G8, p. 31)

The form of address within the above extract is actually quite convoluted. It begins in the imperative ('... make a copy of this table') and then moves to what appears to be a general description of the process of the 'game' in narrative form. The text then shifts back into the imperative ('Throw the dice 50 times') and back again to description. The next line, 'When you have thrown the dice 50 times', introduces the reader into the narrative, and the final two lines return to the imperative. The resulting text gives the impression of mimicking speech in its grammatical inconsistency and thereby veiling the discursive authority of the textbook. Again, what is important is the facilitating of progress through the task.

What might be described as the same task appears in the final section of Y1.12 but is labelled an 'experiment' (the bottom of page 143 in Plate 7.4). Of course, this task comes right at the end of the Y chapter whereas the G task is at the beginning of its chapter. Nevertheless, the distinction between 'game' and 'experiment' is

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 7.4
(Y1: pages 142-3)

significant, as is the difference between the advice that the experiments are ‘best done by two people working together’ and ‘you need a partner and two dice’. The Y readers are working on a serious mathematical experiment, the G readers are playing a game. Even so, both of the G readers must produce a table (and, presumably, a diagram), whilst there is no such requirement made of the Y readers. The product in the G task is the physical object, the table: the Y product is knowledge. Overall, the apparent level of explicitness in the two sets of instructions is very different as is the representation of the dice, with the Y task including a more abstract icon which is incorporated into the verbal text like a symbol.

An earlier ‘experiment’ in Y1.12 occurs in section B which is titled ‘Experiments’ (Plate 7.3). As with the previous section in the Y text, there is a casualness about the treatment of the public domain. In this case, the notion that a football referee might use a drawing pin instead of a coin is clearly absurd for reasons of safety, visibility and the possibility of a difficulty in deciding which way up the pin had landed on a muddy, grassy surface. The public domain setting might be absurd, but mathematics can take it seriously and arbitrate on the fairness of the fictitious proposition to use a drawing pin instead of a coin. Again, there is the incorporation of the icons into the verbal text, as if they were symbols.

The next page (Plate 7.3) provides instructions on how the experiment is to be conducted beginning, as with the dice ‘game’ in G8.05, with a prediction:

Before you start the experiment, get everyone in the class to vote for either ‘point up’ or ‘point down’ as more likely.

(Y1, pp. 137)

This instruction is analogous to that in task A1 in G8.05 and is a rare example of a group activity in the Y series. However, the Y1 task is far less formal. There is no table to be copied and completed and there are no everyday expressions to be rehearsed. The Y instruction produces the experiment as a collective task which everyone is doing at the same time. The G task is individualised and regulated both by the table to be copied and by the exemplary terms to be inserted in the table. The ‘vote’ in Y1 is simply a reassertion that there is an uncertainty in the situation to be investigated, its substantive insignificance (in terms of mathematical content) is ensured by its lack of a task number (the instruction to have a vote precedes B1) and by the fact that there is no return to the result of the vote after the experiment has been completed.

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 7.5
(Y1: pages 138-9)

Task B1 in Y1.12 incorporates three, numbered and bordered instructions. There is also a marginal drawing signifying a hand poised in the act of 'throwing' a drawing pin; this is a rare instance, in the Y series, of a drawing of a hand. The first two instructions employ the same resource as the G tasks, that is, tables are completed in manuscript to signify that this is to be the reader's work. The final instruction represents the graph on which the reader's results are to be drawn, but, this time, there is no incidence of manuscript. This graph is to be an exemplary index of a known phenomenon as is made clear in the subsequent exposition (Plate 7.5). The shape of the graph may zig-zag at first or it may, like the second and third graphs, curve fairly regularly downwards or upwards. It must, however, 'settle down' eventually to the value called the 'long-run relative frequency'. The first three graphs point to that which is exemplified in the reader's own graph. This is not whether it zig-zags or slopes down or up to start with, nor the particular value of the 'long-run relative frequency', nor the number of throws (the graphs have no labels on their axes). That which is to be abstracted is the settling down after an initial variation. Any reader's graph will do as an exemplar; there is no need for manuscript. The fourth graph moves slightly in the direction from the general to the more particular by titling the axes and by scaling the vertical axis. This is what the reader's graph will actually look like (although the lines of the graph paper have not been put in so that there remains some uncertainty about the precise value of the asymptote). The indexing of the 'long-run relative frequency' completes the indexing of the crucial features of the reader's graph.

Task B2 then confirms the collective nature of the experiment. The task also introduces one of the reasons for differences in the results, the value is to be read 'for *your* drawing pin' and, of course, there may be variation between pins within the class. The final task relating to this experiment confirms this by asking which of two graphs relates to which of two different drawing pins, the ratio between point length and head area being considerably different in each case. These last two tasks indicate that there is something about the results of the experiment that is contingent upon the specificities of the drawing pin which is used: long-run relative frequency is a property of the drawing pin which may be determined either by experiment or by a consideration of the shape of the pin. But the esoteric domain has still not been broached. This is an 'experiment' which connotes science more readily than mathematics, because mathematics is analytic rather than empirical. This connotation serves the same function as 'survey' in the first setting in this chapter, that is, it indexes a quasi-discursive space within the mythical/public. The specialist term,

'long-run relative frequency' is appended to this space, as is the graphical index which expressions facilitate the objectification and description of the public domain setting. The science connotation, opposing the game playing denotation in the G8 text also effects a mapping between the dominant (Y) and subordinate (G) voices and a hierarchy within the public domain. Doing science in school connotes the intellectual; playing games connotes the manual (it is, furthermore, a manual game). Thus the invoking of science is implicated in both distributing strategy (by indexing quasi-discourse in respect of the Y reader) and positioning strategy (via the public domain mapping).

Section A of G8.05 continues with a bordered 'discussion point' (Plate 7.6). The 'game' is produced, retrospectively, as a collective task (although 'group' need not signify 'class'). Differences between members of the group are likely, in fact, to be quite common since, for example, the expected value for the frequency of occurrences of the score (2) having the second highest probability is only 1 standard deviation below the expected value for the score having the highest probability (1). This contrasts with the Y experiment for which 200 repetitions are required, each having only two possible outcomes. If the drawing pins are 'fair' ($p(\text{'point up'}) = p(\text{'point down'}) = \frac{1}{2}$) then 87% of the class would be expected to obtain 'long-run relative frequencies' between 0.45 and 0.55. The 'discussion point' in the G text also introduces some consideration of the structure of the 'game': a score of 6 cannot arise, it 'comes up 0%'. Finally, the 'discussion point' refers back to the first task in the section—the prediction.

The likelihood of variations amongst G readers is acknowledged in the section of exposition following the 'discussion point'. The 'percentage of 0s' is analogous to the long-run relative frequency in the Y text, although the technical term is not used here and the ratio is now expressed as a percentage rather than as a decimal. Furthermore, the expected value is given as a fact with no indication as to how it has been arrived at: did the author just keep on throwing the dice; how did s/he know that the percentage would not change if s/he had carried on even longer? As is the case in the Y 'experiment', the value is a property of the objects being used (provided that there is nothing wrong with them). It is not a requirement of the task that the readers obtain the 'correct' value, because 'different people will have a different percentage of 0s'; this is, after all, a 'game' and not an 'experiment'. The 'correct' value is actually provided in the text. The pedagogic value of the tasks lies, therefore, less in the acquisition of new knowledge than in the rehearsal of already, but presumably uncertainly, acquired skills: copying and completing tables; using 'words like fairly

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Plate 7.6
(G8: pages 32-3)

likely', etc; computing percentages; drawing lines; and marking positions on a scale. This is why each reader must produce their own table. The first four tasks in section A of G8.05 are not, in other words, really about probability at all, but about practising skills for their own sake. This is in marked distinction from the Y1 'experiment' in which already acquired skills are utilised as resources in the generation of new knowledge, albeit apparently public domain knowledge.

The G text exposition continues with the introduction of the term 'probability':

We call this the probability of getting a result of 0.

This means that if you threw the dice 200 times (for example) you would expect a result of 0 about 17% of 200 times, about 34 times.

You would not expect exactly 34 results of 0, but you would expect about 34.

(G8, p. 32; italicised text in red in original)

As is the case in the early part of the Y text, the aleatory factor is completely elided in this exposition, despite the fact that it is a fundamental assumption underlying the notion of probability and, indeed, limiting prediction to the probabilistic. No elaboration is felt to be necessary as to why 'you would not expect *exactly* 34 results of 0', or on why the long-run relative frequency graphs have varying shapes at the beginning. This elision of the aleatory is compounded in the G chapter through the introduction of the weather forecasting setting at the start. There is a sense in which the precise details of the weather may be construed as predetermined but unknown, rather than aleatory. In any event, chance can, apparently, be eliminated by a sufficiency of repetitions. Probability/long-run relative frequency in the G chapter and in the first part of the Y chapter are represented as something closer to certain, predictive knowledge.

Task A5, in G8.05, includes a table showing the probabilities of all possible scores (and the impossible score of 6) and then asks:

If you played the game 200 times

- (a) roughly how many results of 1 would you expect?
- (b) roughly how many results of 2 would you expect?
- (c) would you be surprised if a result of 3 came up 70 times?
- (d) would you be surprised if you got a result of 4 20 times?

(G8, p. 32)

Parts (a) and (b) are rehearsals of the calculation performed (implicitly) in the preceding exposition. Giving the correct answers to parts (c) and (d) is contingent upon an appropriate interpretation of the word 'about' used in the contextual definition of probability. The probability of getting more than 50 results of 3 is actually less than 0.001, whilst the probability of getting exactly 20 results of 4 is about 0.08. The answers to (c) and (d) should, therefore, be 'yes' and 'no' respectively, but in the absence of any knowledge of normal distributions, the only basis on which a response can be given is the everyday usage of the word 'about'. There is, thus, an assumption of everyday competence, on the part of the G reader. Furthermore, there is an assumption that this competence will be sufficient to enable the task to be completed. No such commonsense judgements are called for on the part of the Y reader, even the class 'vote' which preceded the 'experiment' makes no subsequent appearance in the chapter.

The final task in section A of G8.05 refers to a marginal drawing of what might be the teacher (white, male, balding, bespectacled and mustachioed) who is making the mythical domain statement, 'The probability of getting a result of 6 is 0%': the teacher is on your side, giving you the answer. The task instructs the reader to 'Write down what this means in a simpler way'. The section has previously led up to the introduction of the term 'probability', but this final task constitutes an implicit criticism of such jargonistic language. Translating mathematical language into everyday terms is something that the Y reader would be unlikely to be required to do. Despite the fact that 'probability' returns in the subsequent section of G8.05 and that task B2 requires another translation (of a statement made by the same character, this time wearing a jacket and tie) in a more or less opposite direction, task A6 'signs off' the section with a kind of Parthian shot which devalues the technical language.

7.1.2 Summary

I shall summarise the points raised so far in the analysis of these chapters. Firstly, there are certain similarities between the texts. The trajectories of neither text have yet included the esoteric domain, so there is a necessary invisibility of specialised mathematics and of the gaze (introjective at the start of G8.05, projective at the beginning of Y1.12). The mathematical interpretants (probability, long-run relative frequency) are represented as properties of the objects under consideration (dice, drawing pins, cars, electric light bulbs) and not as constructs which are articulated within mathematical topics and projected onto or introjected from the public domain. There is, in the Y text, a splitting in the representation of number within the esoteric

domain which is realised as a preference for fractions and decimals. Percentages are retained in the largely public domain G text. However, this splitting is, again, not highly visible in the sections discussed, precisely because of the lack of entry into the esoteric domain.

There are also a number of differences between these texts. Firstly, the foregrounding of the public domain, although present in both texts, is stronger in G8.05 than in Y1.12. The local realism of the opening G setting (the weather forecaster) locates mathematics within the public domain, the 'real' world, with a positivity that is certainly not present in the satirical treatment of the public domain in the Y opening. The public domain for the G reader is local and serious and necessary: the public domain for the Y reader is comparatively arbitrary. The weakening of the public domain authority, in the Y text, foregrounds the quasi-discursive space which is opened up in the text.

The presentation of the setting in section A of G8.05 as a 'game' is sustained for the whole section. Thus, A1 is introduced by 'Here are the rules of a simple game'; A5 includes the line 'If you played the game 200 times'. Performing the tasks is tantamount to playing the game, to engaging in a manual, leisure activity. The Y text also introduces a game, football, but in this case, the task constitutes a commentary on (and, ultimately, an arbitration upon) the game or, at least, on a highly improbable distortion of a part of it. Doing the mathematics is quite different from playing this game. Nevertheless, both texts centre the message within the public domain more or less strongly.

The real distinction is that the G text locates mathematical message within popular culture—television weather forecasts, dice games. Even the mathematical indices (the diagrams and tables) are incorporated into the 'game' with nothing to suggest their generalisation. The Y text, on the other hand, locates message within specialised forms of knowledge which are indexed by surveys (denotatively) or by science (connotatively) which also generalise local realisations of the drawing pin graphs. Mathematical message is embedded within the procedures of local popular culture in the G text, but it is quasi-discursive in the Y text. The Y text generates a commentary on something else, generalising from motoring, manufacturing and 'refereeing'. Message is distributed via localising, in respect of the G text, and generalising, in the Y chapter.

These distributing strategies are supported by positioning strategies which denote different kinds of public domain role for the reader. The G reader is identified with a passive television viewer and an algorithm-following game player (that is, a game player without strategies). The Y reader is identified with a consumer researcher (not simply an interviewer) and an experimenter (connotatively, a scientist). The G roles connote participation in domestic and leisure settings. The Y roles connote description of such settings. Mathematics, albeit disguised within the public domain, is about participation for the G reader and about objective description and, therefore, control for the Y reader. Insofar as objective and rational control connotes a masculine phantasy (Walkerdine, 1988), there is a connotative mapping of the Y reader onto the masculine and, by default, the G reader onto the feminine. In any event, the dual system of dominant voice description and subordinate voice participation is sustained by these texts, as it was in the algebra texts. This is achieved even though there is no entry into the esoteric domain.

Finally, two points must be made regarding mode of signification. The first concerns the opening pictures in each text. The Y picture of the three individuals and the car is entirely incidental to the exposition (the exposition and subsequent tasks do not concern the scene in the picture, but the collected results of the survey). It is, in fact, the only chapter-opening picture in the Y series which is not explicitly referred to in the subsequent text and it is one of only two narrative cartoon chapter-openings¹. The cartoon mode of expression of the Y1.12 opening has comic connotations (as do the playful names of the cars in this section) and so, as has been noted, constitutes a mild satirising of the public domain setting. The picture opening G8.05 is not a comic cartoon, but a drawing of the kind that would be more likely to be found in a straight narrative in, for example, a teenage magazine. The public domain is indeed very serious for the G8 reader who is about to leave school, but not for the Y1 reader who has only recently begun secondary education.

Age difference, however, does not exhaust the difference. Y5 (the reader of which is the same age as the reader of G8) also contains a small number of comic cartoons, these, however, are confined to public domain settings, mostly to three of the intercalary sections concerning 'Money matters'². There are no instances of comic cartoons in G8, even though 94.8% of the textual time is outside of the esoteric domain. The 'straight' cartoon opening G8.05 connotes non-comic, popular

¹ The other being the opening to Y1.11 which includes a joke on a particular form of representation of gradient.

² There is also some caricaturing of the figures in a picture which forms part of a task concerning the organising of groups of people into coaches, which is, again, a public domain setting.

literature, strengthening the public domain centring of the section relative to that of the Y chapter. These kind of drawings and these kinds of popular literature contrast with the icons and their connotations in the G1 chapter on 'Formulas' discussed in Chapter 6. In particular, there seems to be a trajectory within the G series: from cartoons to drawings; from children's to teenage literature; from the ludic to the earnest. This trajectory will be further explored in Chapter 9.

The second point relating to mode of expression concerns the divergent treatment of the dice icons in the two chapters. In G8.05 the dice are shown pictorially, so that three faces of each die are visible. In the Y chapter, by contrast, the pictorial resemblance is reduced in that only a single face, the relevant one, is visible. Furthermore, the icons are aligned with and inserted within the verbal text, so that there is a shift towards a more symbolic mode of signification (although the mode remains, basically, iconic). A similar shift is made in respect of two of the drawing pin icons. This is, again, a move away from the experiential to the formal, from the local to the general.

The introductory sections in G8.05 and Y1.12 can thus be described as being, respectively, relatively strongly and relatively weakly located within the public domain, in terms of content, style and mode of signification. The G section positions the reader as a participant in everyday and manual leisure activities and distributes to the reader message which is elaborated in procedural mode. The Y sections position the reader as the subject of intellectual activity (consumer research, science) and distributes message which is elaborated as quasi-discourse. The texts thus correlate with the manual/intellectual hierarchy and, indirectly, with gender through the participation/control distinction in the reproduction of mathematical knowledge.

7.2 Polyhedral dice

Section B of Y1.12 concludes with a brief exposition, task and 'class activity' under the heading 'Rolling a cuboctahedron' (Plate 7.5). In order to maintain, as far as possible, the comparative nature of the textual reading, I will discuss this Y1 task with a G1 task with which it exhibits a certain content resonance (Plate 7.7); I shall return to the G8 chapter later.

The G1 task, 'Detective dice', appears as an intercalary 'investigation' on page 48 of the G book. Both 'Detective dice' and 'Rolling a cuboctahedron' involve cutting

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 7.7
(G1: Page 48)

out and gluing together polyhedra which are subsequently to be used as dice. Both Y1 and G1, furthermore, relate to the same age cohort (year 9)¹.

The G task has an alliterative title, 'Detective dice', expressed in manuscript outline characters. The alliteration in the title is redolent of the banner headlines in the popular press, so that it carries jokey, social-class-biased connotations². The content of the title suggests, perhaps, a game or perhaps a puzzle rather than the development of mathematical knowledge; there are certainly no esoteric significations in the title. The mode of signification which is used for the title (the manuscript outline writing) connotes children's own writing rather than the authoritative print of the textbook. In three respects, therefore, the title points away from mathematics, away from the esoteric domain and into the public. Comic book connotations are present in the decorative corner incorporating the classification of the task, 'Investigation for 2 people'. As is the case with the G8.05 tasks, the instructions are very explicit, starting with the advice on materials etc: 'You need worksheet G1-4, scissors, glue, blue-tack, a partner'. This is emphasised in blue type and on the slant below the decorative corner. The task itself opens with four notated pictures arranged like a strip cartoon. Again, there is the comic book connotation. The pictures show close-up cartoons of hands in the process of cutting out and folding the net of a tetrahedron, placing a piece of blue-tack on the inside of one face and gluing the tetrahedron together; arrows are included to indicate the direction in which the net is to be folded. The wording is, if anything, even more explicit: 'Crease along the lines. Fold inwards. Check each one folds properly'; 'Glue the grey flaps on one net. Stick it together, with the flaps inside. Then stick the other net together'; etc. This is a manual assembly task for which the entire emphasis is on the product and for which the reader is attributed minimal competence: this is how to hold a pair of scissors; this is the end of the tube that the glue comes out of. The text also makes it clear that the task to come is a game: your partner must not see where you stick the 'blu-tack' [sic].

In a text which is generally not very wordy, the text following the cartoon strip is explicit to the point of being verbose. There is, again, the device of using manuscript

¹ Since there are three G books relating to this group (G1-3) but only two Y books (Y1-2) and since both tasks come near the end of their respective books, the Y task will refer to a slightly later stage within the school year (just under halfway through the year in comparison with just under a third of the way through the year for the G task). Furthermore, there is no necessary assumption, in the scheme, that all of the G books will be completed by the end of year 11. There is, in Bernstein's (1977) terms, weak framing in the G series in relation to the Y series, since the latter is articulated with a public examination system and must be completed.

² A relationship between the G and Y texts and the popular and quality press will be further explored in Chapter 10.

to indicate that the reader is to do the writing. The resulting tables must be transparent with respect to the answer.

There are clear similarities between 'Detective dice', in the first G Book, and the opening section of G8.05 (discussed above), in the last. Both emphasise the manual, although this is a stronger association in the G1 text; both provide highly local, algorithmic instructions; both weaken the authority of the textbook (through the use of manuscript and speech-like text in G8); and both localise the mythical domain component (the tables and the diagram and discussion in G8). To this extent, there is very little evidence of any career within the G series.

The Y text is very different. Firstly, the title is expressed, as throughout the Y series, in bold typeface in contrast to the manuscript outline characters of the G text. The Y title also includes the name of the mathematical object that is to be used. It is a cuboctahedron and it is mathematically described in the first paragraph of the introductory exposition, alongside of which is an icon denoting the object. The image is an icon, rather than a graph, because it incorporates darker shading on three of its faces. Nevertheless, it signifies a definitive cuboctahedron, because it is in red which distinguishes it from the one which the reader is to make. The inclusion of the name of the polyhedron, its mathematical definition (or, at least a simplified mathematical definition) marks out the setting as esoteric domain. However, the task is to involve rolling the cuboctahedron 'like a dice'. This incorporation of non-mathematical signifiers ('rolling', 'dice') shifts the text into the metaphorical domain, as does the inclusion of an icon rather than a graph. The use of simile, however (the cuboctahedron can be rolled *like* a dice) maintains a semantic distance between the non-mathematical (dice) and the mathematical (cuboctahedron). This strongly contrasts with the G text in which the mathematical term for the object is not introduced, but is directly substituted by the non-mathematical 'dice', 'Give your two dice to your partner'. This is a metaphorical substitution, aided by the numbering of the faces of the anonymous tetrahedron. However, the G text is more appropriately described as mythical domain, because the public domain game/puzzle setting clearly predominates¹.

The instructions for task B4 in the Y text are, again, very different from those of the G task. The cuboctahedron is a far more complex object than the platonic

¹ The trope 'metaphor' may occur in any of the domains, including the esoteric (in which the substitution may be between topics). The distinction between the metaphorical and mythical domains rests on a judgement as to whether the principal setting is esoteric or public rather than on the figurative tropes that are available.

tetrahedron, in the sense that the former has fourteen faces of two different kinds as opposed to the four congruent faces of the tetrahedron. Physically, the cuboctahedron net is far more tricky to assemble than that of the tetrahedron. Nevertheless, the instructions are minimal. There are no pictures of hands holding scissors or glue, no arrows showing the direction in which folding is to be made; the reader is not even told to stick the flaps inside. The flaps are numbered, but there is an assumption of competence implicit in the lack of an instruction to glue them to adjacent faces. The instructions reproduce manual competence in the reader so that the manual in what is just as much a manual task as the production of the tetrahedra in 'Detective dice' is pushed into the background.

Part (b) of the Y task¹ is an invitation to speculate on the structure of the object in relation to its long-run relative frequency. This is similar to the 'class vote' preceding task B1 and, again, it is not subsequently referred to. Part (c), in two-and-a-half lines, is analogous to the second half page of the G1 task. The reader has discretion over the number of repetitions (simply 'a large number of times') and the mode of recording the results. Again, the Y reader is reproduced as competent in the manual aspects of the task. The final, bordered, 'class activity' produces communal and cooperative involvement in the task (as opposed to the frequently competitive or quasi-competitive involvement in games in the G series) and suggests a possible linking between the long-run relative frequency and other properties of the cuboctahedron. This actually strengthens the notion that what is to become 'probability' is actually an objective property of the object to which it refers rather than a mathematical construct. This linking is a projection from three mathematical topics: polyhedral geometry, ratio, probability. Again, however, the exclusion of esoteric domain text renders relatively invisible the gaze and the introduction to transtopical discourse.

The principal positioning strategy implicated in the (re)production of voice in these two tasks generates an interactivity mapping. The G text emphasises and pedagogises the manual aspects of the task. In contrast, the Y text underplays this, assuming a manual competence, and stresses the intellectual. This constitutes a mapping between the Y/G voice hierarchy, on the one hand, and the intellectual/manual hierarchy, on the other. The mapping is reinforced by the connotations of participation in games in the G text, as opposed to producing quasi-discursive commentary in the Y task, and of popular newspaper headlines in

¹ Which, in the edition represented in Plate 7.05, includes a misprint: 'octahedron' for 'cuboctahedron' in its final line.

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 7.8
(Y1 pages 140-1)

'Detective dice', which contrasts with the literal title of the Y task. Again, the use of the device of manuscript in the G task points towards local context specificity as does the highly explicit nature of the instructions. There is some differentiation in message strategies. In particular, in the mathematical (esoteric domain) denotation in the title of the Y task contrasts with the exclusion of the mathematical signifier, tetrahedron, in the G task. This contrast is reinforced by the use of different figurative tropes in the two texts, simile in Y1.12, metaphor in G1, p. 48. As with the introductory sections of the Y1.12 text and of the G8.05 text, discussed earlier, neither of the 'dice' tasks enter the esoteric domain, so that even the proto-discourse implicit in the linking of polyhedra, ratio and probability in the Y text remains virtually invisible.

7.3 Probability in action

Section C of Y1.12 (Plate 7.8) opens in the metaphorical domain. The decision to interpret a section of text as metaphorical or mythical domain depends upon an assessment of which is the dominant setting. In this case, the football setting has clearly been left behind, so that the most obvious interpretation is that the pin has been dropped 1000 times within the context of a mathematics experiment. Within this interpretation, the setting is clearly esoteric, whilst the pin and the experiment are imposed, public domain expressions¹. By the end of the exposition, the text has moved into the esoteric domain. There is, thus, a rendering visible of what has now become an introjective gaze. The exposition also introduces the notion of 'chance' which was absent from the previous sections of the chapter.

However, there remains an important elision, or rather, chiasmus. The text opens within an empirical or inductive frame in the metaphorical domain (the relative frequency is achieved through a 'large number of empirical repetitions'). The text then shifts to a theoretical or deductive frame in the esoteric. That is, esoteric domain probability is a construct. This construct can be projected onto the non-mathematical on the basis of a notion of chance and deductively calculable likelihoods². Probability, then, is not contingent upon actual trials. That this is the case is confirmed by the use of the terms 'impossible' and 'certain'. Clearly, an event, such as a tossed coin landing on its edge, may be highly unlikely and may not occur even in a 'very long run'. Its 'long-run relative frequency' would therefore be zero.

¹ As I have defined the esoteric domain of school mathematics as being exclusively concerned with formally defined objects and their relationships (Chapter 4), there can be no physical experiments within this domain.

² For example, where a system is deemed capable of generating one of n equally likely outcomes, then the probability of each is $\frac{1}{n}$.

However, such an outcome is not, in principle, impossible, and so its probability would not be zero. Nevertheless, 'the coin lands on its edge' is generally disregarded as an 'event' for the purposes of probabilistic investigation which, in school mathematical terms, is predicated on the notion of equally likely events¹. Y4.10, for example, opens with a description of a 'spinner', which comprises a disc divided into three 120° sectors and an 'arrow' free to rotate, like a compass needle, about the centre of the disc. The probability that, after being spun, the arrow will come to rest in any one of the sectors is defined as $\frac{1}{3}$, even though there would appear to be an empirical possibility of the arrow stopping precisely on the boundary between two sectors. The text forestalls such difficulties in a parenthetic note:

(We ignore the possibility that the arrow stops exactly on the boundary line between two sectors. If this should happen, the arrow can be spun again.)

(Y4, p. 75)

The probability, $\frac{1}{3}$, can now be recognised as relating to the specific practice in which the spinner has been implicated and is, in no sense a property of the spinner itself. This practice is constituted by a projection of the propositions of mathematical probability. These propositions are indexed by the diagram at the end of the Y1 exposition in Plate 7.8. Essentially, the esoteric domain constitutes generalisable statements precisely because its propositions tend to be analytic in the sense that they define their own objects. Public domain propositions are more frequently empirical and so local. This is most apparent, perhaps, in the case of mathematics, but it is asserted that this distinction characterises the esoteric/public domain classification in all specialist knowledges. That is, public domain statements are, of necessity, either conventionally empirical or what Bakhtin (1986) has referred to as 'secondary speech genres'; in either case, they are contingent upon local experience.

The Y1 text has completed a longer term trajectory from the public domain (at the start of sections A and B) to the esoteric. Because of the elision of the public-empirical/esoteric-theoretical chiasmus, however, the chapter has reproduced an incompletely visible introjective gaze. This has the effect of introducing the possibility of the empirical into the esoteric domain and constituting a potential point of contradiction within school mathematics discourse; a more adequate visibility might

¹ The drawing pin case (landing point up and landing point down are not equally likely) is modelled, in Y4.10, by a 'spinner' where a needle is fixed to the centre of a disc and is free to rotate through 360°. The disc is divided into two sectors, where the angle of one is proportional to the probability of the pin landing point up, the other being proportional to the probability of the pin landing point down. This model is mathematically constructed on the basis that, when spun, the needle is equally likely to come to rest in sectors of equal angles.

be achieved through the explicit 'stripping-off' of the empirical in moving from the public to the esoteric.

On the face of it, the mathematical content of the introduction to section B of G8.05 (Plate 7.6) is similar to the above Y1.12 content; certainly, both emphasise the empirical. However, there are a number of differences. Firstly, the difference in the labelling of the scale corresponds to the splitting of the esoteric domain topic representation of number which was referred to earlier. That is, the Y text labels its scale in decimals, whereas the G text uses percentages. Indeed the title of the G section is 'Percentages'. Only the Y text is using the mathematically preferred form of representation. Secondly, only the Y text moves into the esoteric domain by dropping all non-mathematical significations in the diagram; the G text is always referring to something non-mathematical. Thus, the diagram in the G text concretises impossibility and certainty, using essentially the same index as that in task A4 (Plate 7.1). The Y diagram, on the other hand, abstracts from the relative frequency diagram concerning the faults in cars ((Plate 7.3). The Y diagram is about the semantics of probabilities: 0 signifies impossibility, 1 signifies certainty, 0.8 signifies greater likelihood than 0.7, etc. Furthermore, having established the esoteric domain semantics, the Y text invites the reader to produce public domain discourse by projection, in task C1:

- C1 (a) Think of an event whose probability is 0 (something which can never happen).
(b) Think of an event whose probability is 1.

(Y1, p. 140)

The G text incorporates an apparently equivalent task at the bottom of the first page of section B. The task is bordered and labelled 'discussion points':

Is the probability that the sun rises tomorrow **exactly** 100%?

Can you think of other things with a probability of 100%?

What other things have a probability of 0%?

'Your chance of winning the pools are 1 in a million!'

This means the probability is **nearly** 0%!

What other things have a probability of nearly 0%?

(G8, p. 33)

In fact this is very different from the Y task. Here, the referential domain is the public. The probability that the sun rises tomorrow is, perhaps, not quite 0% because it might go supernova or an asteroid might hit the earth and stop it rotating. It is just possible that I might win the pools (someone might have completed a form in my

name), but it's very very unlikely: 0% probability is a pretty good approximation. The referential domain in the Y task is the esoteric, an assertion which is not gainsaid by the probability (not 0%) that a very similar discussion might actually occur in both the Y and G classrooms. It should be noted that the Y task is an individual task, that is, a task which (re)produces readers as equivalent to each other.

Task C1 in Y1.12 takes the text out of the esoteric domain and into the mythical; a trajectory which is to be completed by the reader. The subsequent task reverts to the drawing pin setting, reprising the computation in the exposition opening the section, with 'relative frequency' being replaced by 'probability' (again, an introjection of the empirical into the analytic, esoteric domain). The final part of the task requires that the two probabilities (that the pin lands point up and point down respectively) be added. The result of this addition should, of course, be 1, assuming that the two possible outcomes are mutually exclusive and exhaustive. A similar addition is required in the next task, which introduces a new setting involving a 'spinner' (Plate 7.8). Again, the answer to part (b) ('Add the three probabilities together') should be 1. In terms of relative frequencies, the sum of the proportions of the total number of trials relating to each different outcome must be unity. In terms of probabilities, the sum of the probabilities of an exhaustive set of mutually exclusive event must be unity, because it is certain that one and only one such event will occur. This is an important, esoteric domain discursive statement which is not made fully explicit anywhere in the three Y chapters on 'probability' (although it does appear again in Y4.10). The discourse, here, remains invisible.

The final four tasks in section C of Y1.12 concern two public domain settings (Plate 7.8). The first (task C4) concerns product testing by a manufacturer of fluorescent tubes and is redolent of the bulb company in section A. The reader, this time, is in the place of management or its hired intellectual labour: the task involves the computing of probabilities, not physically making tubes. Tasks C5-7 all concern a gambling machine. Dave is shown as scruffy, bespectacled and hirsute and making notes using a clipboard; the fruit machine player is also male. As with the earlier exposition and tasks associated with consumer research, the setting is not one of participation in the game, but is concerned with erudite commentary upon it: the reader is placed alongside Dave (or at his shoulder), not with the fruit machine player, who looks a little bemused. The reader also uses Dave's figures to determine the wisdom of participation: rational science versus popular culture. But the text remains outside of the esoteric domain.

G8.05 section B continues, after the opening exposition, with two familiar settings which contrast with the imposed mathematical task (Plate 7.6). The first invokes a room with a tiled floor and asks, 'What is the probability [that a drawing pin] will fall on a red tile?' There is a dissonance between the setting and the task requirements. The next task concerns a statement made by the man (now wearing a tie) who appeared in the previous section, 'The chances of a dry summer are about 1 in 10.' The man's statement is couched in everyday terms; the task involves translating it into mathematical terms, moving from the public to the mythical domain. As with the drawing pin on the floor, however, there doesn't seem to be any point to the task. You might make a game out of the drawing pin (why not a coin?) and the tiles, but it doesn't say it's a game. You might want to make some kind of prediction on the basis of the man's pessimistic statement about (presumably British) summers, but this would, in the context of the everyday, be bizarre. In any event, transforming '1 in 10' into a percentage would not make a computation any easier.

The final page in G8.05 (Plate 7.9) returns to the games settings. The bordered exposition at the head of the page reintroduces the dice to illustrate that 'sometimes you can work out a probability'. 'Working out probabilities' is, of course, precisely what the reader has been doing (or, at least, told that s/he has been doing) throughout the chapter. However, the text is now invisibly signifying *a priori* probabilities: the game of rolling a dice might be described as the projection onto the object of six, equally likely, mutually exclusive, exhaustive events so that the probability of the outcome of the game being any one of these is $\frac{1}{6}$. This projection is elided in the exposition which presents the probability of 'getting a 1' as a property of the dice. Task B3, mythical domain by virtue of its carrying over of the value $\frac{1}{6}$, facilitates a shift into the preferred (in this text) mode of representation of number. Percentages are preferred even though $\frac{1}{6}$ cannot be expressed as a precise decimal percentage. This same shift is reproduced in each of the next two tasks, fraction first, then percentage. Fraction, because it is more directly suggested by the everyday mode of expression '1 in 100', etc; percentage last because that's where you want to end up.

The settings for B4-7, the final tasks in G8.05, concern participation in a raffle, in card and dice games, in the school fete game, 'pick a straw'. All use the second person to indicate participation: 'your chance of winning is 1 in 100'; 'suppose you pick one card ...'; '... you get a six ... you get an ace ...'; 'suppose you pick a straw'. The final part of B7 asks for a less self-centred estimate ('suppose 50 people pick straws ...'), but it does not position the reader in the place of a researcher, like

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 7.9
(G8: Page 34)

Dave in the Y tasks. Knowledge of probability enables you, perhaps, to be a little more circumspect about your likelihood of success in playing such games; but the text never enters the esoteric and is quite firmly centred in the public domain.

The Y text, by contrast, continues in section D (Plate 7.4), with its rationalising of fruit machine playing. Although this text also remains clear of the esoteric, it nevertheless constructs a quasi-discursive space within the mythical domain which sustains control over the setting. The culmination of the opening exposition and three of the tasks concerns the 'profit' made on fruit machines which is the rationality of the entrepreneur, not the player. Task D2 introduces a different setting again concerning quality control in manufacturing industry (this time it is matches being put in matchboxes): once more, rationality is for management not the shop floor.

The final section in Y1.12 returns to the 'experiments' context, this time using dice rather than drawing pins. One of these two tasks was mentioned earlier. The text has not returned to the esoteric, but ends in the metaphorical: these dice are not being used for playing games, but for investigation into a mathematical topic. The reader has some control over these experiments in the sense that the instructions, like those for the cuboctahedron, simply require a 'long run of throws' without specifying a number. In this sense, the last set of experiments and the cuboctahedron task represent a development of the drawing pin experiment for which a run of two-hundred throws was specified. There is, however, no exposition following these 'experiments'; the chapter is, in this sense, left open-ended. The G chapter, by contrast, is closed: well, that's all you need to know about probability and this is what it can help you to do—pick a straw.

7.4 Probability in SMP 11-16: summary

Both of these chapters are predominantly located outside of the esoteric domain. This is usual, in respect of the G text, but highly unusual, in the case of the Y book. However, the texts, nevertheless, exhibit fundamentally different relationships to the public domain. The G text projects its reader into the public domain where they participate (or are potential participants) in the various games described in the chapter. The Y reader, on the other hand, is an analyst of data (relating to consumer research or to manufacturing industry quality control or to the leisure industry). Alternatively, the reader is an experimenter. In each setting, there is a distance created between participation and commentary, so that what the reader is engaging in is apparently context independent or generalisable. However, the text rarely enters the esoteric

domain, so that the generalisability of analytic discourse is actually not achieved. The location of the Y text is within quasi-discourse. This location marks out this particular chapter from the other chapters in the Y series, most of which are located in the esoteric domain. In fact, the analysis of this chapter is presented as representative in respect of the Y series treatment of 'statistics and probability' as a whole. Thus, 'statistics and probability' is marked out from other esoteric domain topics. It might be hypothesised that *SMP 11-16* students following the Y series will experience greater difficulty with 'statistics and probability' than with other topics, such as algebra. However, addressing such an issue requires a rather different data set than that which is available, here, and the empirical validity of the proposition will not be examined in this thesis.

The quasi-discursive spaces occupied by the Y reader stand as surrogates for the esoteric domain, giving the appearance of generalisability. In addition, however, these spaces are recontextualisings of non-mathematical specialised knowledges (for example, consumer research), the dominant voices of which are engaged in intellectual labour. The G text, by contrast, recontextualises popular (leisure and domestic) activities which, though not necessarily class specific, are nevertheless non-intellectual. This contrast is also apparent in the two dice tasks which exhibit an intellectual/manual polarising. There is, in other words, a mapping between the voice hierarchy of school mathematics and the intellectual/non-intellectual hierarchy which is external to school mathematics.

Two additional points should be made about the message structure of school mathematics, as revealed in these chapters. Firstly, the Y text reveals a splitting in the esoteric domain which would tend to push 'percentages' towards the public domain: decimals and fractions are preferred to percentages. The G text, on the other hand, titles a whole section, 'Percentages'. It is worth noting that 'percentages' are widely used in the retailing settings and in other areas relating to money (for example, with respect to wages and taxation) and in the language of the media as these settings are treated in the G texts.

The second point concerns the brief entering of the esoteric domain, by the Y text, which thereby renders visible an introjective gaze. This gaze introduces the empirical into the esoteric domain. Introjection is 'dangerous' in the sense that it may generate contradictions within the esoteric domain and so weaken the classification between the mathematical and the non-mathematical by a weakening of the grammar of the former. In this case, the propositions relating to the esoteric domain topic,

probability, may incorporate public domain significations, thus weakening their analytic character. To the extent that we can no longer talk about probability in exclusively abstract terms and the topic becomes open to challenge within the empirical public domain. This constitutes a shift in the centre of subjectivity of school mathematics out of the esoteric which may lead to a restructuring of the esoteric domain. The scope for introjection in Y1.12 is, however, restricted because of the limited amount of esoteric domain time.

In this and in the preceding Chapter, I have presented close and comparative readings of, centrally, two pairs of chapters from the Y and G series. In doing this, my intention has been to illustrate the workings of the texts as revealed by my language of description, but without highlighting the language itself. A number of features of the SMP scheme have been suggested by these readings. I shall, in the next three Chapters, produce a far more widely based reading of the scheme. This broader reading is designed both to emphasise the key features of the language of description (at the level of text) and to enable some general conclusions about the SMP scheme. A number of these general conclusions have been adumbrated in this and in the preceding Chapter.

Textual Strategies & Resources I: *message strategies*

In Chapters 6 and 7, I have presented close comparative readings of a very limited sample of the Y and G series which, themselves, constitute only a part of the *SMP 11-16* scheme as a whole. These comparisons have substantially followed the order of the text. It is hoped that this form of presentation has reproduced a sense of the inductive work in the development of the language of description. The comparisons have also enabled a contrast to be made between the same mathematical topic treated differently by the two textbook series. The current Chapter and the two which follow it are motivated, partly, by the need to give more concrete meaning to the theoretical categories which comprise the textual level of the language of description. Specifically, these Chapters have been sequenced according to the main categories of textual strategy which were introduced in Chapter 4. In presenting the analysis in these Chapters, I shall draw from as wide a base as possible within the two series of books, with additional emphasis being placed towards the beginning and end of each series.

This and the next two Chapters will enable me to complete the presentation of my language of description to the level of its current development (some possibilities for further development will be discussed in the concluding Chapter to this thesis). I will also foreground the principal findings of the analysis of these texts, some of which have been adumbrated in Chapters 6 and 7. I shall begin with a brief reminder of the major features of the language of description itself.

8.1 Introduction: a brief reprise of the model

In Chapter 4, I introduced the *langue/parole* dialectic as the basic metaphor underlying the relationship between *activity* and *text*. That is, text is generated within an activity and must be interpreted with reference to an activity. Activity is described in terms of practices and subject positions. Practices are constituted within the four domains, esoteric, mythical, metaphorical and public, of which the esoteric domain is the regulating domain of the activity. School mathematics—the activity under consideration—has been described as an activity which exhibits high discursive saturation (DS⁺). Text is described in terms of the (re)production of practices and

subject positions as message and voices. This is achieved by textual strategies, via the recruitment of textual resources.

There are three principle categories of textual strategy. Firstly, message strategies (re)produce the specialised and non-specialised domains of practice. In (re)producing the non-specialised domains, these strategies are (re)producing the action of the recontextualising gaze. Message strategies also (re)produce the level of discursive saturation of the activity. In this case, the relevant message strategy is discourse, which (re)produces DS⁺. However, since the (re)production of subordinate subject positions entails zero or minimal access to the regulating principles of a DS⁺ activity, message strategies must also (re)produce DS⁻ practices, even where the activity is DS⁺. In this case, the relevant message strategy is procedure. In Chapter 7, I also introduced the hybrid message strategy, quasi-discourse, which (re)produces practices which have some of the characteristics of DS⁺.

Insofar as the Y and G *SMP 11-16* texts always address a particular category of voice explicitly, message strategies are always also distributing strategies, which is the second category of textual strategy. Message strategies as distributing strategies will be discussed in this Chapter. In Chapter 9, I shall consider message distributing strategies which achieve distribution via generalising or localising. Finally, Chapter 10 is concerned with the third category of textual strategy, voice positioning strategies. These operate directly on voice. This is achieved either by organising the voice structure within the activity, or by mapping the voice structure onto hierarchies which relate to other activities or which are more general features of the social (such as social class).

Textual resources are contingent upon the medium and local context of pedagogic action in a way that strategies are not. In other words, the categories of strategy that have been introduced above are derived from the theoretical model, whilst resources have a certain (but not total) arbitrariness with respect to this model. The model demands that there be generalising and localising strategies, discourse, procedure, etc, but not that these be realised via photographic text, or symbolic text, or that significations will be recontextualised from shopping or nursing.

This assertion of a degree of arbitrariness of textual resources is not to say that they are irrelevant. The distinction between a visible and an invisible pedagogy, for example, may be made at the level of textual resource via the use of literary tropes. Thus, both metaphor and simile are elliptical in the sense that they are silent on the

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Plate 8.1
(G2 pages 20-1)

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Plate 8.2
(G2 pages 22-3)

precise nature of the metonymic linking between the signs that they implicate. Simile, however, explicitly announces the presence of such a linking and so should not be read as a localised labelling¹. This distinction was made in respect of the discussion, 'Polyhedral dice', in Section 7.2 of Chapter 7. Thus, the limitation to the arbitrariness of resources is that there is always a selection and this selection is relevant in respect of distributing and positioning strategies. Pedagogic textual strategies constitute repertoires of resources from the reservoirs of resources which are available within the specific context of pedagogic action.

8.2 Specialised text: esoteric domain

As has been stated, strategies are categories of textual 'utterance' that (re)produce specific features of a particular activity in the same way that we might describe categories of *parole* as (re)producing particular features of *langue*. Utterances are always context-specific instances (although generalising strategies may minimise this) so that, the relational nature of activity/*langue* notwithstanding, no single utterance can (re)produce the structure as a whole. In relation to activity practices, message strategies must (re)produce the domain structure. Thus there must be specialised and non-specialised text and DS⁺ and DS⁻, as indicated above.

Plate 8.1 shows the first two pages of G2 chapter 4 for which the Teacher's Guide gives the following 'aims':

To give practical experience of drawing regular polygons, and to become familiar with their names.
Making 'patterns' is, we hope, an enjoyable way of practising drawing skills, and pupils should be encouraged to invent their own patterns.
Simple ideas of angle are also called upon.

(G2TG, p. 12)

The chapter concerns the repetition (practising) of practical, manual skills and of a lexicon within an 'enjoyable' context: 'It might be nice to display particularly pleasing patterns' (ibid). Students are to be 'encouraged to invent their own patterns'. This is a creativity which is doubly spurious in that it is neither mathematically structured, nor is it pedagogically valued. There are no explicit criteria regulating what is to count as a 'particularly pleasing pattern', the patterns are simply vehicles for manual work: a sugaring of the pill. It is noteworthy that 'simple ideas of angle' are only to be 'called

¹ At the level of the empirical text, it is generally less a case of there being a simple choice between metaphor (A is B) or simile (A is like B) that a tendency to hide or make visible the possibility of non-identity.

upon'. The mathematics, in this respect, constitutes a resource in the rehearsal of manual skills.

Task A4 clearly constitutes an *algorithm* (a linear sequence of instructions) for the production of a hexagon. It is a sequence of symbolic and iconic instructions which facilitates the drawing and nothing else. The 'angle measurer' is an *operational matrix*, that is, it structurally delimits possibilities other than in a linear sequence. The measurer defines uniquely the circle and therefore the size of the hexagon and locates the vertices. The Teacher's Guide notes that the 'use of an angle measurer may need revising' (ibid). The ruler is also an operational matrix, defining the sides of the hexagons as straight lines. Nowhere is a hexagon defined in terms of straight lines, it is fixed only by the explicit deictic, '**this regular hexagon**', and by the indexical representations of hexagons¹.

Another algorithm is summarised by the contents of the indexical box on page 21 labelled 3. Boxes 1 and 2, on the previous page, offer some background to the algorithm². There are six angles which are all the same and which make 360° altogether, 'So ...'. This background enables the recognition of the features of the algorithm, facilitating its extension to other polygons. In the next section, there are boxes almost identical with boxes 1-3, as shown in Plate 8.2. The '360°' is associated with the angle measurer that is to be used in the drawing and the number of angles represents the specificity of hexagons, pentagons and other polygons (the names of which the student reader is to 'become familiar with'). The chapter repeats these two algorithms for octagons and for decagons and the review, 'Review: chapters 4 and 5', invokes the algorithms in the production of a nonagon pattern:

- You need an angle measurer.**
- 1 This shape has 9 sides.
It has a rather peculiar name.
It is called a **regular nonagon**.
 - (a) There are 9 angles at the centre.
They add up to 360°.
What size will each angle be?
 - (b) Use your angle measurer
to draw a regular nonagon.

¹ The lack of any definition of 'hexagon' in the esoteric domain renders invisible the distinction between icon and index. In this case, that is, it is not clear that such mathematical objects cannot be iconised.

² The exposition preceding these boxes seem to present a bizarre argument: 'Measuring angles is not always accurate. You can easily be a few degrees out. Before you draw an accurate shape, it is best to **calculate** the angles' (G2, p. 20). This advice ignores the fact that you cannot measure the angles of a hexagon unless you or somebody else has already drawn one and, furthermore, if 'measuring angles is not always accurate', then it would seem to be an odd method to choose for drawing the objects.

- (c) Draw a pattern inside your nonagon.
(G2, p. 59; diagram (index) of nonagon omitted)

It is not entirely clear why 'regular nonagon' is more peculiar than any of the names in chapter 4, but clearly, parts (a) and (b) of this review task invoke the algorithms in B1 and B2 of the chapter (Plate 8.2). This last task is repeated as a whole page review in G5, 'Review: polygon patterns':

You need an angle measurer.

A regular nonagon has 9 equal sides.

- 1 The angles at the centre of the nonagon add up to 360° .
There are 9 of these angles.
Copy and complete:

*Each angle at the centre of a regular nonagon
is $360^\circ \div \dots = \dots$*

- 2 Use the angle measurer to draw a large regular nonagon.
Then draw this pattern.

(G5, p. 56; indexical nonagon diagrams omitted;
bold text in green in original; italicised
text in manuscript in original)

Sections C (octagon) and D (decagon) of chapter 4 are reduced to a single page each and the task statements in these and in the two review items cited above are correspondingly reduced in comparison with the extensive double page spreads for the opening two sections of chapter 4. This reduction denotes a degree of internalising of the algorithms by the student reader. There has been, however, no other progression in the trajectory defined by the chapter and review tasks.

Furthermore, the G2 chapter and review tasks are unconnected, in explicit terms, with anything else in the G series. There are no more polygons and almost no reference to angular measure¹. There are clocks and dials and other circular scales in every G book and pie charts appear in G2 and G5², but pie charts are produced using a centigrade scale and without other reference to angular measure. Discourse is thereby completely absent from this rare instance of esoteric domain geometry in the G series³. The two categories, *algorithm* and *operational matrix*, defined above, are the two realisations of the textual strategy, *procedure*.

¹ There are single page review sections on angle in G1 and in G3, substantially outside the esoteric domain in each case.

² The Teacher's Guide for G2 suggests that, as an 'extension', a pie chart scale might be used instead of an angle measurer as a 'slightly different method of construction'. The pie chart scale is a transparent plastic disc marked on a scale from 0 to 100 the incorporation of which operational matrix would necessitate an alteration to both algorithms and would reduce the possibilities for polygon construction insofar as 100 has fewer factors than 360.

³ There are two other chapters, G7.03 and G8.08.

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Plate 8.4
(Y1: page 81)

Plate 8.3
(Y1: page 76)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 8.5
(Y1: page 87)

The development of the geometric ratio, π , in Y1, chapter 7 incorporates a rather different strategy. The Teacher's Guide describes the chapter as follows:

The idea of ratio, as developed in chapter 6, plays an important part in this chapter. The ratio $\frac{\text{circumference}}{\text{diameter}}$ in a circle is approached as a limiting case of the ratio $\frac{\text{perimeter}}{\text{diameter}}$ in regular polygon.

(Y1TG, p. 30)

Unlike the G texts, the Y series is multiply and often explicitly recursive, so it is not generally possible to mark out the beginning of a particular topic in an unambiguous way. However, we can pick up the developing discourse on geometry at the start of section C of chapter 6, which opens with the exposition and tasks shown in Plate 8.3. This is mythical domain text, because the setting concerns non-specialised objects (pictures) onto which specialised expressions (scale factor, enlargement, 1.6) have been projected. This section and the next remain in the mythical apart from a brief point of clarification concerning the extension of the term 'enlargement' to cover substantive reductions (esoteric domain). Section E, however, opens in the esoteric domain as is shown in Plate 8.4.

The nature of this trajectory will be discussed in the next section. What is being established, mathematically, is a relationship between the geometrical transformation 'enlargement', the comparative term 'similar' and geometrical ratios. This sets the basis for the following chapter, the opening of which is shown in Plate 8.5. The chapter opens with a formal definition of terms, that is, a definition in terms of esoteric domain objects. The exposition also articulates with 'enlargement' and 'similarity' and with geometric ratios from the previous chapter. The ratio, $\frac{\text{perimeter}}{\text{diameter}}$, is initially referred to as the 'p-number' of a polygon and sections A and B of the chapter tabulate and graph p-numbers against number of sides up to 48; the graph is followed by an exposition:

As the number of sides increases, the polygon looks more and more like a circle. The p-number gets closer and closer to the p-number of a circle, which is just over 3.14.


[...]

The p-number of a circle is denoted by the Greek letter p, which is written π and pronounced 'pi'.
The value of π , correct to 5 decimal places, is 3.14159.
The perimeter of a circle is called the **circumference** of the circle.
So the ratio $\frac{\text{circumference}}{\text{diameter}}$ is π . Or, in other words,

π is the multiplier from diameter to circumference.

[...]

(Y1, pp. 90-1; indexical diagrams omitted)

The exposition gives a new conception of a circle as the limit of a series of polygons having increasing numbers of sides (the circle appeared earlier in the book as a locus). It also introduces the term 'circumference' and the expression π , which is described as both a ratio and as a multiplier. At the end of the chapter, several approximations to π are given including its decimal expansion to thirty-five decimal places¹. The strategy employed in the Y text is discourse in which mathematical signs (esoteric domain) are articulated so that the principles of mathematical practice are rendered more visible. The signifier 'circle' here denotes a mathematical object which is defined, within the esoteric domain, as the limiting value of a polygon as the number of sides increases. An extensive array of connotated metonyms has been attached to 'circle'. This array includes: polygon, perimeter, side, circumference, radius, diameter, 3.14159, π , ratio, graph, $\frac{\text{circumference}}{\text{diameter}}$, multiplier, and  (which is also implicated in the exposition)². The G text (Plates 8.1 & 8.2), by contrast, does no more than present algorithms and operational matrices which facilitate the completion of the immediate task in hand. The algorithm for the calculation of the angle subtended at the centre by the sides of regular hexagons, pentagons, octagons and decagons is not even generalised³.

The relationship between the circumference and diameter of a circle is introduced in the G series in G7, chapter 3. This chapter remains almost entirely outside of the esoteric domain (as does the G series generally), but there are three instances of esoteric domain exposition⁴. The first gives an algorithm for the calculation of radius from diameter, or vice versa, using the expression, 'of', in preference to the mathematically more usual, 'x':

The diameter is the width of a circle.

The distance from the centre of a circle

¹ The notion of an irrational number does not appear explicitly in the Y series. The irrational nature of π is implied by the use of approximations rather than an exact value and by the expression of its decimal expansion to thirty-five places of decimals on what appears to be a very long strip of paper which curls round after the thirty-fifth place (concealing the subsequent digits) and snakes off the edge of the page.

² The introduction of π as 'the Greek letter p' also facilitates the metonymic chain: circle $\rightarrow \pi \rightarrow$ Greece \rightarrow Classical Europe, which resonates with the image of 'classical architecture' described in Chapter 6.

³ Although the generalisation is implicit. The final task in the chapter, D4, suggests: 'Try drawing regular shapes with more sides. You could use 12 sides, or 15. Then draw a pattern inside them' (G2, p. 25).

⁴ There is also one esoteric domain task.

to the edge is half the diameter.
We call this distance the **radius**.

$$\text{radius} = \frac{1}{2} \text{ of diameter}$$

(G7, p. 15; graphical index omitted; bold text in red in original)

The second provides an algorithm for the calculation of circumference, incorporating the equality symbol, even though the algorithm represents an approximation:

The distance round the edge of
a circle has a special name.
It is called the **circumference**
of the circle.
The circumference is a bit longer than 3 times the diameter.
If you only want a rough answer for the circumference of a circle
you can use

$$\text{circumference} = 3 \times \text{diameter}$$

(G7, p. 16; bold text in red in original)

The final section of esoteric domain exposition gives another algorithm:

The rule

$$\text{circumference} = 3 \times \text{diameter}$$

gives a rough answer for the circumference,
but it is always too short.

To be on the safe side, you can add **10% to the rough answer**.

(G7, p. 18; bold text in red in original)

The only deviation from the immediate mathematical object and its defined features occurs with the implication of the percentage algorithm in the final extract. This is invoked as a routine and not incorporated into a principled articulation such as does occur in the Y text. In fact, adding on 10% is tantamount to using the algorithm:

$$\text{circumference} \approx 3.3 \times \text{diameter}$$

This algorithm gives an approximate error of +5.0%, whilst the original algorithm generates an approximate error of -4.5%, so the new algorithm, whilst avoiding an underestimate, is actually less accurate. Furthermore, there will be cases in which the 'safe side' is an underestimate rather than an overestimate¹. The failure to declare the

¹ For example: A 5 cm diameter reel of gold wire has 20 turns of wire on it; 320 cm of wire are needed; is there enough? Using $\pi \approx 3.3$, the answer is 'yes', which is, of course, incorrect.

principles behind the algorithm will, in such cases, result in inappropriate computations.

In summary, specialised, that is, esoteric domain practices may be (re)produced as DS^+ or as DS^- . DS^- is realised as procedural message, that is, by message strategies which are algorithms or which are operational matrices. These strategies create no distance between the setting in hand and the more general esoteric domain message. A feature of procedures is that they cannot, of themselves, facilitate generalisation (discussed in Chapter 9) and may, indeed, inhibit it. This is illustrated by the 'safe side' algorithm for calculating the circumference of a circle which is cited above. Discourse is the textual strategy which (re)produces DS^+ . Discursive message makes visible the principles of the structure of topics, articulating esoteric domain topics and signs. Discourse thus enables a distancing between esoteric domain message, in general, and the specific textual setting, so that it facilitates generalisation. Discourse is not another level of text, rather, it constitutes a complexity, a relative completeness of message articulation. In DS^+ practices, there is always a sense in which any substructure within the esoteric domain can be described with respect to any other substructure.

Discourse is almost entirely absent from the G series. The 'rough answers' obtained using the circle algorithms, cited above, are not even marked as instances of 'approximation'. This, despite the fact that 'approximation' appears as a topic elsewhere in the G series. Message in the G series is disarticulated by textual strategies which constitute a collection of context-specific algorithms and operational matrices. This results in the disarticulation of the DS^+ practices of school mathematics, (re)producing them as DS^- . This disarticulation is redolent of Fordist techniques in the labour process (see Braverman, 1974; Matthews, 1989) and of the context-specific 'preconceptualisation' described by Hales (1980). There is no determinism, here. The actions of the G readers are not predetermined or preconceptualised. Rather, a textually wilful substitution of procedures for discourse delimits their access to mathematical practices. The discursive structure of the practices which constitutes the dominant subject position of the activity is invisible in the G texts.

By targeting their readers via the labels Y and G, these message strategies also distribute the esoteric domain. Thus, discursive message is confined to those texts (Y) associated with the dominant voice; the texts associated with the subordinate

voice (G) comprise procedural message. At the level of activity, DS⁺ is exclusive to the dominant subject position; DS⁻ is constitutive of the subordinate subject position.

8.3 *Non-specialised text: recontextualising*

No activity can be entirely esoteric because this would render pedagogic action impossible. Activities, therefore, incorporate a gaze which constructs a public domain. The public domain is structured according to esoteric domain principles, but is relatively non-specialised in terms of expression and content. That is, neither signifiers nor their denotations are specialised. Where content is comparatively specialised, but expression is not, the practice is metaphorical domain. Where expression is specialised, but content is not, the practice is mythical domain.

These domains of practice are (re)produced as recontextualised message. Recontextualised message is realised via recontextualising strategies which ‘grasp’, or which appropriate non-specialised signifying resources and restructure them, in a more or less radical way, such that they conform to esoteric domain grammar. As with specialised discourse, recontextualising strategies may realise procedures. DS⁺ can only be fully realised within the esoteric domain. I have therefore denoted the more complex form of text which is outside of the esoteric domain as *quasi-discourse*.

The G texts are substantially composed of recontextualising strategies. For example, Plate 8.6 illustrates the recontextualising of domestic contexts. At the top of the page in Plate 8.6, we can see that Jim’s lampshade (and, indeed, Jim himself) has been constituted as a resource for the development of a new algorithm (the one that was discussed above in Section 8.2). Jim uses the previous algorithm (circumference $\approx 3 \times$ diameter) to calculate the amount of tape that he will need, rather stupidly neglecting to add on some extra to take account of measuring inaccuracies etc. He then manages to buy exactly 90 cm, which is not enough. In the recontextualising, it has been necessary to constitute Jim as incompetent and also to restructure retailing procedures relating to such commodities. It is likely that tape of this kind would be sold on a roll or by the metre. Introducing too much ‘reality’ into the setting, however, would detract from the mathematical point being made, that is, that the algorithm being used is an underestimate.

At the bottom of the page, in task C3, we are to believe that Zola will line a baking tin by calculating its circumference using the algorithm, circumference $\approx 3 \times$

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Plate 8.6
(G7: page 18)

diameter plus 10% of answer, and then measuring the appropriate length. We might suspect that, left to her own devices, she (and it almost has to be 'she' if cooking is involved) would pull a length of baking sheet from the roll and wrap it around the tin to get the measure, she might even guess; it would hardly matter if she cut too little and had to add another piece.

On the following page, the following task (C6) is set:

- Eve packs rolls of carpet.
 When the carpet is rolled up
 its diameter is about 45 cm.
 Eve puts 3 bands of sticky tape
 round the roll.
- (a) About how much tape does she need for one band?
 - (b) How much does she need for each roll of carpet?
 - (c) The sticky tape comes in 100 m rolls.
 Roughly how many carpets can Eve pack with 1 roll of sticky tape?
- (G7, p. 19; marginal drawing omitted)

This time, the setting has been created out of the recontextualising of a work context. Eve is a manual worker in a factory or warehouse. She packs just one kind of carpet (different pile thicknesses would result in different sized rolls) and uses just a single turn of tape for each band (the answer in the Teacher's Guide gives 'Accept 145-155 cm' for part (a)) and there is no waste on the roll of tape. Clearly, these and other 'assumptions' made in the formulation of the task produce an unlikely image of Eve's working practices¹, but, again, anything more 'realistic' would call for a different algorithm (or, of course, the application of discourse).

These tasks are mythical domain because they incorporate specialised expressions, diameter, circumference. The 'myth', as will be discussed below, constructs mathematics as a necessary condition for participation in domestic and manual labour, the mathematics parading as a generic and essential tool as it does in the utilitarian epistemology discussed in Chapter 2. But it is generic only because any number of contexts can be recontextualised to fit its procedures (algorithms and operational matrices). Thus public domain settings are constituted as surrogates for mathematical discourse. The public domain is, in this sense, driven by the esoteric. The absence of discourse from the G series texts means that recontextualising must be achieved by the author of the texts. The gaze is not rendered visible to the G reader.

Discourse has been incorporated in the Y1 chapter, 'Polygons and circles', as has been discussed above, Plate 8.7 shows the first exit from the esoteric domain

¹ The familiar use of the name as well as the nature of the task suggest that it is Eve's working practices and not those of management that are the object of analysis.

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 8.7
(Y1: pages 92-3)

following the introduction of π . The two boxes in the middle of page 92 summarise the discourse. These have the look of algorithms, but extensive metonymic articulations within the esoteric domain have been put in place in the earlier expositions in this and previous chapters, as has been illustrated above. The graphs within the boxes incorporate a flowchart index which was used in the first chapter in the book (Y1.01) and also in Y1.03 (as described in Chapter 6). Y1.01 also introduced the notion of an inverse operation; the relationship between the two graphs is a tacit reminder of this notion. The restriction to algorithms in the G7 chapter demands a high degree of similarity in the tasks, those in section C, for example, are all about wrapping something around something circular. The introduction of discourse in the Y chapter facilitates a much more wide-ranging gaze. The Y text is concerned with, for example, the computation of diameter from circumference (in the G7 chapter, a new algorithm must be provided for this in section D), the introduction of a semicircle, and wrapping a rectangle around a cylinder.

The recontextualisings in Plate 8.7 are of a different nature from those in the G chapter. Although the text (apart from the boxes) is mythical domain, mathematics is not mythologised as a necessary condition for participation, because there is no participation. The setting in the first task (B6) is historical¹. Tasks B7-13 recontextualise objects, but the open form of narrative does not specify any particular setting. The computation in task B11, for example, is linked to the marginal drawing (of a pot of paint and carelessly placed, paint-filled brush) only via the signifier 'paint pot'. There is no recontextualising of a painting and decorating task. The exposition and tasks at the start of section C (Plate 8.7) recontextualise a process (although not a precise setting) from the general field of economic production. However, this is accompanied by an iconic rendering of task C1 which incorporates humour. This is clearly not a factory in which you, or indeed anyone else, might work. The joke is a putting together of that which must be kept apart, insects and food². Humour, as a resource, has an alienating effect as has the use of the historical setting in B6. The 'myth' is not one of participation, but of omniscience. Mathematics has a valid description for just about everything. This will be explored further in Chapter 9.

¹ In this extract, the historical nature of the setting also ensures that it is the engine that is being foregrounded rather than the manual labour with which it is associated. The miller is far too tied up with his work to notice the mathematical actions of the mill and of the horse, although, judging by the direction of the gaze of the latter in relation to the labelling of the mill, the horse may have its suspicions.

² The joke also effects a racist alienation of cultures which classify insects, even chocolate coated insects, as edible.

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Plate 8.8
(Y4: pages 108-9)

Recontextualising, in these examples, consists of the *projection* of specialised objects and relationships onto non-specialised contexts. It might be analysed as a sequence incorporating interpellation, de-location and re-location. The nature of specialised objects and relationships is made visible in esoteric domain discourse which thereby facilitates recontextualising at the level of textuality¹. Where the text does not enter the esoteric domain, recontextualising cannot be made visible; it is achieved by the author, but not by the reader. However, it is still possible to create a distancing from an immediate practice by producing a text which constitutes a commentary on something else. Tasks C1 and C2 in Plate 8.8 illustrate this. In task C1, the immediate setting which is recontextualised is professional photography. We are introduced to a character, Nisha, and to some minimal (although more than is usual in the Y series) detail of her life: her car is getting old; she is a photographer. This localises the context. However, the task is not about photography, but constitutes a commentary on Nisha's practice, a commentary which will enable her to make decisions. Similarly, task C2 localises its immediate setting by referring to the printing of 'this book'. The task, however, does not concern printing as such, but the financial organisation of the production process. In this case, some apparently technical terms are introduced ('production cost', 'storage cost per copy') and the commentary moves from an initial simplification to one of more complexity when warehousing is taken into account. The chapter in which these tasks appear is titled 'optimisation', which term is explained in the exposition at the start of section C (the title of which repeats the chapter title):

Optimising means choosing the best thing to do.
'Best' may mean 'cheapest', 'quickest', 'largest', 'smallest', and so on,
depending on the circumstances.

What is 'best' depends on your point of view. If a driver has a long journey to do, she² may think it best to get to where she is going as quickly as possible. If so, she will travel at the fastest possible speed. But at a fast speed the car uses up petrol at a greater rate, so that will be more expensive than going at a slower speed. Only the driver herself can decide whether time or money is more valuable to her.

(Y4, p. 107; my footnote)

Optimising/optimisation is a recontextualising from economics, another specialised knowledge. The recontextualising has resolved 'optimising' into two components. The objective component, in the first paragraph of the extract, is that which can be measured, mathematically, of course. The subjective component, in the second paragraph, relates to an individual point of view. But 'optimising' is being

¹ The corresponding statement at the level of structure would be: 'DS⁺ facilitates the gaze'.

² The generic use of feminine pronouns, used here, is balanced by the use of masculine pronouns elsewhere; this tactic is used in *SMP 11-16* in preference to gender neutral expressions such as s/he.

presented as a generalisable strategy or set of strategies for the production of commentaries upon a diversity of circumstances. The commentary generates optimal possibilities from which a selection is made, subjectively, outside of the gaze of mathematics. In an economics text, we might see 'optimising' articulated with other specialised objects, cost efficiency, and the like. Within this mathematics text, the term is referred to a vaguely specified corpus of esoteric domain message. 'Optimising' is paraded as if it were DS^+ , but its public/mythical domain location denies this possibility, because settings outside the esoteric domain are not fully regulated by that domain. The construction of a region of apparently articulated discourse outside of the esoteric domain has been referred to as *quasi-discourse*. As is the case with discourse, quasi-discourse occurs to a far greater extent in the Y series than in the G books.

The action of the gaze in the instances of recontextualising described so far (in both books) is projective in direction. That is, esoteric domain referents are being projected onto public domain settings: a circle is projected onto a roll of carpet; mathematical relationships are projected onto the economic practices of a photographer; etc. In each case, it is esoteric domain mathematics that is being used in the explication of a non-mathematical setting. Mathematics itself becomes a language of description. The situation in Plate 8.9 is somewhat different.

The text opens in the public domain. The mode of expression remains non-specialised throughout the two pages, but the content moves to a specialised one as the photograph is replaced by a drawing which puts the car (but, significantly, not the driver¹) into a 'glass box' connoting the projected images of the car onto the faces of the box, that is, the various plans and elevations of the car. The text, however, makes no explicit reference to projections or to elevations. Instead, the reader is provided with 'front view', 'side view', 'back view', and 'top view'—the latter 'often called the plan view'—with the 'glass box' expanding slightly in one or more dimensions in each view. Apart from the introduction of the term, 'plan', there are no specialised expressions in this text. Yet the setting, insofar as it concerns projections, is specialised. Essentially, the mathematical terms 'projection' and 'elevation' have been replaced by 'view'. This is a metaphorical relationship which is introjective in the sense that 'view' is appropriated as an approximate instance of 'projection'. The text

¹ Humans are not to be incorporated into mathematical drawings. It is interesting that the front covers of G5 and Y1 seem to defy this principle: Y1 shows a contour map of a face; G5 includes the view from above of a house which has had its upper floor removed, complete with man, woman and mammal of indeterminate species watching a very thin television.

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Plate 8.9
(G4: pages 42-3)

is metaphorical domain: it comprises a specialised setting incorporating non-specialised forms of expression.

Clearly, the text also incorporates non-specialised expressions which signify non-specialised content, the car, and, in this respect, it is public domain. This illustrates an important feature of the kind of analysis being conducted here. That is, in general, no text is entirely confined to a single domain, because there is almost always a combination of specialised and non-specialised significations. Certain significations, those relating to number, for example, are frequently ambiguous. The text in Plate 8.9 incorporates a public domain resource, the car, but the signification which is foregrounded, is the denotation 'view' and a connotation of 'projection'. The car is a replaceable¹ carrier for a specialised setting. In the mythical domain, mathematics describes the non-mathematical; in the metaphorical, it is the non-mathematical which describes the mathematical.

The introjective nature of recontextualising within the metaphorical domain constitutes a possible contamination of the esoteric domain, because public domain connotations are being attached to esoteric domain significations. In the case illustrated in Plate 8.9, for example, 'plan' (which is a projection) is being confused with a view from the top. The more so by its incorporation into the locution 'plan view'. Y5 employs a similar metaphor, but blocks the introjection by explicitly distinguishing between the public and esoteric domain significations, as is illustrated in Plate 8.10. The 'perspective view' is what you see if you look at the table and this is an approximation to an orthographic projection only if you are 'a long way away' from the object, but 'the orthographic projection is not a view you could ever actually see'.

In the example of the car, the metaphorical domain text comprises procedures: the drawings of the car are operational matrices. Metaphorical domain text may also incorporate quasi-discourse, although this is rare, because this strategy tends to be confined to the Y texts. These generally move from the metaphorical into the esoteric domain (as is the case with the text immediately following that in Plate 8.10) or incorporates a minimum of non-specialised expression to signify mathematical objects (such as the use of block alphabetical characters or 'flags' to signify geometrical figures). As was discussed in Chapter 7, those Y chapters dealing with the topic probability are distinguished in that they include comparatively little esoteric domain space. Quasi-discourse therefore becomes necessary. In Plate 8.11, for example, the

¹ But not entirely arbitrary—it has been selected.

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Plate 8.11
(Y4: page 75)

Plate 8.10
(Y5: page 93)

'spinner' is introduced as a surrogate for a kind of generalised probability generator. The spinner is then used to represent or to describe other probabilistic situations.

Message Strategies

<i>Specialised text esoteric domain</i>	
(a)	procedure
	(i) algorithm
	(ii) operational matrix
(b)	discourse
<i>Non-specialised text—projective recontextualising: public & mythical domains</i>	
(a)	procedure
	(i) algorithm
	(ii) operational matrix
(c)	quasi-discourse
<i>Non-specialised text—introjective recontextualising: public & metaphorical domains</i>	
(a)	procedure
	(i) algorithm
	(ii) operational matrix
(c)	quasi-discourse

Table 8.1

8.4 Message strategies: summary

In this Chapter, I have considered the constitution of specialised and non-specialised text within the four domains and in the two modes, discourse and procedure. These message strategies (re)produce the domains and level of DS of the practices of the activity: they are summarised in Table 8.1. The (re)production of practices is never realised entirely separately from the (re)production of subject positions as has been made apparent in the discussion. For example, it has been mentioned that esoteric domain message, discourse and quasi-discourse tend to be confined to the Y series (dominant voice), whilst the G texts (subordinate voice) are largely comprised of procedures and are predominantly located within the public domain. This differentiation constitutes a distribution of practices such that DS⁺ esoteric domain practices and DS⁻ public domain practices are referred to relatively dominant and

relatively subordinate subject positions, respectively. At the level of text, 'to-be-apprenticed readers' are produced via esoteric domain discourse and non-esoteric domain quasi-discourse, 'to-be-alienated readers' are produced via procedures which are largely non-esoteric domain. Chapter 9 continues the discussion of distributing strategies, but the emphasis shifts to the (re)production of subject positions rather than practices.

Chapter 9

Textual Strategies & Resources II: *distributing strategies*

As I have illustrated in Chapter 8, message strategies are implicated in distributing strategies insofar as they are targeted at specific voices. In this Chapter, I shall focus on generalising and localising distributing strategies other than those which simply divide domains and level of discursive saturation between subject positions. I shall discuss generalising and localising in relation to the use of signifying modes, the selection of settings, and the objectification of readers.

In this Chapter, I wish to give expression to a more substantial proportion of the scheme than is possible through the form of qualitative analysis which is the principle method used in this thesis. In respect of this, I shall include reference to some quantitative analysis which compares the Y and G series in terms of modes of signification. Some reference will also be made to the B and R series, although these remain backgrounded for the purposes of the main study. The quantitative analysis is intended as a support to the semiotic analysis, which is the predominant method used in this thesis.

9.1 *Signifying modes*

9.1.1 *Quantifying signifying modes*

Signifying modes are textual resources that were introduced in Chapter 4; they are listed in Figure 4.4 as: icon (cartoon, drawing, photograph); index (table, graph); and symbol; index and symbol can be non-manuscript or manuscript. It was argued in Chapter 4 that the use of different signifying modes can orientate the text towards generalisation or towards localisation. In particular, the *visual code of presence*, which is intrinsic in iconic signification, itself effects a semiotic localising. Thus a measure of the relative use of iconic and non-iconic modes (and of the categories of iconic mode) in the G and Y books has face validity as an indicator of the extent of localising within each series. This being the case, I decided to produce a quantitative comparison to support the qualitative analysis. It must be emphasised that the operational difficulties involved in quantitative content analysis are readily

acknowledged. The validity of indicators in relation to concepts and the reliability of operational recognition rules can never be guaranteed. On the other hand, quantitative methods can be far more representative in the sense that they can achieve a more exhaustive coverage of the empirical text. Crucially, the quantitative content analysis is being used in support of and, occasionally in dialogue with, the more detailed semiotic analysis and not in any sense as a replacement for it.

Because signifying modes are textual resources, they are, by definition, analytically very low level constructs. By virtue of this, the definitions given in Chapter 4 were, in terms of operational recognition, more or less sufficient to make a start on the practical analysis. One additional qualifier was made at the outset: indices such as arrows and emphatic borders were to be ignored completely unless they are clearly part of (or where they overlapped with) another section of text. This is principally because a decision to include them in the analysis would render decisions concerning spaces enclosed by such indices highly problematic. Further minor qualifications were made in the course of conducting the analysis and are included in Appendix 3.

There are two obvious ways of quantifying signifying modes, that is, to count them in terms of frequency or in terms of area. A measure in terms of frequency would clearly problematise the validity of a comparison between symbolic and non-symbolic text, because an iconic or an indexical text visually terminates itself in a way that a symbolic text does not¹. Furthermore, there is clearly a positive relationship (although not necessarily a linear one) between the size of an icon or index and its visual impact. An area measure will also give an indication of how much of the book or series as a whole comprises each signifying mode. I therefore decided to compare signifying modes in terms of the area occupied by each mode on each page of a sample of each of the G and Y series and also to include a sample from the B series (one of the median tracks). Each page of the books is a portrait sheet of approximately 24 cm by 17 cm. A centimetre grid of this size was prepared on a transparent, plastic sheet, this was laid over each page and the number of grid squares containing any part (however small) of text interpreted within each category of signifying mode were counted. Where a square contained more than one mode, a decision had to be taken as to which dominated in terms of coverage.

¹ Regester (1991) uses a frequency count of 'visuals' of different kinds in her analysis of history textbooks. However, she is comparing the visuals in two categories of textbooks and is not directly concerned with the ratio of visuals to verbals.

The intention was, firstly, to compare the G series with the Y series. However, it is also apparent from the most cursory inspection of the two series that there are trajectories, in terms of modes of signification, within each. I therefore decided to analyse the beginning and end of each series. The beginning and end of a series of books are, to an extent, naturally punctuated by, respectively, the first and last books in the series. However, Books G1 and G8 constitute a smaller proportion of the main G series as a whole¹ (approximately 25%) than do Books Y1 and Y5 with respect to the main Y series² (approximately 40%). I therefore decided to analyse Books G1 and G8 and an equivalent proportion of the Y series, the latter being defined by the first and last 100 pages of the Y course. Book Y5 ends with a 40 pages of 'General review', which comprises tasks but not exposition and, in a sense, comes after the end of the course. I decided to omit the 'General review' section and analyse the 100 pages immediately preceding it, that is, pages 48 to 147 inclusive, together with the first 100 pages of Y1. A random sample of 40 pages from Book B3 (total 92 pp) was also analysed³. B3 is the middle book in the 5-book series and so nominally represents the central book in the book-based part of the SMP scheme. The results of this analysis are given, in Tables 9.1-3, and discussed in Sub-section 9.1.2.

Two further counts were made. Firstly, an alternative measure of the relative use of photographs in the books was made by counting the number of photographs in each. In addition to the G and Y series, the photographic content of the B and R⁴ series was also measured, in this way. Photographs vary considerably in size and some of them are overlaid on other photographs. I decided to count, as individual photographs, images that had obviously been produced separately and then overlaid. Where an enlargement of an image was overlaid on the 'original', they were not counted separately. The numbers of photographs in each book in the G, B, R and Y series are given in Table 9.7.

Secondly, since the type sizes in the G and Y series appear to be different—the G type appearing to be bigger—the number of non-manuscript, alphanumeric characters

¹ ie Books G1-8, excluding ancillary materials such as worksheets and cards, supplementary booklets etc.

² ie Books Y1-5, excluding worksheets and the two extension books.

³ Pages 54 and 55 were omitted *a priori* from the sample from Book B3. This was because the symbolic text on these pages is produced as if it were newspaper text (although not fully iconically). These pages thus contain very high numbers of characters and are wholly unrepresentative of Book B3 and of the B scheme as a whole. The occurrence of one or more of these pages in the random sample would have dramatically distorted the sample. The choice of 40 pages was made in relation to an additional form of analysis which is described below.

⁴ Books R1-3 are intended to substitute for books B3-5 for 'more able' students who initially follow the B, or middle, track.

Mean Page Coverage by each Signifying Mode

	Icon	Index					Symbol		Total
		MS			non-MS		MS	non-MS	
		Graph	Table	Graph	Table				
Book	Cartoon	Drawing	Photo	Graph	Table	Graph	Table		
G1	44.8±15.3	67.7±15.4	4.0±5.6	5.4±4.2	4.6±4.2	11.5±8.4	0.5±1.0	10.2±5.9	104.2±10.2
G8	0.3±0.6	43.7±12.8	32.9±16.5	5.6±6.2	2.3±3.4	23.7±12.6	3.1±2.9	7.3±3.6	127.0±10.3
B3	6.7±6.2	23.7±13.6	6.1±9.8	2.8±4.4	0.0±0.0	53.1±15.0	4.2±4.6	1.8±1.9	133.4±15.5
Y1	8.6±5.0	29.3±9.6	1.1±1.6	1.4±1.5	0.0±0.0	32.7±9.5	3.9±2.3	4.0±2.4	157.8±9.7
Y5	4.0±3.3	6.9±5.0	2.1±2.8	0.0±0.0	0.0±0.0	73.4±11.7	5.0±3.5	0.8±0.8	151.1±10.7
									252.9±9.2
									245.7±8.4
									231.7±8.0
									238.7±6.2
									243.3±5.9

means & 95% confidence intervals; all data to 1 decimal place

Table 9.1

Table refers to mean numbers of cm squares per page containing the relevant signifying mode (1 page is approximately 408 square centimetres)

The data relates to the whole of each of Books G1 and G8 (pp 0-61 in each case), to pp 1-100 of Book Y1 and to pp 48-148 in Book Y5 (ie the last 100 pages before the 'General Revision' section which concludes the Y series) and to a random sample of 40 pages from Book B3 (the central book in the B series, pp 54-5 excluded).

MS = 'manuscript form'

Mean Page Coverage by Symbol, Index, Icon

	Book G1	Book G8	Book B3	Book Y1	Book Y5
Symbol	114±11	134±12	135±16	162±10	152±11
Index	22±10	35±14	60±17	38±10	78±12
Icon	116±20	77±17	37±16	39±11	13±7
Non-Icon	136±15	169±16	195±17	200±10	230±9.7
Page Coverage	253±9	246±8	232±8	239±6	243±6

means & 95% confidence intervals; all data to nearest whole number

Table 9.2

Ratios of Signifying Modes

	Book G1	Book G8	Book B3	Book Y1	Book Y5
non-symbol:symbol	1.2	0.8	0.7	0.5	0.6
icon:non-icon	0.9	0.5	0.2	0.2	0.1

all data to 1 decimal place

Table 9.3

Table refers to mean numbers of cm squares per page containing the relevant signifying mode (1 page is approximately 408 square centimetres)

The data relates to the whole of each of Books G1 and G8 (pp 0-61 in each case), to pp 1-100 of Book Y1 and to pp 48-148 in Book Y5 (ie the last 100 pages before the 'General Revision' section which concludes the Y series) and to a random sample of 40 pages from Book B3 (the central book in the B series, pp 54-5 excluded).

Character Densities in Books G1, B3 & Y5

Book G1				Book B3				Book Y5			
page	characters	symbol space	character density	page	characters	symbol space	character density	page	characters	symbol space	character density
0	194	48	4.04	2	506	92	5.50	48	1121	168	6.67
1	306	78	3.92	3	422	89	4.74	49	941	144	6.53
5	744	114	6.53	6	916	147	6.23	52	562	108	5.20
6	531	87	6.10	7	263	50	5.26	53	1114	179	6.22
7	420	83	5.06	10	635	124	5.28	56	1129	176	6.41
8	377	73	5.16	13	1162	242	4.80	57	1921	263	7.30
9	161	41	3.93	15	538	95	5.66	58	1037	180	5.76
10	424	78	5.44	18	607	90	6.74	59	958	163	5.88
11	503	85	5.92	22	384	75	5.12	64	1162	181	6.42
13	286	60	4.77	23	815	212	3.84	65	0	0	—
17	901	184	4.90	25	253	61	4.15	69	974	137	7.11
18	487	94	5.18	30	873	182	4.80	70	1356	179	7.46
19	537	100	5.37	31	618	139	4.45	72	850	133	6.39
21	498	104	4.79	33	638	180	3.66	74	586	85	6.89
22	486	102	4.76	34	1006	219	4.59	78	513	93	5.52
23	574	109	5.27	35	490	83	5.90	79	985	183	5.38
24	586	113	5.19	36	1358	238	5.71	83	701	131	5.35
25	658	123	5.35	37	647	121	5.35	87	792	121	5.35
26	285	55	5.18	39	616	140	4.40	92	585	85	6.38
31	718	150	4.79	41	924	163	3.67	95	390	65	6.00
32	0	0	—	43	1127	179	6.30	97	650	112	5.65
33	484	86	5.63	44	820	146	5.62	102	716	130	5.51
36	859	155	5.41	48	840	115	7.30	106	1362	235	5.80
37	307	67	4.58	50	582	96	6.06	108	1242	221	5.62
38	602	114	5.28	53	556	97	5.73	111	1181	238	4.96
41	310	51	6.08	57	778	118	6.59	113	1176	239	4.92
42	720	158	4.56	58	801	130	6.16	114	1215	234	5.19
43	300	79	3.80	59	1034	173	5.98	115	1232	218	5.65
44	499	88	5.67	60	245	55	5.34	116	1469	228	6.44
45	512	95	5.39	64	780	146	5.34	117	954	182	5.24
47	1006	155	6.49	65	553	154	3.59	121	1297	201	6.45
49	641	148	4.33	68	965	158	6.11	130	465	89	3.22
50	787	167	4.71	72	586	87	6.74	131	815	149	5.47
53	496	109	4.55	73	703	112	6.28	133	1332	203	6.56
54	522	116	4.50	76	315	71	4.44	137	695	122	5.70
55	330	57	5.79	82	1203	198	6.08	138	585	143	4.09
56	527	107	4.93	85	826	138	5.99	140	871	173	5.03
58	783	159	4.92	86	982	164	5.99	141	1114	195	5.71
59	1022	196	5.21	90	508	134	3.79	145	531	98	5.42
60	905	165	5.48	92	646	121	5.34	147	614	119	5.16
mean	532	104	5.11*	mean	714	133	5.39	mean	929	158	5.89†
std error	56	7	0.11*	std error	42	8	0.15	std error	57	9	0.12†

* Omits page 32

† Omits page 65

Table 9.4

Each table represents a random sample of 40 pp. from the relevant book. Pages 54 & 55 of Book B3 were omitted from the sample because the symbolic text on these pages is produced as if it were newspaper text and is thus wholly unrepresentative of B3 and of the B scheme as a whole. Page 32 in G1 and page 65 in Y5 contain no symbolic text and so data relating to these pages had to be omitted from the character density calculations.

on each page was counted for a random sample of 40 pages from Book G1 and for another of 40 pages from that portion of Book Y5 which was sampled in the previous analysis (ie pp. 48-147). The number of characters on each page of the random sample of 40 pages from book B3 were also counted. Characters are letters or numerals or mathematical or other symbols which are included under the heading 'symbol' in the earlier analysis of signifying modes¹. The mean number of characters per page was counted for each sample and a measure of 'character density' per page was computed by dividing the number of characters by the amount of symbol space on the relevant page (taken from the analysis of signifying modes). The mean character density per page for each sample was also computed. The results are given in Table 9.4.

9.1.2 Localising via signifying modes

Four sections of text for analysis were selected from the G and Y series on the basis that they constituted the beginnings and ends of each series. The sections of text were: i) Book G1; ii) Book G8; iii) pages 1-100 of Book Y1; iv) pages 48-147 of Book Y5. Viewed in this way, a comparison of the absolute quantities of each signifying mode in each section of text—that is, without reference to the variations within each section—is valid as a description of trajectories within each series and of differences between the beginnings and between the ends of the two series.

However, it is also conventional to test for the statistical significance of the results of quantitative analysis with respect to a null hypothesis. In this case we must construct an imaginary text from which samples—assumed to be (but actually not) random—were taken and calculate the probabilities of obtaining the actual results shown in Tables 9.1 and 9.3. In consideration of the questionable relevance of the null hypothesis, in this case, I have decided to discuss the results of this analysis, firstly, as repeated comparisons of 100% samples of the 'beginnings' and 'ends' of each series. In this discussion, the confidence intervals given in Tables 9.1 and 9.2 will be ignored. Subsequently, some qualifying comments will be made on the basis of significance tests made using an estimate for χ^2 . The data relating to Book B3 is not central to the analysis and was obtained rather differently from that relating to the

¹ It was decided that a fraction, such as $\frac{3}{4}$ would count as three characters (because of the way in which such text is produced), but that the percent symbol, %, would count as one (for the same reason). There were, in fact, few instances of either. Symbols signifying calculator keys were counted as single characters.

Ratios & Order of Means of Signifying Modes

mode	G1	G8	B3	Y1	Y5	order
icon	8.9	5.9	2.8	3.0	1	G1>G8>Y1>B3>Y5
non-icon	1	1.2	1.4	1.5	1.7	G1<G8<B3<Y1<Y5
cartoon	149.3	1	22.3	28.7	13.3	G1>Y1>B3>Y5>G8
drawing	9.8	6.3	3.4	4.2	1	G1>G8>Y1>B3>Y5
photo	3.6	29.9	5.5	1	1.9	G8>B3>G1>Y5>Y1
index	1	1.6	2.7	1.7	3.5	G1<G8<Y1<B3<Y5
ms graph	3.9	4.0	2.0	1	0.0	G8>G1>B3>Y1>Y5
ms table	2.0	1	0.0	0.0	0.0	G1>G8>B3>Y1>Y5
non-ms graph	1	2.1	4.6	2.8	6.4	G1<G8<Y1<B3<Y5
non-ms table	1	6.2	8.4	7.8	10.0	G1<G8<Y1<B3<Y5
symbol	1	1.2	1.2	1.4	1.3	G1<(G8,B3)<Y5<Y1
ms symbol	12.8	9.1	2.3	5.0	1	G1>G8>Y1>B3>Y5
non-ms symbol	1	1.2	1.3	1.5	1.5	G1>G8>B3>(Y1,Y5)
total	1.1	1.1	1	1.0	1.1	(G1,G8,Y5)>(B3,Y1)

The table shows the ratios of each signifying mode per book expressed to the smallest non-zero value as base (which appears in the table as 1); » indicates a factor greater than 10.

Table 9.5

Ratio of Means of Signifying Modes

Comparison	icon	cartoon	drawing	photo	index	ms graph	ms table	non-ms graph	non-ms table	symbol	ms symbol	non-ms symbol	total
G1:G8	1.5	149.3	1.5	0.1	0.6	1.0	2.0	0.5	0.2	0.9	1.4	0.8	1.0
G1:Y1	3.0	5.2	2.3	3.6	0.6	3.9	0	0.4	0.1	0.7	2.6	0.7	1.1
Y1:Y5	3.0	2.2	4.2	0.5	0.5	0	-	0.4	0.8	1.1	5.0	1.0	1.0
G8:Y5	5.9	0.1	6.3	15.7	0.4	0	0	0.3	0.6	0.9	9.1	0.8	1.0

The table shows the ratios of signifying modes between the books in the lefthand column.

Table 9.6

G and Y books. Nevertheless, some mention of this data will be made within each part of the discussion.

Tables 9.5 and 9.6 show direct comparisons between the sections of texts (to be referred to as 'books', for simplicity) in terms of the ratios of the mean values represented in Tables 9.1 and 9.2. Table 9.5 expresses the mean value of each signifying mode for each book as a ratio with respect to the smallest non-zero value as base. The final column shows the ordering of the books. Table 9.6 shows the ratios of the mean values of each signifying mode relating to the books shown in the lefthand column, for example, the first figure in the second column (1.5) is the ratio of iconic signification in G1 to that in G8 (ie 1.5:1).

Most of the sequences in the righthand column of Table 9.5 are in a direction which, on the face of it, suggests an increase in generalising strategies within each of the series and an increase in generalising strategies in moving from the G to the Y series. Thus icons are decreasingly incorporated as resources within the sequence G1 → G8 → Y1 → Y5. However, there are some notable exceptions which will be referred to in the following discussion.

The use of cartoons is a minimum in Book G8 and not, as we might have expected, in Book Y5. With reference to Table 9.1, it is clear that cartoons represent a substantial (greater than 5% of the average page coverage) resource only in Book G1 and that they are almost entirely absent from Book G8. By contrast, in the 'photo' column of Table 9.1 Book G8 stands out as the only book for which photographs constitute a substantial resource. We can describe a trajectory within the G series which constitutes a substitution of photographs for cartoons and drawings. In other words, there would appear to be an overall increase in localising strategies within the G series. This reading is supported by the data in Table 9.7 which points to a massive increase in the use of photographs in the last two books of the G series.

In addition to a strengthening of the visual code of presence, the movement from cartoons to photographs also connotes a trajectory from the playful and fictional world of childhood to the real world of adulthood. For example: Plate 9.1 includes a G1 image of shopping¹ incorporating puns ('J. Nare'/*Jane Eyre*²; 'Pseudo'/Cluedo³) and imaginary magazine and trade names; Plate 9.2 shows a G7 image which

¹ This G1 chapter does include a photograph, apparently of a greengrocer's shop, but it is a rare instance in the early G books.

² By Charlotte Bronte.

³ A popular board game.

incorporates the real thing. Plate 9.3 implicates a G1 image of work, again, playfully satirised; in Plate 9.4, from G8, both the drawing and the three photographs are far more 'real' than the G1 cartoon. The G series may thus be understood as constituting a double localising trajectory. Firstly, moving from a weak to a strong visual code of presence and, secondly, moving from the freedom of play to the local certainty of the real world.

Numbers of Photographs in *SMP 11-16* Books

G Series		B Series		R Series		Y Series	
Book	Photos	Book	Photos	Book	Photos	Book	Photos
G1	3	B1	0	R1	6	Y1	4
G2	1	B2	5	R2	1	Y2	5
G3	0	B3	4	R3	12	Y3	3
G4	6	B4	21			Y4	2
G5	11	B5	6			Y5	6
G6	18						
G7	80						
G8	75						
pp/photo	2.5	pp/photo	15.9	pp/photo	23.8	pp/photo	40.6

Table 9.7

On the other hand, the connotative value of cartoons may be interpreted as localising within childhood. In other words, although cartoons involve a weaker visual code of presence and are thus less localising than photographs, their connotative values are highly localised within childhood. In this sense, both G1 and G8 localise through their implication of icons. G1 via the connotations of the particular category (cartoons), G8 via the enhanced visual code of presence of its preferred category (photographs).

It is apparent from Table 9.1 that not all of the reduction in cartoons and drawings between G1 and G8 can be accounted for by the increase in photographs, because there is an overall reduction in icon space between these two books. Corresponding to this decrease, there has been an increase in both indexical and symbolic space, most substantially in terms of non-manuscript graphs (a doubling) and non-manuscript symbol (an increase of about 25%). Thus, two trajectories can be described: an exchange of cartoons and drawings for graphs and an exchange of cartoons and drawings for symbols. These trajectories represent shifts from the modes exhibiting greatest localising to those exhibiting most generalising. The overall trajectory, therefore, is towards an increase in generalising. However, the analysis of

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 9.2
(G7: page 1)

Plate 9.1
(G1: page 19)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 9.4
(G8: page 37)

Plate 9.3
(G1: page 27)

domain trajectories in Chapter 5 concluded that only about 5% of Book G8 comprises esoteric domain text. Indeed, approximately one third of the graphs in G8 are road maps and most of the others are diagrams of boxes. The strategy which generalises via the implication of graphs and symbols is, in other words, opposed within the G8 text by strategies which localise via public domain settings (see, further, Section 9.2).

Within the Y series, there seems to be a reduction in symbolic space between Y1 and Y5. However, this is entirely accounted for by the increase in index space. The principle trajectory in the Y series represents an exchange of icons, mainly drawings, for non-manuscript graphs. But, as is apparent from the data in Table 9.2, the increase in indices in Y5 as compared with Y1 exceeds the decrease in icons. This accounts for the difference in symbol space. Again, with reference to the analysis of domain trajectories in Chapter 5, the amount of esoteric domain space in Y5 is more than ten times that in G8 at over 50%. Y5 graphs are generally mathematical objects. Although Y5 does include projections of the globe and of other objects, in each case it is the nature of the projection which is foregrounded, rather than the resulting 'map'. The generalising, icon \rightarrow index trajectory between Y1 and Y5 is therefore not countered within the text as it is in the G series.

As noted above, Table 9.5 indicates that the differences between the use of different signifying modes by each book considered are generally such as to suggest an increase in generalising strategies within each series (G and Y) and between the G and Y series. Thus, there is a reduction by a factor of 1.5 in iconic space between G1 and G8 (Table 9.6), a reduction by approximately 2 between G8 and Y1 (Table 9.2) and a reduction by 5.9 between Y1 and Y5. Indexical space increases by a factor of 1.6 within the G series. Thus the indexical space within G8, the last book in the G series, is approximately the same as that in Y1, the first book in the Y series. Indexical space is increased by a factor of 2 within the Y series. Symbolic space increases within the G series¹ and is greater in Y1 than in G8 (the subsequent slight reduction between Y1 and Y5 has been explained above). B3 (nominally, the median book in the textbook scheme) is close to Y1 in terms of iconic space, close to G8 in terms of symbol space and between Y1 and Y5 in terms of indexical space. The difference between the G and Y series in terms of symbolic text is probably underestimated by the use of textual space as a measure. This is illustrated by the data

¹ The factor of this increase is small. However, since the starting point (symbol space in G1) is approximately 45% of the total page coverage, factors are of somewhat less importance than additive quantities.

in Table 9.4 which indicates a statistically significant difference¹ in character density between G1 and Y5 symbolic text.

The use of manuscript in indexical and symbolic space was described (in Chapter 4) as constituting an iconicising of the indexical/symbolic and hence measures localising. In absolute terms, the amounts of manuscript space are small (always less than 10% of page coverage). Nevertheless, there are substantial differences between the books as is shown in Table 9.8. Basically, manuscript text is about three times more common in the G series than in Y1 (and B3) and has almost disappeared by Y5². This confirms the increase in generalising strategies within the Y series and between G and Y. Tables do not occupy a great deal of space, their representation being only about 2% of page coverage. When the quantities of manuscript and non-manuscript tables are added, the mean representation of tables in G1, G8, B3, Y1 and Y5 are, respectively, 5.1, 5.4, 4.2, 3.9 and 5.0 square centimetres per page. So, there appears to be more use of table space in the G series than in the Y series. However, since manuscript tables generally use far more space than non-manuscript tables and since most of the tables in G1, nearly half of those in G8, but none of those in Y1 or Y5 (or B3) are manuscript, this measure probably fails to give a valid comparison. The larger manuscript tables are, of course, also constituted iconically, so that they are, in this respect, localising.

Incidence of Manuscript Text

Book	ms text	
	mean space	% of page coverage
G1	20.2	8.0
G8	15.1	6.1
B3	4.6	2.0
Y1	5.4	2.3
Y5	0.8	0.3

Table 9.8

Finally, with reference to Table 9.7, it should be mentioned that 18 of the 21 photographs in B4 and 9 of the 12 in R3 are incorporated into tasks and exposition in which photographic enlargement is used as a metaphor for mathematical enlargement. In these texts, it is the photograph as photograph that is foregrounded and not its

¹ $\chi^2 \approx 4.856$ which indicates significance at the 0.05 level.

² Manuscript text in the Y5 sample occurs once as 'handwritten' algebra and three times as headings to the public domain 'Money matters' sections in the book.

**Differences of Means of Signifying Mode Space per
Page Between Book B3 & Books Y1 & Y5**

<i>B3—Y1</i>			<i>B3—Y5</i>		
<i>mode</i>	χ^2	+/-	<i>mode</i>	χ^2	+/-
icon	0.06	-	ICON	7.23	+
non-icon	0.22	-	NON-ICON	13.16	-
cartoon	0.21	-	cartoon	0.60	+
drawing	0.44	-	drawing	5.27	+
photo	1.00	+	photo	0.59	+
index	5.24	+	index	3.14	-
ms graph	0.40	+	ms graph	1.63	+
ms table	X	X	ms table	X	X
non-ms graph	5.17	+	non-ms graph	4.47	-
non-ms table	0.01	+	non-ms table	0.09	-
SYMBOL	7.93	-	symbol	2.98	-
ms symbol	2.11	-	ms symbol	0.81	+
NON-MS SYMBOL	7.00	-	non-ms symbol	3.49	-
ms text	0.05	-	ms text	1.90	+
total	1.86	-	total	5.26	-

Table 9.11

**Differences of Means of Signifying Mode Space per
Page Between Book Y1 & Book Y5**

<i>Y1—Y5</i>		
<i>mode</i>	χ^2	+/-
ICON	15.71	+
NON-ICON	18.51	-
cartoon	2.34	+
DRAWING	17.04	+
photo	0.42	-
INDEX	27.44	-
ms graph	3.40	+
ms table	X	X
NON-MS GRAPH	28.89	-
non-ms table	0.29	-
symbol	1.76	+
ms symbol	6.33	+
non-ms symbol	0.84	+
ms text	3.3	+
total	1.13	-

Table 9.12

In the above tables, bold type indicates significance at at least 0.05 level, capitals indicates significance at at least 0.01 level. The +/- column indicates the direction in the difference, + indicating that the space per page consisting of the relevant signifying mode in the first book named in the heading is greater than that in the second named book.

The estimation for χ^2 is given by the formula
$$\frac{(M_1 - M_2)^2}{\frac{S_1^2}{N_1 - 1} + \frac{S_2^2}{N_2 - 1}}$$
 where M and S are the sample means and

standard deviations of the samples and N are the sample numbers.

scene, so that there is a weakening of the visual code of presence¹. However, even without taking this factor into account, a reading along the bottom rows of Table 9.7 confirms that the G series as a whole makes far greater use of photographs: the occurrence of photographs in the G books is 6.4 times as frequent as that in the B series, 9.5 times that in the R series, and 16.2 times that in the Y series. Furthermore, Table 9.7 suggests a small but relatively steady presence of photographs in the Y series in comparison with the increasing trajectory in the G books. Similarly, Table 9.1 suggests a small presence of cartoons throughout the Y series which, again, stands in contrast to the decreasing trajectory in the G books. This illustrates the very different extents to which these signifying modes are incorporated into localising strategies in the two series. Specifically, they mark the respective forms of localising in G1 and G8 whereas both are present, but marginalised in the Y books.

Index of Differences Between Books

Icon	G1→G8	G8→Y1	Y1→Y5	
Cartoon	G1→G8	G1→Y1	G8→Y1	G8→Y5*
Drawing	Y1→Y5			
Photo	G1→G8	G8→Y1	G8→Y5	
Index	(G1→G8)	Y1→Y5		
non-ms graph	Y1→Y5			
Symbol	G1→G8*	G8→Y1	(Y1→Y5)	
MS text	G1→B3	G1→Y1	G1→Y5	G8→Y1* G8→B3* G8→Y5 (Y1→Y5)

Bracketed differences are not significant at the 0.05 level; asterisked differences are significant at the 0.05 level, but not at the 0.01 level; other differences are significant at the 0.01 level.

Table 9.13

As was mentioned earlier, estimates for χ^2 were made in relation to the data for which the means and 95% confidence intervals are shown in Tables 9.1 and 9.2. For this purpose, the data was treated as repeated comparisons between random samples drawn from normal populations. An estimate for χ^2 (1 degree of freedom) was calculated for each comparison; these values are shown in Tables 9.9-12. In these tables, the results of the tests are given at two levels: all comparisons shown in bold typeface are significant at the 0.05 level, in addition, those which are capitalised are

¹ This argument, of course, weakens the validity of the use of the occurrence of photographs as a sole indicator of localising. The same is true of the qualifications made in respect of increasing generalising in the G series as a result of the increase in indexical and symbolic space. However, the use of quantitative content analysis is being used here simply in support of the more detailed semiotic analysis and is not intended to stand alone.

significant at the 0.01 level. Table 9.13 displays those differences between books (in terms of signifying modes) which are referred to in the above analysis.

By referring the comparisons in Table 9.13 to Tables 9.9-12, it is clear that all of them apart from the three bracketed items are statistically significant and that those marked with an asterisk are significant only at the 0.05 level. Thus the tests may be taken to qualify, or, perhaps, to append caveats to, statements made concerning the indexical trajectory within the G series and the symbolic and manuscript trajectories within the Y series.

9.1.3 Signifying modes: summary

This quantitative analysis of signifying mode space has pointed to trajectories within each series and to differences between them. Within the G series, there is a trajectory which exchanges iconic text in G1 for symbolic text within G8. Insofar as this comparison is reliable, the G series thus becomes less iconic, signalling a progressive weakening of localising strategies¹. On the other hand, the trajectory which exchanges cartoons and drawings in G1 for photographs in G8 constitutes a strengthening of the visual code of presence and indicates an strengthening of localising. It has been suggested, however, that cartoons connote childish play, whilst photographs (at least, the particular photographs used in the later G books) connote adult reality. In this respect, the G series moves from one form of localising to another, matching the chronological age of the student. Trajectories relating to the use of localising strategies within the G series can thus be described as moving in different directions. There is a trajectory from strong to weak localising (the substitution of symbolic for iconic text); there is a trajectory from weak to strong localising (the substitution of photographs for cartoons and drawings); and there is a trajectory exchanging one form of localising for another (the substitution of photographs for cartoons). This last trajectory can be described as the progressive replacement of education by domestic and working responsibilities. The G books signify this change by moving from the child's world of cartoons to the apparent reality of photographs.

Within the Y series, the principle trajectory is represented by an exchange of iconic for indexical. Thus, drawings in Y1 are replaced by (non-ms) graphs in Y5.

¹ This trajectory is, to an extent, countered by the localising use of public domain settings (see Section 9.2).

This signals a progressive weakening of localising strategies¹. The use of cartoons and photographs remains small and more or less constant within the Y series. The trajectory which moves from stronger to weaker localising is, at least as far as the analysis in this section is concerned, unopposed.

Comparing the G with the Y texts, we find that there is a substantial and statistically significant reduction in iconic text between G and Y and corresponding increases in indexical and symbolic text. The latter is greater when the statistically significant difference in character density per page between G1 and Y5 is considered. Within the category of iconic text, cartoons and photographs are substantial resources only within the G series. Here, they comprise approximately 18% of textual space in G1 and 13% of textual space in G8. The largest proportion of Y series textual space which consists of these modes is 3.6% (cartoons in Y1). Finally, manuscript text is never a very substantial resource, in terms of textual space (Table 9.8), but is minimal in the Y series in comparison to the G books.

Overall, the analysis of signifying modes points to the comparatively extensive and effectively constant use of localising strategies in the G series. These strategies are characteristic of procedural message (re)producing DS⁻ practices. The Y series, on the other hand, makes far less use of these localising strategies from the start. At the end of the Y series, there is even less use of these localising strategies and far greater use of generalising modes (indexical and symbolic text). The Y texts are thus characteristically discursive, (re)producing DS⁺ practices and the trajectory which progressively weakens localising points to an apprenticing of the reader.

9.2 *Generalising & localising via setting*

The previous Section analysed the texts in terms of the mode of signification. In this Section, I shall be more concerned with that which is signified. Specifically, I shall consider the ways in which the G and Y books invoke public domain contexts in generalising and localising strategies. I want to introduce the term *setting* to refer to a recognisable space for social practice. Examples of settings are 'domestic' (refinable as, for example, 'shopping', cooking, gardening, DIY², etc), 'work', 'school',

¹ This weakening of localising strategies is not countered by public domain settings as is the case in the G series (see Section 9.2).

² Do-It-Yourself.

‘travel’¹. The setting may also be school mathematics, which, in this study, indicates esoteric domain message. The category ‘setting’ facilitates the organising of public domain text for the purposes of the discussion which follows. It is entirely consistent with the model being employed here to present a description in which only that activity which is of immediate concern, school mathematics, is brought sharply into focus². Nevertheless, the description of that activity is partly achieved through an interrogation of the way in which it acts selectively on its carriers, which are precisely the settings through which it is realised.

Since the category ‘setting’ is not being defined sufficiently tightly to enable a quantitative analysis to be carried out. It has, therefore, been decided to present illustrative cases from the SMP texts. However, it is asserted that similar descriptions could be made using virtually any extracts from these books. The settings that occur most commonly in these texts are the domestic and work settings and these will be discussed below. The school setting is also important and this will be discussed in Chapter 10.

9.2.1 The domestic setting

The domestic space may be glossed as that social region which is concerned with consumption, production and reproduction within the context of a kinship network. Domestic settings are common and often textually extended, in the G series, and rare and generally brief, in the Y series. I shall discuss examples from three categories of domestic setting, shopping, DIY and cooking. Although the domestic setting is the focus of this setting, its rarity of representation in the Y texts requires that I make reference to some text relating to other settings in order to highlight the contrasts between localising and generalising strategies. Essentially, the domestic is used to localise in G, but not in Y.

Pervasive throughout the G series are settings related to shopping, almost invariably produced as narratives ‘from the shopper’s perspective’. Here is an example from Book G4, the shoppers being represented, in this instance, in the third person:

¹ That ‘setting’ is not coterminous with ‘activity’ is exemplified in the plausible overlapping of ‘travel’ with each of the other examples of settings: ‘travel’ may be indexed as a movement between social spaces.

² This was indicated in Chapter 2, where the empirical focus of the thesis was indexed as the recontextualising field of the school site.

Soap powder is sold in 'Euro-sizes'.
 These are Euro-sizes E5, E10 and E20.
 In the McGee family
 there are 2 adults and 3 children.
 The McGees use six E10
 packets of soap powder each year.

C5 What weight of soap powder do the McGees use in a year?

C6 The E10 packet costs £2.59.
 How much does soap powder cost the McGees each year?

C7 The E20 packet contains twice as much
 powder as an E10 packet. It costs £3.89.
 (a) How many E20's would the McGees use in a year?
 (b) How much would powder cost them if they used E20's?

C8 (a) How many E5 packets would the McGees
 use in 1 year.
 (b) The E5 packets are on special offer.
 At the moment, an E5 packet costs 95p.
 How much would the McGees soap powder
 cost in a year if the used E5 packets?

(G4, p. 18¹; drawing omitted)

As was indicated in Chapter 2, there is evidence that 'best buy' decisions are efficiently and appropriately made in context (that is, in the supermarket) with very little reference or debt to school mathematics (see Lave, 1988). However, recourse to such research is hardly necessary to reveal some of the violence that has done to the setting in this case. For example, the long timescale of this narrative (a year) ought, perhaps, to make allowances for inflation², and the answer to C8 (b) given in the Teacher's Guide (£11.40) suggests that either the 'special offer' lasted all year (rendering it rather 'ordinary') or the McGees bought a year's supply all at once. This, despite the family's apparent storage problems indicated by their customary use of the middle sized packet rather than the 'better value' E20 size. The conditions have to be fixed in order to allow the exercising of the mathematical intention of the chapter, which is 'to provide pupils with experience of ratio problems' (G4TG, p. 14). This recontextualising is, however, apparently belied by the realism of the task formulation. The reader is provided with some 'factual' information about shopping (Euro-sizes) and is introduced to a family. The task, itself, apparently concerns the optimising of the McGees' domestic routine. The setting, in other words, is highly localised in terms of both task detail and the community space of the family. Thus the narrative is comparatively closed: this might be the reader's family, either now or in the future; or it might connote a 'soap' family with which the reader may identify.

¹ The vulgar use of the apostrophe to indicate a plural is representative of a common inelegance in language.

² The possibility of prices going down is mentioned in the 'discussion points' in the Teacher's Guide, as noted below.

This is a mathematics task, but the text is silent on the matter of how it is to be read and on how the task is to be carried out. There is, deliberately, no explicit pedagogising of method in this chapter, as is common in the G series:

There are many methods of solving problems where ratio is involved. We have deliberately not set out a 'standard method'. After pupils have done a few questions, we hope that discussion with them will bring out these various methods, and thus help them tackle the next batch of problems.

(G4TG, p. 14)

There is an assumed transparency in the task text with regard to its reading and a repertoire of 'strategies' within the classroom. The repertoire is activated by the 'realistic' task and made explicit in 'discussion'. Thus, pedagogic practice, here, is a celebration of essential competences through official recognition and sharing in discussion. At the end of the chapter there is a bordered set of 'discussion points, the first two of which are, 'Why are things cheaper when you buy them 'in bulk'?' and 'Sometimes it is not sensible to buy in bulk. When is it silly to buy huge packets?'; the Teacher's Guide gives the following answers to these discussion points:

Cheaper in bulk: easier to handle, less packaging costs, less transportation costs, take up less space etc.

Large packs not best when you have little money, you can't store large packs, food might go off before you can eat it, prices might go down etc.

(G4TG, p. 16)

However, the inclusion of such discussion only serves to accentuate the domestic setting which has apparently been incorporated by school mathematics. In other words, we might ask why shopping should appear in a mathematics textbook. One reason might be to initiate a route into the esoteric domain. Alternatively, the performance of the shopping tasks might be constructed as mathematically therapeutic. However, the text does not leave the public domain and, furthermore, there is very little (if any) explicit articulation of these shopping tasks with other tasks and exposition in the G series. Thus the tasks seem to be constituted for the benefit of optimising shopping practices themselves. Mathematics, even if this is signified only by the subject or book title, is being constructed as a prior condition for optimum participation in the public domain. This is the *myth of participation* that was introduced in Chapter 8.

There are two points to be made, here. Firstly, there is no pedagogising of method. Rather, methods are constructed as residing within the reader or within the setting: the reader's competence in participation is constructed as the prior condition for mathematics, whilst the latter is constructed as the prior condition for the reader's competent participation in the setting. The myth thereby deconstructs itself¹. Secondly, the setting, itself, is a recontextualising of shopping. The criteria for successful completion of the tasks within the mathematics context are not the same as those structuring the domestic activity. There is, in other words, no basis for the reader's competence. The myth of participation is precisely a shibboleth.

The 'best buy' problem is the setting for the whole of chapter 1 of Book G7. Localising in this chapter is enhanced by the extensive use of photographs of 'real' supermarket commodities and prices which 'were accurate in 1987' (G7TG, p. 7). Furthermore, the teacher is, here as throughout the series, urged to carry the localising further:

[Discussion] is central to the chapter. The pupils' own stories of 'bargains' and 'best buys' should be exploited. [...]

The prices will, inevitably, be out of date. It would be useful to bring in (or list) actual tins, packets and so on. You could make a collection of labels if you prefer not to use actual tins etc.—this is not quite so good, but better than nothing!

Comparing prices in different supermarkets may be possible.

All the prices used in the chapter were accurate in 1987—you will (like the authors!) need to search hard to find tins or packets where 'biggest gives more for your money' is not true.

(G7TG, p. 7)

The stated intention of the chapter has moved completely into the public domain in the expression of the 'aims' in the Teacher's Guide:

To develop various strategies for working out 'which packet gives you most for you [sic] money.'

To show that 'value for money' depends on more than this, and to encourage examination and discussion of other factors.

(G7TG, p. 7)

Here, the intention explicitly concerns participation in a domestic setting which is, again, produced as providing the rationale for the tasks. Again, mathematics is the prior condition for this participation (or, at least, for effective participation). This chapter is distinguished from the previous one discussed in that it does include

¹ See Derrida, 1981; Sturrock, 1979.

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Plate 9.6
(G7: page 5)

Plate 9.5
(G7: page 2)

prescribed methods, presented as algorithms with reference to specific cases (Plates 9.5 & 9.6). The tasks following these expositions allow for the extension of the algorithm to other commodities and the second algorithm is extended slightly in a marginally more general formulation on the final page of the chapter:

This is an E20 pack of Persil automatic.
In 1987 it cost £4.99.
To work out the cost per kg of the Persil
you need to divide the price by the weight.

(G7, p. 6¹)

Nevertheless, the algorithm is entirely restricted to commodities sold by weight and is nowhere extended to the more general topic of ratio (although this topic is mentioned in the G4 Teacher's Guide).

Whereas the G4 example provided a highly localised setting through the introduction of a family, the G7 chapter localises through the use of 'actual' commodities and real prices, etc. The earlier example presented a vision of future participation, the latter one makes the reader the grammatical subject of the narrative: there are few cases of a third person, everything is expressed in terms of what 'you' can buy and which gives more for 'your' money. The trajectory from G4 to G7, supported by the shift from drawings to photographs, is towards a greater realism and, perhaps, immediacy of participation. Again, there are 'discussion points' covering almost identical areas and having almost identical answers as those in the earlier chapter:

Smaller packets may also in general be
preferable to people on low incomes if they
simply cannot afford the larger size—or have
nowhere to store food etc.

(G7TG, p. 8)

There are very few shopping settings in the Y series, here are two instances taken from the first 'review' section in Book Y1:

Shopkeeper A sells dates for 85p per kilogram. B sells them
at 1.2 kg for £1.
(a) Which shop is cheaper?
(b) What is the difference between the prices charged by the two
shopkeepers for 15 kg of dates?

(Y1, p. 55)

¹ This E20 packet weighs 6.2 kg, the E20 packet in the G4 example weighs 5.6 kg.

Britannia best British flour cost £0.71 for 12.5 kg.
 Uncle Sam's best American flour cost \$1.30 for 3.5 lb.
 If 1kg = 2.2lb and £1 = \$1.85, which brand of flour
 was cheaper, and by how much per kilogram?

(Y1, p. 56)

Here, there is a clear semantic distance between the reader and the shopping setting and the narratives have become far more open. The first example uses a recontextualised algebraic resource, generalising through the use of letters which depersonalise the shopkeepers. This example also refers to a slightly exotic commodity and to a highly unrealistic quantity. The second case presents a transatlantic comparison which clearly cannot be relevant to most shoppers who would not be expected to board Concorde in order to get a cheaper pack of flour. In other words, the tasks include some element of strangeness which ensures their dislocation from the domestic setting to which they now only indirectly relate.

These review examples signal a chapter (Y1.04) which incorporates an exposition on the mathematical method which is to be applied in such cases, the 'unitary method'. Like the G7 exposition, the unitary method is introduced by reference to a particular case, that of a length of steel rope (not a domestic setting):

Here is a problem of a very common type.

12.3 m of steel rope weighs 10.6kg.
 How much does 17.5m of the rope weigh?

You can solve this problem by first working out how much 1 metre
 of the rope weighs. This method is called the **unitary method**.
 It can only be used when the rope is **uniform**, so that every metre
 weighs the same amount.

(Y1, p. 43; drawing of rope and border omitted)

The exposition goes on to describe and carry out the computation. The first point to note is that the exposition marks out the 'problem', but not the setting, as being of a common type. The tasks that follow concern the weights of bolts and of gold, the alcoholic content of wine, the revolutions of a wheel, the thickness of paper, the iron yield from iron ore, the weights of links of a chain and of ball bearings, the petrol consumption of a car and of a motorbike, the vitamin C content of orange juice, the weights of squares and cubes of metal, foreign exchange rates, and the conversion of units; the last two of these are also signalled by the second of the two tasks quoted above. Most of the tasks employ objects which might participate in a diversity of settings (or in none at all) and, in any event, no setting is elaborated, the narratives are very open because the settings are comparatively arbitrary. It is the mathematical structure that is being foregrounded, a general structure that may be imposed upon many settings. This foregrounding of mathematical structure is a generalising strategy

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Plate 9.7
(Y1: pages 46-7)

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Plate 9.8
(Y2: page 10)

which indexes the possibility of diverse descriptions in contrast with the narrow participation promised by the G text's localising.

The section also includes several instances (again, none of them implicating domestic settings) in which the unitary method cannot be applied. The first is a strip cartoon representing a discussion between a girl and a boy about the relationship between the depth and volume of water in a cone (as is often the case in these books, the girl has the correct answer). The final tasks in the section are shown in Plate 9.7. This last case, task E17, satirises the setting and its characters both through the mode of expression and the content. Thus, although the exposition (which is considerably more extensive than the extract above) is expressed with reference to a specific case, the variation in settings in the following tasks generalises the method. The Teacher's Guide includes a comment at the head of the answers to this chapter which promises future discursive elaboration:

Please note that questions E15 and E16 are introduced to be merely 'counter examples' involving misuse of the unitary method. The important (and difficult) topics of the effects of the enlargement on area and on volume are dealt with later in the series.

(Y1TG, p. 21)

This topic is begun in chapter 1 of Book Y3. The unitary method is developed in chapter 1 of Book Y2 and in subsequent chapters in the series. The Y2 chapter includes more shopping settings as in Plate 9.8. Again, narrative detail is minimal, 'dress material' and 'crisps' being represented as instances of something which can be bought, the latter being represented as possible instances of proportionality and non-proportionality (previously defined in the esoteric domain). There is also a varying in the form of address in the exposition, 'suppose *you* buy ...', '... if *we* buy ...' and, in the subsequent task, 'when a *motorist* buys ...'. These texts exhibit: a variation in settings and in form of address; a minimising of setting detail; textual movement between the esoteric and other domains; and longer term trajectories which facilitate discursive elaboration. These features all serve to minimise the local importance of the setting and to accentuate the mathematical grammar. The setting is backgrounded whilst the mathematics is foregrounded. Mathematics, in other words, is generalised in relation to the public domain. This contrasts with the localising G texts which frequently concentrate on a given setting for an extended amount of textual time, sometimes an entire chapter. Closed narratives, in the G texts, give far more setting detail. There is a greater consistency in projecting the reader into the public domain setting and, thereby, in the incorporation of non-mathematical regimes of truth. This is at the expense of the exclusion of esoteric domain text and discourse.

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Plate 9.9
(Y3: pages 126-7)

The content of background and foreground is reversed between the two series of books.

Participation in a public domain, domestic setting is not entirely absent from the Y texts, Plate 9.9, for example, shows a rare Y instance of a DIY setting. On the right-hand page, the reader is introduced to Jan, Rob and Cathy who are decorating their flat. This introduction connotatively signifies a possible future for the reader, sharing (and decorating) a flat with some friends¹. However, the task involves a recontextualising from management practice, this being made clearer, perhaps, by reference to the other tasks in the chapter. This chapter opens with a problem involving the planning of staff holidays at 'Mayhem's stores'²; the next task involves planning the holidays of a sports centre staff; the next concerns the planning of the building of an office complex; and the task preceding that shown in Plate 9.9 is about managing the manufacturing process in what appears to be a dressmaking factory. The last two tasks in the chapter follow the first by satirising their settings. C1 concerns the problems involved in getting a passport in 'Ruritania' (an optimising of the imaginary) and C2 is about 'Cold Comfort farm museum' (complete with besmocked yokel holding a pitchfork and uttering 'oooo Aaaaarr', an 'ancient farm worker'). The method involves, essentially, the transcription of listed data into a form of graph as illustrated on the lefthand page in Plate 9.9 (a transcription which is far more radical than anything in the G series). The chapter presents an apparently generalisable heuristic through the introduction of a range of public domain settings, but without ever entering the esoteric domain. The Y4 chapter discussed in Chapter 8 (Section 8.3) produced 'optimisation' as an instance of quasi-discourse. The planning heuristic in the Y3 chapter indexes a similar discursive space via the diversity of settings which are presented, although the heuristic is, perhaps, insufficiently developed to constitute discursive message. Nevertheless, the possibility of a quasi-discursive commentary is opened up, so that the decorating task is less about participation in DIY than about exploring the range of application of the heuristic. Mathematics is presented, not as a precondition for participation in the domestic setting, but as generalising its structure, linking it metonymically with an indefinite number of other settings. Thus, the generalising strategy operates by backgrounding the specificities of the setting or, as in the first task and last two tasks of the chapter, by satirising the setting.

¹ The name 'Jan' is ambiguous in terms of gender which ambiguity is not resolved by the use of a personal pronoun later in the text: the group could thus comprise two males and one female or one male and two females.

² A metaphor which presumably signifies an erroneous use of 'mayhem' and also a satirising of the setting.

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Plate 9.10
(G7: pages 22-3)

In contrast, DIY settings are very common in the G series. Plate 9.10 shows one instance from Book G7 in a chapter which, rather unusually, has been given an esoteric domain title, 'Ratios'. As with the McGees' shopping, the narrative is comparatively closed, introducing a familial setting, in this case, fairly strongly gender-stereotyped. It is dad, not mum, who is doing the work. Jane, whilst female, is somewhat inappropriately dressed and is only handing dad the bricks¹. This stereotyping is, however, disrupted in task A6 and the associated drawing. The drawing is 'straight', it is not a cartoon, so that the code of presence is minimally disrupted. The opening task gives further narrative details, 'first they dig a trench ...'. The closed narrative and strong code of presence localises the setting. Furthermore, this is public domain text. There is no mention of ratios and no mention of method which must reside as a competence within the reader-setting. Indeed, the term 'ratio' appears only three times in the whole chapter (which moves from a DIY to a cooking setting): once in the title, once in a recipe for fruit punch and once in the exposition associated with the latter:

This is a recipe for fruit punch.
You mix cider and apple juice
in the ratio 2 to 3.

That means that for every 2 parts of cider,
you add three parts of apple juice.
So if you used 2 litres of cider,
you would use 3 litres of apple juice.

(G7, p. 25; bold text in red in original)

The Teacher's Guide makes it clear why the term has been generally avoided:

The phrase 'in the ratio 2 to 3' is used only on page 25, since most ratio applications are not stated in this form. Pupils using Stage Three assessment will need this usage emphasised.

You may need to revise m^2 . The words 'foundation', 'coping stones', and 'ingredients' may need attention.

(G7TG, p. 12)

For the most part, the esoteric mathematics is to be backgrounded in favour of the relevant domestic setting. The use of the term, 'ratio', which is used in the title of the chapter is, in fact, only a requirement for those readers involved in the highest level of formal assessment relating to the G scheme. Otherwise, mathematical signifiers are confined to ' m^2 ', which has been met before. On the other hand, the non-mathematical terms 'foundation', 'coping stones' and 'ingredients' are important,

¹ The drawing illustrates the violence of recontextualising in the ergonomically inappropriate positioning of the heap of bricks (they're obviously not new ones) and other materials and tools. Dad, inexplicably, has placed his mortar on the opposite side of the wall from that on which he is standing.

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Plate 9.12
(G7: page 26)

Plate 9.11
(G7: page 24)

foregrounding the settings at the expense of mathematics. As with many of the G series public domain settings which have been discussed, method is not fixed:

We have not suggested one 'correct' method. You will need to discuss with your pupils the various methods which might be used in particular circumstances. [...]

There is no wish here to impose a particular method or to suggest that one is 'better' than another.

(G7TG, pp. 1-13)

No method at all is provided for tasks A1-6, but four methods are illustrated in the context of task A7 (Plate 9.10), these are shown on the following page (see Plate 9.11). These algorithms are all strongly contextualised by the particular task and, in any event, apparently represent only a sample of the 'lots of ways to work this out'; 'what matters is getting the answer right—and checking that your answer looks about right'. The rationale for the tasks, the methods and the criteria for evaluation reside within the reader-setting articulation. Plate 9.12 shows some more methods and, this time, incorporates the iconic. This set of methods is concerned with an ice cream recipe.

The primacy of the setting is further and explicitly emphasised in both the Teacher's Guide and in the student's text:

[...] In very many of the questions, the assumption that the relationship is strictly linear ('twice as much takes twice as long' etc.) is very much open to discussion and is brought out at the very end of the chapter [...]

(G7TG, p. 12)

The chapter closes with a 'discussion point':

'It take 4 minutes to boil an egg.
How long will it take to boil 2 eggs?'
Questions like this are meant to fool you!
But there are other times when the methods in this chapter won't work.
For example, it might cost £20 to have 500 posters printed.
But it won't cost twice as much for 1000 posters.
Why can't you just double?
What other examples are there like this?

(G7, p. 28)

The reader is, in fact, given no chance to be fooled, but is alerted to the trick straight away. This contrasts with several examples in the Y series, where the trick has to be discovered. Mathematically, this is the same qualification as is being made by the Y1 text in Plate 9.7. The Y text, however, focuses on the mathematical structure, 'the cone widens out, so each 1cm of depth has a **different** volume'. This

deviates from the requirement of ‘uniformity’, introduced earlier. Furthermore, the Teacher’s Guide gives answers to E15-17 which index alternative mathematical structures:

The weight is related to the area of the metal, not simply the length of its side.

[...]

The weight is related to the volume of the metal.

[...]

One man will take *four times* as long as four men, not a quarter of the time. Six men will take a sixth of the time one man takes.

(Y1TG, p. 23)

Although the G text also mentions mathematical structure, this occurs only when it addresses the teacher. Even then, ‘linear’ is translated as “‘twice as much takes twice as long’ etc.’ The answer to the ‘discussion point’ quoted above remains firmly in the public domain and incorporates further localising strategies:

Many things cost comparatively less when ordered in bulk.

It should be possible to find examples of printing within the school: report forms, brochures etc.

Pupils could be asked to find examples—personalised stationery, Christmas cards, photocopying perhaps—where rates for different quantities are sometimes advertised.

(G7TG, p. 13)

The G7 ‘ratios’ chapter represents the public domain settings as driving the mathematics, whereas the Y texts consistently present mathematics as making sense of the public domain (mythical domain) or as illustrated by the public domain (metaphorical domain). The Y texts mark out the limitations of the application of the unitary method. It achieves this explicitly, in the case of the ‘cone of water’ discussion. In the tasks concerning the metal squares and cubes, the limitations are implied by the introduction of dotted lines on the associated ‘graphs’, which both reveals the inadequacy of the postulated ‘answer’ and suggests a more appropriate solution. The inadequacy of the ‘unitary method’ in E17 is indexed by the absurdity of the final utterance in the cartoon. The task is presented as a puzzle, with the solution being encoded in the equivalently absurd logic which moves from ‘4 men take 12 hours’ to ‘1 man takes 3 hours’ (although, looking at the men, this might be plausible) There is no unitary method in the G7 text, however. Arrays of ‘ways’ are presented, *post hoc*, as methods that might have been used by the reader. The mathematical equivalence of the ‘ways’ is not addressed, nor are they given ultimate

privilege over other methods that the reader might have used, 'what matters is getting the right answer'. The limitations of these methods (which are those of the reader her/himself) are given in the structure of the setting, for example, 'many things cost comparatively less when ordered in bulk'. The reader must, at least potentially, already be familiar with such knowledge, because there are no clues within the formulation of the 'discussion point'. The G text is, again, localising the settings within the postulated material experiences of the reader and it is within these experiences—within the public domain—that the limitations of mathematics as well as its specific procedures reside.

The G text presents the public domain setting as paramount, but this is not to gainsay the recontextualising achievement of the gaze. To refer back to task A1 (Plate 9.10): there is an elision of the process whereby dad arrives at the requirement for '8 barrow-loads of concrete' which, in any event, we might expect to result in a measure of cubic metres or yards. The recipe for the concrete, itself, is suspect. More sand than aggregate ('gravel') is unusual, and the mixture is far too strong for foundations¹. Using 'shovel' and 'barrow-load' as units is appropriately 'manual' and 'amateur', but it is not clear that such a choice would facilitate the mixing of concrete, especially as the size of a shovel is defined in terms of the amount of cement in a bag². Finally, on the following page of the G7 chapter (Plate 9.10), the reader is asked to estimate the number of bricks need to build a half-brick wall $1\frac{1}{2}$ m high and 4 m long. Such dimensions for a half-brick wall, especially without end or intermediate piers, seem positively dangerous³.

Thus, firstly, the G text is presenting domestic settings which, in articulation with the reader's individual experiences, embody mathematical algorithms and the limitations of these algorithms. Secondly, there is no explicit consideration of any unifying mathematical structure, but, rather, the presentation of various context-

¹ The *Reader's Digest New D-I-Y Manual* (Frewing, N.J. & Wilkins, T, (eds), nd, London: RDA) suggests 1 part cement, $2\frac{1}{2}$ parts sharp sand and $3\frac{1}{2}$ parts coarse aggregate. There is no obvious reason for the practically inappropriate recipe, because using the quantities, 7 shovels of gravel, 5 of sand and 2 of cement would not render the task arithmetically more complex; nevertheless, this illustrates the arbitrariness of the practical details.

² The *Reader's Digest New D-I-Y Manual* provides recipes both in proportions (see earlier footnote) and in the units in which the materials would be ordered (bags, in the case of cement, kg for sand and aggregate). This publication also offers the following instructions labelled 'What to specify for a ready-mixed load': 'Mix C7.5P to BS5328; medium or high workability; 20mm maximum aggregate'. As in the G text, there is a certain invisibility of the generative grammar. However, we can assume that this message, esoteric as it is, will get us our concrete: it is not meant for us, but for the concrete mixer.

³ We can tell that this is a half-brick wall from the information given in the text, including the pattern of stretchers and the number of bricks required per square metre.

specific algorithms, any of which (along with any others that yield the right answer) are acceptable. Thirdly, the settings are localised by the comparatively closed narratives. This is achieved through, for example, the large amount of detail given and the use of characters (often named and/or represented in an icon) which have some affinity with the reader. Mathematics, in other words, is *within* the domestic world, is substantially disarticulated by context-specificity, and is a precondition for optimising participation within domestic practices: this is the myth of participation.

However, domesticity is recontextualised, so that the generative/evaluative grammars of these 'real life' practices are elided and negated. The text announces itself as mathematics, but offers no ingress into mathematical discourse. The text promises an optimising of domesticity, through its localising strategies, but has shifted the material basis of domesticity from the home to the classroom, so that what is being optimised is no longer domesticity. Furthermore, the means of the proposed optimising is algorithmic, but the text frequently assumes that these algorithms are already available to the reader and confers official sanction on those that 'work'. Under these circumstances, the myth of participation deconstructs itself by inverting its own prioritising: rather than mathematics facilitating participation in the domestic, it is domestic competence that facilitates participation in mathematics. Such participation is denied, however, as the text almost never leaves the public domain.

The domestic space is that space which is most highly localised and individualised in relation to the student-reader *vis a vis* school mathematics. Yet this space is minimally indexed by the Y texts. Where the text does intrude into the domestic setting, the narratives are generally open and recontextualising is highly visible. Text is mythical, but rarely public domain. Rarely is the participation of the reader indexed. It is not localised and individualised public domain competences that are celebrated, but the power of mathematics to describe, define and control. Thus, in Plate 9.13, a rare instance of a cooking setting incorporates no characters (and certainly not the reader) there is not even a specification of a particular, exemplary joint of pork. It is always the mathematical expressions (proportionality, relationship, the line graphs) that are foregrounded. There is no participation, here, even the absurdity of a joint weighing 0 lb needing 35 minutes fails to get a mention, although this consequence of the formula is represented on the graph (A). Rather than few detailed, closed narratives which localise, the Y texts implicate many open narratives, foregrounding the common mathematical structure and backgrounding the setting. But the mathematical structure resides within the esoteric domain which constitutes the gaze, rather than in the always already abstracted setting. The myth of

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 9.13
(Y2: page 152)

participation in the immediate and local is replaced by the *myth of universal description*.

9.2.2 Work Settings

‘Work’ is understood, here, as the social space relating to commodity production and distribution within an economic context. Work settings are more or less equally common in the G and Y series and will be discussed further in Chapter 10, in relation to *positioning strategies*. Here, the concern is still with localising and generalising, which are modes of *distributing strategy*.

In Chapter 6, the setting concerned with the ‘Blagdon Hire Company’ was discussed in relation to the introduction of algebra in the G series. Here, a substantial part of the chapter was centred on Peter, a junior clerical worker in a tool hire shop. It was noted that the apparent age and junior status of Peter (and the unskilled classification of his job) constituted metonymical links with the reader, who might hope for such a position upon leaving school. The use of icons of Peter and his environment and customers, the naming of Peter and other characters and the comparatively extensive textual time occupied by the setting (almost an entire chapter) also serve to close the narrative and localise the setting.

Chapter 6 in Book G3 incorporates similar localising strategies. This time, the title of the chapter, ‘Stacking and filling’, accords with the public domain setting (‘Asco’s supermarket’) which extends for the whole chapter. In section B, the reader is introduced to Gillian, who drives a fork-lift truck at the supermarket. Section C concerns the work of John, who ‘has to pack large trolleys at Asco’s’. Section D introduces another packer, Hilary, and also reintroduces John. There are cartoons of Gillian driving her truck and of John and of some of their workmates having a coffee break and (inexplicably) hanging up bunting whilst Gillian drives past on her truck. The narrative is, again, comparatively closed (apart from the decorations, perhaps), localising the setting.

Mathematically, the chapter is about the enumeration of rectangular arrays. We can calculate the number of shirts or blouses which would fill a display, the number of boxes on various pallets and stacks of pallets or in trolleys, the number of packets which will fit into a box. There are no algorithms given in the chapter, again, there is an assumption of competence within the reader-setting articulation, again effecting the deconstruction of the myth of competence. Potential generalising is introduced

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 9.14
(G2: pages 4-5)

through the use of racks, stacks, trolleys and boxes of different dimensions, but since there is no instance of discourse, or even explicit procedure, and no shift out of the public domain until the final section (mythical domain), the generalising is never achieved. Furthermore, the localising within Asco's operates effectively in the opposite direction. As with the domestic examples in the G series, the localising in this chapter produces the (deconstructed) myth of participation, but, again, this is negated by the recontextualisation. The wide variety of pallet and trolley sizes, the clear space around each stack of pallets and the fact that the dimensions of trolleys and boxes are exact multiples of the dimensions of whatever is being stacked in them, illustrate the constitutive nature of the recontextualising gaze¹.

There are no instances of such extended settings in the Y series, although a work setting does occasionally extend beyond a single task. For example, the left-hand page of Plate 9.9 concerns dressmaking. The dressmakers, themselves, and the customer are anonymous and the reader is cast in the role of a production manager, also anonymous. Furthermore, although this task is localised and the narrative is relatively closed, it is included, as discussed earlier, as one of a number of such tasks which vary in terms of setting, but which are analysed in the same way. Localising within each task is offset by the generalising in the diversity between tasks and the production of quasi-discourse.

A substantial number of the work settings in the G series localise the settings by producing narratives concerned with the kind of part time or entrepreneurial work that might be performed by school students. Plate 9.14, for example, shows a section in Book G2 which introduces John's and Charlie's bird table construction business and Anne's picture painting business. Elsewhere in the same book Sandra and her friends earn £20 washing cars at the weekend, Ken is making up 5kg bags of potatoes and Jane is filling plant pots with soil. In book G3, Vicky is working in a shop on Saturday, in G4, seven friends are making money washing cars and in G6, John did a paper round. Most of these instances include not only the names of the individuals concerned, but also cartoons or drawings of them. This does not represent a great number of instances in the series as a whole, but there are almost no such cases in the Y series and, those that do appear are very different. For example:

¹ It is also not entirely clear why someone (Hilary) would be packing butter etc *into* boxes in a supermarket.

A girl who delivers newspapers works for $13\frac{1}{2}$ hours each week
and earns £8.50 a week.
What is her rate of pay per hour (to the nearest penny)?

(Y2, p. 36)

The girl is anonymous: there is neither name nor illustration. The task constitutes a description of the work from the mathematical perspective of 'rates' (which is the chapter title). The previous task is even more brief and concerns a typist, 'What is his rate of pay per hour?'¹. The task following the newspaper deliverer is about the relationship between the mass and volume of a piece of iron. Localising is minimised via the minimising of detail, so that the narratives are comparatively open. Generalising is achieved by the multiplication of settings. The opposition between the G and Y series in terms of localising/generalising strategies is apparent in work settings as in domestic settings. The G texts localise via narrative detail and closedness and by association with the reader; the Y texts generalise by minimising narrative detail and by multiplying settings, foregrounding the mathematical structure of the recontextualising gaze.

9.2.3 Localising & generalising via setting: summary

The absence of discourse in the G books renders invisible the principles of the esoteric domain that is the mathematical structuring of the text. Here, the message is procedural and, in this sense, devoid of content. A semantic void is established, as it were, around the procedures, corresponding to the unsayable of DS- practices. Within the esoteric domain (rare, within these books) the tasks must be entirely confined to the following of necessarily short sequences of instructions which must be contingent upon already acquired or essential competences. Within the public domain the procedures can be less explicit to the extent that their formulation presumes and articulates with the reader's local experiences and competences. In either domain, the semantic void is filled by the reader her/himself.

Domestic and work settings are ideal resources in this respect, because they connote the substantive network of relationships and practices in which the reader is or can expect to become enmeshed. Narratives are closed so that settings are highly localised. A failure to localise in this way would problematise the mathematical competence that is assumed to reside in the articulation between the reader and the

¹ The introduction of a male typist reveals a 'political correctness' with respect to gender. This is a recurring (although not entirely consistent) motif in the *SMP 11-16* books, as was noted in Chapter 7. This 'political correctness' marks out the *SMP 11-16* series from the previous *SMP* edition (see Dowling, 1991c) and also marks out gender from social class, the latter frequently being represented by stereotypical images.

setting. The reader must recognise the setting in order to recognise their own competence. Mathematics is localised to the setting and to the individual reader. Mathematics is mythically established as the precondition to the reader's optimum participation in the settings, but only as a signifier. Its esoteric domain signification is absent. The reader's presence in the public domain, however, substitutes for this signification and deconstructs the myth of participation. The trick, of course, is that the restructuring of settings under the action of the recontextualising gaze results in the production of spurious contexts which pretend to be, but which are structurally different from, domestic or work contexts. Insofar as the empirical reader recognises her/himself, they cannot access the mathematics because the mathematics has structured something other than that which they can recognise. Insofar as the empirical reader does not recognise her/himself, then still the mathematics remains inaccessible, because of its invisibility.

The Y books, on the other hand, generalise through the implication of many, comparatively open narratives. In a sense, the reader must not recognise her/himself in the setting, because of the effectivity of recontextualising. The Y texts facilitate mathematical metonymies between settings and between mathematical propositions. They are discursive or, at least, quasi-discursive. In the Y texts, the message to be distributed is the highly connected, discursive message of the dominant voice. DS⁺ practices are distributed to the dominant subject position, which is the subject of the gaze and which generates the myth of mathematics as omniscient. In the G texts, it is the location of the subordinate voice within the spurious public domain which is to be achieved. The myth cannot be generated from this place, but can only be received as the message of authoritative discipline.

9.3 Localised & generalised readers: the Teacher's Guides

I shall now describe the localising and generalising of readers within the Teacher's Guides. However, it may be helpful to rehearse the general structure of the description so far.

The student reader is inserted into narratives and settings via the production of metonymic links which attach more or less readily to the text and to the reader. Section 9.1 of this Chapter describes this insertion via the incorporation of the iconic code of presence; Section 9.2 describes the implication of settings in this respect. The localising strategies of the G books implicate comparatively closed narratives, drawing the reader into the settings by insisting upon recognition. Here, metonymic

chains are (speaking metaphorically) comparatively direct. The generalising strategies of the Y books minimise the demand for situational recognition and so allow the reader to stand at some distance from the setting. Here, metonymic chains between the reader and public domain settings are comparatively indirect in the sense that there is a significant degree of uncertainty concerning the location of the reader *vis a vis* the setting. On the other hand, whilst the G texts obscure metonymic links between public domain settings and the generative grammar of the esoteric domain, it is precisely these links that are foregrounded in the Y texts.

Thus, there is a sense in which the Y texts are for and about mathematics, whilst the G texts are for and about the reader. The G texts demand that the reader inserts her/himself into the semantic void which these texts produce around their procedures¹. We might expect, therefore, that the specificities of the reader, in terms of attributes and location, should become of great importance in the G texts, but that they should be comparatively absent from the Y texts. Operationally, these differences might be expected to be most apparent in the pedagogic commentaries of the Teacher's Guides, to which I shall now turn.

9.3.1 Localising & generalising the student

The Teacher's Guide to Book Y1 (Y1TG), in common with most of the Teacher's Guides, comprises a brief introductory section. In the case of Y1TG, this section is approximately five-and-a-half pages in length, of which three-and-a-half pages concern the *SMP 11-16* scheme more generally, and two pages comprise an 'Introduction to Book Y1'. The remaining 36 pages are the 'Notes and answers for Book Y1', divided into sections corresponding to the chapters of Book Y1, each having a, generally brief, introductory paragraph followed by lists of answers. Although the amount of text relating to pedagogic practice is very limited, there are a small number of comments present that prioritise mathematics in relation to the particular student:

Section G is particularly important, and if it is felt that the chapter is taking a long time, or if pupils find the first few questions in section G very difficult, then the section can be postponed. However, whenever the work in section G is introduced it is likely to be difficult for many pupils.

(Y1TG, p. 18)

¹ The extent to which this occurs is an empirical question which cannot be addressed in the present study, which is limited, empirically, to the description of textual subjectivities and objects.

As was noted in Chapter 6, this extract suggests that concessions may be made in the face of student difficulties, but only in terms of pacing. However, postponement may not be perpetual, because the content is ‘particularly important’ and, in any event, will not overcome all difficulties. Again:

Please note that questions E15 and E16 are introduced to be merely ‘counter-examples’ involving misuse of the unitary method. The important (and difficult) topics of the effects of enlargement on area and on volume are dealt with later in the series.

(Y1TG, p. 21)

The use of ‘awkward’ numbers in the equations in section E is intentional: it focuses attention on the process of solution.

(Y1TG, p. 26)

... The method used for increasing an amount by 35%, for example, is to multiply by 1.35. Although more difficult to grasp, this method (and the corresponding method for percentage decreases) has distinct advantages over the more usual method in that it easily extends to such problems as ‘what is the overall effect, in percentage terms, of two successive percentage increases of 30% and 35%?’ or ‘what amount, when increased by 15% becomes £250?’

(Y1TG, pp. 38-9)

To obtain the correct answer is not enough. Attention is to be placed on the mathematical processes involved which are valued because of their generalising potential. In the end, students’ difficulties will just have to be overcome. These are to be subordinated to the mathematical message.

On the face of it, the Y1 chapter on ‘Investigations’ (Y1.08) allows for more individualised approaches:

Obviously, the investigations in this chapter are not important pieces of mathematics in their own right. So the teacher is not trying to teach, or the pupils to learn, any important mathematical ‘facts’. The class discussion—which is essential—can focus on the ideas and efforts of the pupils themselves, comparing approaches, discussing the explanations offered, and so on.

(Y1TG, p. 33)

The extract above ends the notes on the following investigation:

Two people are playing noughts and crosses.
‘Nought’ goes first.
How many different ways are there to make the first two moves (one move each)?

(Y1, p. 98)

The notes in the Teacher’s Guide suggest:

Let the class work at the problem for a while without general assistance. The point about the meaning of 'different' will come up in individual questions, but don't 'prompt' anyone who does not ask it for themselves.

(Y1TG, p. 32)

However, this approach is very different from the canonisation of competences that characterises much of the G text:

When the results of the class's labours are examined, the person with the biggest number may think of himself or herself as somehow 'the winner', but this 'score' may be reduced when some cases which are not really different from others in the list are eliminated.

Now the attention must be focused on the real nature of the problem, which is not simply to count as many cases as possible, but to have a way of knowing that all possible cases have been considered and none has been counted more than once. This can be done in two stages:

- (1) possible first moves;
- (2) possible second moves for each first move.

At each stage we try out every position on the grid in a definite sequence, for example

1	2	3
4	5	6
7	8	9

Then we ask if it is the same as any other already counted.

For your benefit, here is one way of listing a complete set of all possible 1st and 2nd moves.

[...]

(Y1TG, pp. 32-3)

It appears that the unaided work is intended to reveal incompetences, which are subsequently to be remedied by the provision of heuristics. This is apparent in a reference to this chapter in the general introduction to Book Y1:

Depending on the level of confidence of the class and the extent of their previous experience of investigations, it may be a good idea to introduce the first problem in chapter 8 (Cutting a cake) before reaching the chapter, and without any of the assistance given in the chapter. Those who try to solve the problem and 'get lost' are likely to appreciate more the need for a methodical approach.

(Y1TG, p. 10)

The notes for the first of the investigations outline further general 'strategies':

Investigation 1 introduces a strategy which is often useful in tackling a problem: simplify the problem and see what can be learned from solving the simpler problem. This investigation (together with investigations 2 and 5) also offers scope for:

tabulation — making a record of the results of experimentation, which makes it easier for the results to be viewed as a whole;

looking for relationships, and expressing them in a form which other people can understand;

using the relationships discovered to help in solving problems;

trying to find explanations for why the relationships are true, and making such explanations clear to other people.

(Y1TG, p. 32)

Even where mathematical content is not central, there are still mathematical processes to be elaborated. These are to be transmitted and acquired, rather than revealed as already present in the student as competences.

G1TG has only slightly fewer pages (40 pp.) than Y1TG (46 pp.), although the corresponding student's books, G1 and Y1 have 61 and 156 pages, respectively. This is partially because G1TG includes an 11 page commentary on pedagogic practice in relation to the G scheme, whereas the equivalent section in Y1TG is less than two pages in length. Furthermore, whereas approximately 85% of the page space of the notes and answers section in Y1TG is devoted to answers, the rest comprising headings and introductory paragraphs, about 50% of the space on the corresponding pages in G1TG are given over to pedagogic considerations for each chapter. Purely in terms of textual time, in other words, pedagogic considerations are far more visible in G1TG¹ than in Y1TG.

The differentiation of the SMP curriculum at the start of the third year of secondary schooling is explicitly made in terms of 'ability', which term is used seven times in the Guide in referring to a range of 'abilities' on what is, apparently, at least an ordinal scale:

The Y series is for the most able group of pupils (roughly speaking, the top 20—25 per cent or so, although the proportion is likely to vary from school to school). The B and R series are for the 'middle' group (the next 35—40 per cent [or] so) and the G series is for lower ability pupils (apart from those with special learning difficulties).

(G1TG, p. 4)

The exclusion of those with 'special learning difficulties' appears to be made on the basis that printed materials are, in some respect and to some extent, unsuitable for such students (G1TG, p. 6). Having excluded this group, the G1 Teacher's Guide

¹ G1TG is unique amongst the Teacher's Guides in the *SMP 11-16* scheme in terms of its comparatively extensive introductory section. However, all of the G series Guides, but none of the other Guides, contain extensive chapter introductions relating to pedagogy.

describes its student constituency as 'lower ability' or 'less able'. These expressions are used eight times (in total) in the Guide; the expression 'lower attaining pupils' is used twice and 'weaker pupils' is also used twice. The Guide also gives various indications as to what 'lower ability' means. 'Low ability' is a property of the students which is unlikely to change since, once the G course is under way, there is no facility for transferring to the higher tracks (as there is for students following the B course). At the start of the scheme, however, there may be some shuffling in the setting:

A pupil who is started on *Book G1* but then appears to find the work too easy can be given booklets from levels 3 and 4 as individual extension work, and may be able to transfer to *Book B1* when setting is finally decided.

(G1TG, p. 5)

Thus, the relative ease with which a student performs mathematics is a measure of her/his ability: mathematical performance is an indicator of competence. Furthermore, it is necessary to match the mathematical level to the competence level:

Of course no assessment will be appealing if pupils cannot manage to do it! At the same time it is important that it is not so simple that pupils find it beneath them. In order to make the assessment sufficiently challenging to pupils we have in general fixed the 'pass mark' at 70%. Passing a test when 70% or more is required is generally reckoned to show that the candidate has mastered [sic] the work contained in the test, so setting the requirements this high has the double benefit of challenging the pupils and allowing them to display mastery.

(G1TG, pp. 13-14)

This need for matching is clearly of far less importance than in the Y scheme where, as the earlier extracts illustrate, certain 'important' mathematics is bound to be difficult for at least some of the students whenever it is introduced. However, the provision of 'extension' books in the Y series (YE1 and YE2) does reflect a need to 'stretch the most able pupils' (Y1TG, p. 6) so, here too, mathematics measures 'ability'. The extract which offers the possibility of transfer to the B scheme at the start of the course also suggests that level 3 and 4 booklets might be used as individual extension work. These booklets are considered appropriate for all but the 'lower ability' students in the year before G1 is introduced. This suggests that mathematical performance is dependent upon some articulation of age and 'ability', so that each of these independent variables can, in a sense and to a limited degree, compensate for the other¹. This possibility of compensation is explained by the understanding of 'low ability' pupils as slow:

¹ This is also found elsewhere in school mathematics, for example, in the documents relating to the UK National Curriculum, see Dowling (1990b).

... we feel strongly that lower attaining pupils can continue to learn provided that the pace of learning is appropriate, and the content is relevant to the pupil.

(G1TG, p. 6)

The G series proceeds slowly, moving at a pace in keeping with the pupils' understanding.

(G1TG, p. 6)

However, there may be absolute limitations on performance in terms of more complex tasks:

At first sight [problem solving] may seem far-fetched as an aim for the low attainer, but we believe it forms an essential part of the course. A 'problem' to one pupil may be routine to another. We do not wish to introduce artificial problems, and so have written in problems only where it seems appropriate¹. The sort of activity we would call a 'problem' might be to give a pupil a parcel, some scales, a post office guide to postal charges and some stamps. The pupil is then asked to stamp the parcel. Each part of the problem is simple, but for the less able it is the joining together of the simple tasks which is difficult.

(G1TG, p. 9; my footnote)

'Low ability' students are, in this respect, simple-minded, being generally limited to separate, simple tasks. Finally, 'low ability' students are perceived as needing diversity in terms of pedagogic materials. This perception is implicit in the structure of the G scheme, which comprises: 8 books of 61 pages each plus associated worksheets; 'G booklets'; 3 'G resource packs' each containing 32 cards; 'G supplementary booklets'; and 'topic booklets' (shared with the B series). This gives a total of 31 'G' items as listed in the price list (many more, if the cards in the 'resource packs' are counted individually) and compares with 17 'B' items and 7 'Y' items. This need for diversity is also made explicit in the Teacher's Guide:

We have taken the view that lower ability pupils need a wide variety in their mathematics — variety of presentation, content, method of working and so on. The G materials reflect this view.

(G1TG, p. 7)

We have provided a variety of G materials because we believe that pupils of lower ability need a varied mathematical diet. In the same way, we believe that any assessment they undergo should also be varied. A variety of assessment instruments are provided — short written tests, tests of practical and oral ability and mental tests.

(G1TG, p. 13)

¹ Here, as elsewhere, is evidence of strained writing: a scenario in which authors of materials admit to having included problems where it did not seem appropriate is bizarre.

Even such diversity as is directly provided by the materials is insufficient:

We hope that pupils will enjoy the wide range of mathematical activities in the G material. But there are a number of vital mathematical activities that cannot be written down. We have tried to include suggestions about these in the notes on each chapter, where appropriate.

(G1TG, p. 8)

The Guide provides a page-and-a-half of exposition on six 'vital mathematical activities': 'discussion'; 'mental mathematics'; 'approximation and estimation'; 'practical work'; 'problem solving'; and 'further consolidatory work'. This emphasis on diversity and the consequent brevity of any instance of 'activity' might be described as an interpretation of 'low ability' students as having a short attention span. A relationship between the two independent variables, competence and age, is, again, apparent, because the booklet scheme for use in the first two years of secondary schooling also constitutes diversity through the provision of 115 booklets of eight or sixteen pages in length. A short attention span is constructed as a property which is common amongst younger students and which is still present in the older 'lower ability' students.

The G1 Teacher's Guide thus produces 'ability' as an independent variable indexing a property of the student. Some function of this variable with the other independent variable 'age' (or, at least, 'year cohort') produces mathematical performance as a dependent variable. Performance is measured (or assumed) in terms of: i) the level of difficulty experienced with the mathematics curriculum at any given level within its own hierarchy; ii) the pace at which transmission can proceed; iii) the complexity of tasks that can be attempted; iv) the length of attention span. The relationship between age, ability and performance is summarised in Table 9.14.

**Relationships Between Age, Ability & Mathematical Performance
in SMP 11-16**

		Mathematical Performance			
		Difficulty (level of maths)	Pace	Complexity of tasks	Attention Span
Age	High	high	X	X	long
	Low	low	X	X	short
Ability	High	high	high	high	long
	Low	low	low	low	short

'X' indicates that age is not a factor

Table 9.14

9.3.2 Localising & generalising pedagogic action

The question to be addressed now is: what are the pedagogic consequences of the attribution of 'low ability'? Trivially, perhaps, one consequence is that there should be some measure of matching between ability (the independent variable) and level of mathematics, pace, task complexity and task/topic duration. This is apparent from the extracts cited above and from the structure and content of the G scheme as discussed earlier. It has also been mentioned that diversification also extends to mode of pedagogic action as well as topic and mode of presentation. Thus 'low ability' students need the various forms of pedagogic action, 'vital mathematical activities', that have been listed above. Of these forms of pedagogic action, 'mental mathematics' is given some attention in Y1TG:

It is assumed throughout that unless there is an instruction to the contrary calculators will be used for all but the simplest calculations which can be done mentally.

We strongly recommend that teachers encourage mental calculation, and from time to time give short sets of questions to be answered mentally. We also suggest having occasional practice sessions on written arithmetic, but that the scope of these should not extend beyond addition, subtraction, multiplication by 2, 3, ... 9 and division by 2, 3, ... 9 of whole numbers and money.

(Y1TG¹, p. 9)

In this extract, mental arithmetic is presented as a skill which, like written computation, needs some practising. There is no indication as to the nature of its value, but the limiting of written arithmetic to dealing with single digit numbers might be taken to imply that the value is measured in terms of utility and that the value added by dealing with two and more digit numbers is not worth the effort. In G1TG, however, mental mathematics extends well beyond arithmetic and its value, furthermore, seems to derive from its role as a kind of therapy:

In a sense all mathematics is mental; however, we have tried to encourage more emphasis on the mental aspects of the work. By this we do not only refer to 'mental arithmetic', but to the whole range of mathematical activities. For example, 'seeing' what will happen if you cut a folded piece of paper, or reflect a shape, are valuable mathematical activities. Many mental skills need practising, and we have indicated suitable places in the chapter notes.

(G1TG, p. 8)

The suggestion that there is a sense in which all mathematics is mental points to a psychologistic interpretation of mathematics as cognitive states or processes and,

¹ In fact the requirements for arithmetic without the use of a calculator at Level 4 (the average level for 11-year-olds) of the UK National Curriculum slightly exceed that suggested here.

although practising skills seems to suggest a utility value, it is not at all clear that this is the value being placed upon “‘seeing” what will happen if you cut a folded piece of paper’. In the latter case, some kind of logico-mathematical experience seems to be being indexed: ‘mathematical activities’ are cognitive therapies. Indeed, any potential use-value of mental mathematics is denied in the advice relating to the ‘mental tests’, which form a part of the formal assessment for the G series:

The mental tests provide an opportunity for pupils to show their skills at doing mathematics in their heads. The tests are to be worked without a calculator and it is not intended that pupils do any working with paper and pencil. If however pupils do rough working this should not be penalised.

(G1TG, p. 15)

The tests are ‘opportunities’ for the pupils to celebrate the success of pedagogic action, but there are to be no penalties if pupils decline this particular opportunity. The interpretation of mental mathematics as pedagogic therapy is in keeping with the nurturing pedagogic practice represented throughout the Guide:

Pupils of lower ability need encouragement and help in learning mathematics. The particular difficulties which a pupil finds are often specific to that pupil. So when using the books, we hope that teachers will explain and discuss the work with pupils, ask them where they have seen examples of this kind of activity outside the classroom and so on.

(G1TG, p. 7)

The nurturing pedagogic practice suggested in the first sentence of this extract is reinforced in the ‘aims’ outlined in the chapter notes:

To help develop ‘strategies’ with numbers ...

To help pupils develop their skills of estimating ...

To give pupils a feel for what is ‘more likely’ or ‘less likely’ ...

To help develop flexibility in number calculations ...

To reinforce the equivalence of ...

To give some primitive ideas about equally likely events ...

... To help pupils see when multiplying is the appropriate operation ...

To introduce simple ideas of statistical inference ...

To develop strategies and thinking ahead by games playing ...

... to encourage drawing skills ...

To provide a stimulating context for the practice of drawing skills.

(G1TG, pp. 19-36; my italics)

By contrast, the chapter notes in Y1TG (which, as noted above, are very brief) include few references to the students, but relate to the mathematical content. That is, to particular conventions which have been used and, sometimes, to the particular route through the content being taken in the relevant chapter and in relation to other chapters in Y1 and subsequent books in the scheme. The following extract, for example, comprises the whole of the introduction to the answers for chapter 7, 'Polygons and circles':

The idea of ratio, as developed in chapter 6, plays an important part in this chapter. The ratio $\frac{\text{circumference}}{\text{diameter}}$ in a circle is approached as a limiting case of the ratio $\frac{\text{perimeter}}{\text{diameter}}$ in a regular polygon.

(Y1TG, p. 30)

This extract also refers to the development of an idea, but, in this case, the idea is discursive rather than cognitive as is evidenced by the comment in the introduction to the chapter 6 answers: 'The ideas in it arise frequently in later work.' (Y1TG, p. 28). The 'ideas' are immanent in the practice rather than developing in students heads. The therapeutic pedagogic practice of G1TG starts and frequently ends with the individual student, as is suggested by a comment in an earlier extract: 'The particular difficulties which a pupil finds are often specific to that pupil'. This individualising of mathematics is explicit elsewhere within the Guide:

... Within the assessment items themselves we have not in general insisted on any particular methods being used. In this the assessment again reflects the written G materials, and the belief that teaching one single algorithmic approach to solving a problem is not the best we can do for pupils.

G1TG, p. 13)

Individual students have individual difficulties and are encouraged to employ individual approaches. The class, as a whole, is then constituted as a reservoir of strategies which can be shared in 'discussion'. Interaction, in the form of discussion, is given considerable emphasis in the G Series Teacher's Guides, the answer sections for most chapters including advice on possible areas and forms of discussion:

Discussion between pupils, and between pupil and teacher, is perhaps the most useful mathematical activity possible; 'talking through' with the teacher may be the only way to make the work relevant. Discussion should always precede a teacher-led lesson, and discussion can often follow a class game or investigation. Pupils may be asked how they solved a particular problem, and the different methods used by pupils can then be compared. Often for these pupils, there is no single 'correct way' of doing things. Rather there is one method which suits a particular pupil best for a particular problem.

(G1TG, p. 8)

As earlier extracts illustrate, 'discussion' in the Y books is valued less for its sharing potential than for its corrective effect. There is an apprenticing, for the Y student, but no negotiation of mathematics. The uncompromising position of the Y text is highlighted in the following extract (which was also reproduced earlier):

... The method used for increasing an amount by 35%, for example, is to multiply by 1.35. Although more difficult to grasp, this method (and the corresponding method for percentage decreases) has distinct advantages over the more usual method in that it easily extends to such problems as 'what is the overall effect, in percentage terms, of two successive percentage increases of 30% and 35%?' or 'what amount, when increased by 15% becomes £250?'

(Y1TG, pp. 38-9)

Mathematics, in the G series, is often to be individualised in terms of its procedures. The centrality of the student is also apparent in the emphasis placed upon 'relevance', in G1TG, in relation to the students' individual and collective experiences. The introduction to the G materials lays stress on the notion of relevance:

... But we feel strongly that lower attaining pupils can continue to learn provided that the pace of learning is appropriate, and the content is relevant to the pupil.

(G1TG, p. 6)

... We hope that teachers will explain and discuss the work with pupils, ask them where they have seen examples of this kind of activity outside the classroom and so on.

(G1TG, p. 7)

Furthermore, the Guide includes the suggestion (in its introduction as well as in the chapter notes, as mentioned in earlier sections of this chapter) that teachers will produce additional materials, which are of greater immediate relevance:

We hope that much in the G materials will act as a 'model' for work of your own devising. work on timetables, map-reading, shopping and so on is far more motivating form pupils if it is seen to be 'real'. Blagdon¹ can never substitute for your own town! So in a sense, we hope that some chapters in the books never get used by pupils. They are written to be replaced by work which is firmly based on the pupils' own environment. Of course, replacement may not always be possible, but work based on the pupils' own school, town or surroundings may be added to a particular chapter. Some of the later topic booklets in the fourth and fifth years particularly can be thought of as models for a booklet based on the pupils' own environment.

(G1TG, p. 8; my footnote)

¹ The name of an apparently fictional town referred to frequently in the SMP texts. There is no obvious reference to Blagdon in Avon or Blagdon in Devon.

The subsequent section of the Guide is subtitled ‘the maths we haven’t written’ and is arranged under the headings of the ‘vital mathematical activities’ that were mentioned earlier. The section includes several references to the need for relevance:

... ‘talking through’ with the teacher may be the only way to make the work relevant...

... Questions of the type ‘about how much ...?’ ‘how far do you think ...?’ may be asked about much of the work, and are often best based on the pupil’s own environment and experience. Pupils should be encouraged to look at their answers to questions and ask ‘is this a sensible answer?’.

... We do not wish to introduce artificial problems, and so have written in problems only where it seems appropriate...

Discussion of how to solve problems will be almost as valuable as actually solving them. A discussion of how to avoid congestion in the school corridors, which would be the best local school to amalgamate with, where to go to buy a bike cheaply — all these represent the sort of problem whose solution is mathematically valuable. They are, of course, absolutely specific to the pupils’ own environs and interests. ‘Problems’ may arise topically from a newspaper or TV or a local incident. Valuable discussion can come out of unpromising territory.

Some of the chapters in the book, or games in the resource pack may lead to valuable problems; you are equally likely to find them around you, in your own situation.

(G1TG, pp. 8-9)

That the mathematics must be made relevant in terms of the experiences and specific situation of the student is not tantamount to a denial of the recontextualising achievements of the gaze which can be described by reference to an example that is offered of an appropriate problem:

... The sort of activity we would call a ‘problem’ might be to give a pupil a parcel, some scales, a post office guide to postal charges and some stamps. The pupil is then asked to stamp the parcel...

(G1TG, p. 9)

The implied narrative is comparatively open. It could relate, for example, to a domestic or to a work setting. Even so, it is actually quite difficult to imagine a non-school-maths situation in which anyone would actually perform such an task. Within a domestic setting, the necessary range of stamps would be unlikely to be available and the parcel would be checked at the Post Office counter anyway. Within a work context, the probability that an automatic franking machine and/or scales calibrated in terms of postal charges would be available would be quite high. Furthermore, even to the extent that such a task is plausible within a work setting, the individual would be trained to perform it as a routine, it would not constitute a problem. The task, in other

words, is structured in terms of esoteric domain mathematics, number (linear measurement) and arithmetic (combinations of the face values of the stamps), rather than the work or domestic context.

The use of scales in the above ‘problem’ is an example of what is referred to in G1TG as ‘practical work’, which is an important pedagogic mode in relation to ‘low ability’ students, a mode which may actually be used in substitution for ‘written work’:

We hope that it will be possible to give pupils as wide a range of practical experience as possible. Some (like weighing) we have included in the work. In other places it is very desirable to have practical work alongside, or in place of, written work. So, for example, alongside book G1 we would hope that watches and clocks would be used in chapter 4, ‘Time’, and that dials, meters and scales of various types can be brought into the classroom for use in chapter 1, ‘Estimating and scales’.

(G1TG, p. 9)

‘Scissors’ are included in the list of ‘essential equipment’ and the Guide provides advice on how to obtain filter papers (‘we suggest you ask your science department’) and ‘items such as dice, counters and mirrors’ (the names and addresses of some suppliers of educational aids are listed). Furthermore, ‘practical tests’ are included in the formal assessment for the G scheme and most of the chapter notes include suggestions for ‘practical activities’. ‘Practical’ work of the kind described in G1TG is clearly localising insofar as it does not incorporate discursive distancing from itself. The suggestion that practical work may substitute for written work seems to place little value in such discursive distancing.

9.3.3 The Teacher’s Guides: summary

Whereas Y1TG focuses its attention on mathematical practices, G1TG theorises the student. The G1 Guide firstly marks out its student reader in relation to the general readership of the scheme: the G student has ‘low ability’. ‘Low ability’ is theorised as reflecting competence at a comparatively low level of mathematics, low pace of working, low task complexity and short attention span. The individual realisations of these properties in individual students must be matched by pedagogic action. The G1 Guide further prescribes an appropriate pedagogic practice for ‘low ability’ students as a form of therapy. This therapy is individualised in terms of the ‘mathematical’ strategies, which may be idiolectical, and in terms of settings, which must be ‘relevant’ in respect of individual experiences and circumstances. There is, in a number of the extracts cited from G1TG and elsewhere in the G series Teacher’s Guides, a considerable emphasis on discussion within which students share their

strategies. The G students are thus constituted as a resource. Discussion between Y students, on the other hand, is generally corrective. Emphasis is also placed, in the G Guides, on localising, 'practical' tasks.

We can describe Y1TG as proposing generalising strategies in the sense that there is minimal reference to the student who, furthermore, must be brought to the discourse. There is no apology for the fact that 'whenever the work [...] is introduced it is likely to be difficult for many pupils'. G1TG, on the other hand, proposes localising strategies in relation to a highly specified student to whom mathematics must be taken. The emphasis on pedagogic advice in G1TG, furthermore, produces the teacher, a dominant voice in school mathematics, as incompetent *qua* teacher, as if pedagogy is only an issue for 'low ability' pupils which are, naturally, of a different species from the teacher. Y1TG produces a generalised student reader in respect of an external mathematics, a potentially dominant voice. G1TG produces a localised and 'disabled' student reader which incorporates its own mathematical limitations with respect to the esoteric domain. However, the G students are constituted as a reservoir of highly localised, public domain 'mathematical' strategies.

9.4 Distributing strategies: summary

Distributing strategies relating to the subordinate voice in school mathematics (the G reader) are predominantly localising, whilst those relating to dominant voices (for example, the Y reader) are generalising. Localising and generalising strategies involve signifying modes, settings, and the reader her/himself as textual resources. Firstly, iconic localising is achieved via an iconic code of presence which inserts the reader into a participating position with respect to the narrative or setting. Such an insertion is inevitably localising in that it focuses on the specificity of the here and now. However, as discussed in Chapter 4, there can be no iconic signification of esoteric domain mathematical signifieds, because these objects and relationships are defined only formally: mathematically, there is no tangible instance of a 'circle'. Thus the iconic insertion of the reader is inevitably an insertion into the public domain. Secondly, localising is achieved by the attachment of reader-oriented metonyms within narratives, so that domestic settings, for example, might be described as exhibiting a high 'valency'¹ *vis a vis* the G reader. Thirdly, localising is achieved via the restriction of message to the procedural, again foregrounding the here and now and obscuring the generative grammar of the esoteric domain and of the

¹ In (recontextualised) chemistry, 'valency' can be glossed as 'bonding power'.

recontextualising gaze. The DS⁺ practices of mathematics are (re)produced as DS⁻ practices.

When the G text addresses the teacher, it does so by theorising and localising the G student, the subordinate voice, ensuring that the locus of pedagogic action is defined as public domain and again facilitating the insertion of the G student into that domain. The text is now *about* the subordinate voice far more than it is about mathematics (there is very little mathematical guidance in the G series Teacher's Guides). The location of pedagogic practice within the public domain is nowhere more eloquently expressed than in the maturation trajectory proposed for the G reader as represented by the iconic exchange of cartoons for photographs in moving from G1 to G8: the playful reality of the child is replaced by the domestic responsibilities of the young adult.

Localising is concerned with participation: iconic participation, or presence; participation in reader-friendly settings; participation in procedures. Localising strategies, therefore, propagate the myth of participation. The myth of participation announces mathematics as a necessary condition for optimum participation in public domain settings. However, localising strategies are realised in procedural rather than discursive message. The lack of articulation of this message of necessity results in semantic vacancies within and around the its procedures. These vacancies are filled by the insertion of the reader her/himself or, rather, by their prior participation in the public domain settings that are involved by the text. Thus, the myth of participation at once constructs mathematics as a necessary condition for participation and participation as a necessary condition for mathematics: it is, thereby, deconstructed.

The myth of participation is revealed as mythical in another way. The construction of the public domain is the achievement of the recontextualising gaze which acts in accordance with the grammar of esoteric domain discourse. The construction of the public domain, therefore, constitutes grammatical transformations in the settings which are appropriated. The criteria for the elaboration of recontextualised domestic, work settings, etc, are, to a greater or lesser extent, constituted by school mathematics, rather than by the domestic or economic spheres. Therefore, whilst mathematics may optimise these settings, there is no reason to suppose that it will do the same for domesticity or work. Empirical disjunctions between school mathematics and the 'everyday' have been argued in Chapter 2 of this thesis and elsewhere (Dowling, 1986, 1989). On the other hand, if participation in domesticity and work is to be a necessary condition for mathematics, then there can be no material basis for

such prior participation, because the domesticity and work that is involved is recontextualised domesticity and work.

Generalising is achieved via the dislocation of the dominant voice (the Y reader) from the public domain. Iconic codes of presence are marginalised, often quite literally, and metonymic attachments within the public domain are mathematics-oriented rather than reader-oriented, drawing the reader into the esoteric domain. But not as a theorised object. On the contrary, the dominant voice is to become the voice of the esoteric domain and of its gaze. Generalising strategies apprentice subjectivities: localising strategies alienate objects. If subjectivities are to be achieved, then the esoteric domain must be (re)produced as DS⁺ and, indeed, this is the case in the discursive message of the Y books. Even where the text remains within the public domain, the reader is kept out by the strategy of quasi-discourse which fills the semantic vacancies in and between public domain procedures. When the Y series Teacher's Guides address the teacher, they do so minimally, because there is a sense in which the text is addressing itself. The Y text does not theorise the student any more than the womb theorises the embryo. There can be no compromises with the practice.

The public domain for the subordinate voice is a fictional world. For the dominant voice, the public domain is fictional, too. However, it contains sufficient metonymic links to allow the potentially dominant voice to enter, thereafter to be immediately recruited into the esoteric domain, as a subject. This corresponds to the apprenticing of the dominant subject position: the interpellation constitutes entry into the public domain; apprenticeship achieves the one-hundred-and-eighty-degree rotation from the public to the esoteric domain. Within the esoteric domain, mathematical practice is DS⁺. In addition, the gaze may be cast in diverse outward directions, so that its action is made visible. There is, however, no promise of participation other than participation in the esoteric domain. This is because the location of the dominant subject position is within the esoteric, not the public domain. Rather, a claim is being made to the validity of mathematical description. It is not that you need mathematics in order to participate in the everyday (utilitarianism), nor that you are doing mathematics by participating in the everyday (mathematical anthropology). Rather that mathematics itself constitutes a language of description with unlimited applicability. This is the myth of universal description.

Finally, it has been noted that the closed narratives introducing domestic and work settings in the G texts connote sets of material relationships within which the

reader is or may expect to become enmeshed. The ideal reader is established within an ideal set of relationships. The emphasis on discussion between G students as a feature of pedagogic action has also been noted. The G student is thus constructed within an interactional peer group. The G reader is, as has been established, also constructed as embodying a repertoire of low-level, everyday competencies which facilitate their idealised entry into the public domain settings that are created for them. The ideal G reader is thus constructed as a resource in a form of pedagogic action which entails the sharing of already acquired, highly localised competencies. In this respect, the G student has no career other than within the interactions that are to be facilitated or which are indexed by the text. Pedagogic orientation is always in the here and now.

The Y texts, by contrast, construct the Y reader as being in deficit. Discussion, in the Y text, is intended to reveal these deficiencies, as with the investigational work in Y1, or to highlight mathematical knowledge. Public domain interactions are comparatively rare. Where they are introduced, they tend to reflect the classroom discussion; Plate 9.7 provides an example of this and another in which the reader is clearly alienated from the setting. The apprenticing pedagogic action of the Y text provides compensation for the deficit in the Y reader in recruiting them as a potential dominant voice. Pedagogic action is, thus, future-oriented.

Perhaps surprisingly, then, there is a sense in which it is the Y text and not the G book that generates a deficit model of its ideal reader. And it is the Y text, not G, which produces a compensatory pedagogic action. The absence of any deficit in the G reader, however, is also a justification for their absence of any career and must be seen in the context of their established classification as a 'lower ability' student.

Textual Strategies & Resources III: *positioning strategies*

In Chapter 9, I described textual strategies which construct voice via the distribution of message within the voice structure, that is, distributing strategies. In this Chapter I will consider those strategies that act directly on voice. There are two general categories of these *positioning strategies*. Firstly, a strategy may achieve the *mapping* of its voice structure onto public domain hierarchies. For example, I will illustrate, with reference to the *SMP 11-16* texts, the mapping of mathematical ability onto an essentially social class hierarchy. In this example, social class is implicated as a *textual resource* by the positioning strategy. These strategies are referred to as *interactivity positioning strategies*, because the resources that are involved are public domain, that is, they are recontextualised from outside school mathematics. Alternatively, voices may be positioned in relation to each other without recourse to public domain resources. For example, students may be arranged on hierarchical scales such as 'ability' or 'age'. Because these latter strategies achieve unmediated relationships between voices, they are referred to as *intervoice positioning strategies*.

I shall begin with a discussion of interactivity positioning strategies, using the example of gender. The main focus of my attention in the first Section of this Chapter, however, will be on the recruitment of social class as a resource within school mathematics. This is partly because there is a 'recognition', by the *SMP 11-16* text of gender and of race and, to a far lesser extent, of ablebodiedness, as politically sensitive areas. This is apparent in an attempt at 'political correctness' in respect of these areas. Social class, by contrast, is not recognised in this way. It is also apparent, from the review of the literature (Chapter 2 and Appendix 1), that social class has received far less attention, recently, in respect of the analysis of textbooks than either gender or race¹. This is, however, not tantamount to a dismissal of these other dimensions, which clearly justify further attention; this is discussed further in Chapter 11.

¹ Ablebodiedness has, it appears, yet to be taken seriously in this kind of research, however, see Brown & Dowling, 1989.

10.1 Interactivity positioning strategies

10.1.1 The recruitment of gender

Strategies within the category of interactivity positioning strategies attach connotations to specific voices through their association with hierarchies which are recontextualised from outside school mathematics. The discussion here, therefore, is concerned primarily with the public domain. Elsewhere (Dowling, 1991c; this paper is discussed briefly in Chapter 2) I have considered school mathematics strategies relating to gender. Referring to a wider range of school mathematics texts than the present thesis, I described the way in which certain texts exclude the feminine, so that mathematics is represented as masculine and, by default, the feminine is non-mathematical. Other texts reproduce stereotypical images of the masculine and feminine and associate erudite mathematics with the former¹. Other texts, such as *SMP 11-16*, exhibit an apparent 'political correctness' (as noted in Chapter 7) with respect to gender. However, these texts present an unreal world of gender equality (although not entirely consistently, as is noted, *passim*, in this thesis). Their failure to confront substantive gender inequalities explicitly renders them ironic.

Gender is a defining and hierarchical category of subject positions within domestic activity. However, the gender hierarchy is pervasively (re)produced within all activities as a resource in the (re)production of their voice structures. The result might be described as patriarchy. A modernist feminist description might regard this pervasion as a patriarchal hegemony of which the patriarch is the subject. However, the model of the social which is being applied here regards patriarchy as the universalisation of a fundamental resource. It is not so much that the patriarch actively penetrates school mathematics, rather, school mathematics recruits, appropriates and recontextualises the patriarchal hierarchy. This is not an idealist conception as is, arguably, 'hegemony'. Gender is understood as a fundamental and material organising principle (others being age and kinship) within a fundamental domain of social structure, the family. School mathematics can hardly fail to implicate it. Crucially, gender within school mathematics is recontextualised gender. The recontextualising may, for example, be realised as a mapping of gender onto mathematical competence (see, for example, Dowling, 1991c; Walkerdine, 1988, 1989).

¹ The SMP series *Revised Advanced Mathematics* was cited as an example of the first kind of text; the original SMP series (Books 1-5) are examples of the second. Both are published by Cambridge University Press.

The earlier work cited above made empirical reference to the original SMP scheme as well as to *SMP 11-16* (and a number of other texts relating to school mathematics). The recontextualising of gender within the original SMP scheme certainly retained an element of gender stereotyping. In this sense, gender was not 'recognised' by these texts as a dimension of social inequality (or, alternatively, this form of social inequality was 'recognised' as necessary). As I have suggested above, however, the *SMP 11-16* scheme, on the other hand, does 'recognise' gender and its recontextualising is often realised in terms of idealised states in which gender is no longer relevant. There are women farmers, women builders (but not, it appears, women tax inspectors or men doing needlework).

Whether this is an 'adequate' tactic in the politics of 'equal opportunities' is not a central concern in the present study. Here, the subject is not tied to one essential identity or to a small number of possible and competing identities (gender, class, ethnicity, etc). Rather, the subject is constructed whenever an activity can be described. As was discussed in Chapter 4, the empirical (human) subject is to be considered as a concatenation of subject positions. Insofar as gender is a resource which is universally involved in positioning strategies in these subjectivities, then it is reasonable to speak of the social as characterised, but not caused, by patriarchy. In my model, the relationship between the social and its cultural (re)production is dialectical rather than causal.

This brief discussion relating to gender has been included for two reasons. Firstly, as has been mentioned here and elsewhere in this thesis, the *SMP 11-16* texts are structured in gender terms, their apparent 'political correctness', notwithstanding. Secondly, insofar as gender can be understood as a simple dichotomy¹, the discussion is intended to clarify the notion of the recruitment of public domain resources by positioning strategies. In this case, school mathematics can be seen (in at least some of its textual realisations) to attach masculine metonyms to the dominant voice and feminine metonyms to the subordinate voice. Thus the school mathematics hierarchy of subject positions is mapped onto a gender hierarchy.

¹ This may be a reasonable approximation in social terms, but clearly not in cultural terms (see, for example, Garfinkel, 1967; Kessler & McKenna, 1978).

10.1.2 The recruitment of social class

My intention, now, is to focus on the recontextualising of another hierarchy, that of social class. Class may be understood as a fundamental organising principle in the social spheres of commodity production and exchange¹. The description of class as an organising principle demands an empirical analysis of commodity production and exchange; this is clearly beyond the scope of the current project. Were such an analysis to be carried out, it is supposed that class would emerge as a complex structure of dominant and subordinate subject positions². Nevertheless, social class stratification is connoted by certain discursive oppositions, such as intellectual/manual³ and by institutional oppositions such as quality/popular press⁴. Social class is thus available as a resource which can be appropriated in the positioning of voices within activities such as school mathematics. Thus, in the same way as school mathematics positioning strategies attach gender-oriented metonyms to dominant and subordinate voices, they may attach class-oriented metonyms in the (re)production of the voice hierarchy. Insofar as this is achieved, it can be argued that Y and G readers are constructed in terms of social class.

In the discussion which follows, I will describe two categories of interactivity positioning strategy which were introduced in Chapter 6. These will be referred to as *connotative mappings* and *denotative mappings*, respectively. Eco notes that:

The distinction between connotation and denotation is not (as many authors maintain) the difference between 'univocal' and 'vague' signification, or between 'referential' and 'emotional' communication, and so on. What constitutes a connotation as such is the connotative code which establishes it; the characteristic of a connotative code is the fact that the further signification conventionally relies on a primary one ...

(Eco, 1979; p. 55)

This is, essentially, the distinction that is being used here. A *mapping*⁵ is the production of a relationship between voice hierarchies within, on the one hand, school mathematics and, on the other, its public domain. For example, high ability

¹ Race being another, that is, in relation to international divisions of labour, see Dowling (1990a, 1991b).

² The complexity of the structure being attested by the diversity of forms that class analysis has taken; see, for example: Dahrendorf, 1959; Miliband, 1969; Poulantzas, 1975; Sohn-Rethel, 1978; also Parkin, 1978.

³ Indeed, Alfred Sohn-Rethel (1973, 1978) uses this opposition as the fundamental organising principle of social class, as was discussed in Chapter 3.

⁴ See Tunstall, 1983, and discussion later in this chapter.

⁵ In mathematics, the term 'mapping' refers to a relationship between the elements of one set with the elements of another, thus, for example, the mapping $x \rightarrow 2x$ (read as 'x maps onto 2x') would map the elements of the set {1, 2} onto the respective elements of the set {2, 4}.

and low ability may be mapped onto respective high and low voices in the (recontextualised) social class structure of commodity production and exchange: high ability → middle class; low ability → working class. Thus a resonance is achieved between ability (school mathematics) and social class (commodity production and exchange). Where the mapping is denotative, it is achieved by direct, which is to say, metaphorical signification. For example, via the incorporation of middle class and working class characters into Y and G texts respectively¹. Where the resonance is connotative, the metaphor remains implicit. It is suggested by the initiation of a metonymical chain, so that the Y and G texts share something (the metonym) with, respectively, the middle and working classes. The attachment of the metonym 'manual' to the G reader, for example, produces a mapping onto 'working class' via the connotation of the metonym. The signification of the mapping, in other words, relies on a prior signification. I shall discuss connotative mapping first.

10.1.3 Connotative mapping: intellectual & manual

It is instructive to look, again, at the covers of the Y and G books. The pictures on the front of Books Y1 and G1 (Plate 10.1 & 10.2) might serve as emblems for their respective series. The Y1 picture, a contour map of a face, is intellectually ambiguous: is it a mathematising of humanity or a humanising of mathematics? It is a janusian celebration of the mythical: mathematics can describe us; we can participate in mathematics. The picture also foregrounds the ambiguity of its own mode of signification. Depending upon how you read it, it is both icon and index. The icon/index is, in other words, enigmatic, signalling the mysteries to be explored between the covers, in chapter 11, 'Gradient'. Contrast this cover with that of Book G1 (Plate 10.2). There is nothing very ambiguous here. The icon signifies three everyday items, foregrounding the quotidian. Mathematics is backgrounded in a quite literal sense: the digital calculator watch is partially obscured by its analogue. The juxtaposition of the three images suggests a historical narrative, not a mathematical one. There is no mystery, nothing intellectual. Even the calculator, in a sense, manualises the intellectual and wristwatches are manual things.

The arcane/mundane opposition between the two series is restated on the covers of each book. The manual theme of G1 is sustained on the covers of G2 (Plate 10.3) and G3 (Plate 10.4). The hands of the technician (a scientist wouldn't be doing this) are foregrounded along with the physical act of weighing the mouse. The manual act

¹ The relationship between such characters and the reader, of course, remains connotative unless a denotative connexion is made.

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 10.2
(G1: front cover)

Plate 10.1
(Y1: front cover)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 10.4
(G3: front cover)

Plate 10.3
(G2: front cover)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 10.6
(G4: front cover)

Plate 10.5
(Y2: front cover)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 10.8
(Y3: front cover)

Plate 10.7
(G5: front cover)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 10.10
(Y4: front cover)

Plate 10.9
(G6: front cover)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 10.12
(G8: front cover)

Plate 10.11
(G7: front cover)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 10.13
(Y5: front cover)

of measuring on a building site also seems a long way from the intellectual classroom. Y2 (Plate 10.5), by contrast, shows an M.C. Escher print, the mathematics ('Points, lines and planes' in chapter 10) is again indexed by the enigma proposed by the icon.

Books G4 and G5 (Plates 10.6 & 10.7) signal very practical skills, again all outside of the classroom. These covers iconise the indexical. The windmills show light and shade and grass bents are drawn in. Mum, dad and dog (the reader's?), watching TV/reading, find their way into an iconised plan of their own house which is graphically mapped onto a map of their town. The girl on the cover on Y3 (Plate 10.8) might be a girl in the reader's class, but she is not the subject of the picture. She appears to be facing herself and this image is repeated in an ever reducing sequence. Something tricky is going on here and it has something to do with mirrors. How is it achieved, why does it work: mathematics will explain on page 54, where another picture of the girl in the mirror is printed.

G6 (Plate 10.9) shows an 'exploded diagram' of what might be a single roomed crofter's cottage. 'People use exploded diagrams to fit things like cassettes together, or to take them apart', we are told on page 26, under the heading 'Exploded diagrams (1)'. It is, perhaps, unlikely that such diagrams are used in the construction or demolition of crofters' cottages, but 'Exploded diagrams (1)' is about the construction of an audio cassette. 'Exploded diagrams (2)' (pp. 53-6) is about a plastic kit of a model Mercedes 540 which 'you can buy' and, presumably, assemble manually. Y4 (Plate 10.10) produces a metaphor which maps the shadow cast by the lampshade onto a the graph of a mathematical function. There is, again, nothing manual about this image, even the hand which draws the graph is absent. Nor is there anything useful about the image, the mythical potential of mathematics as arcane description of the mundane is celebrated, but not, on this Y cover, its use value for participation. Indeed, the image almost dismisses the public domain as having minimal intrinsic value.

The covers of the final two books in the G series (Plates 10.11 & 10.12) take us farther afield than the homeliness of the G5 and G6 covers. The sprinting feline blurring its exotic savannah setting connotes speed, the title of chapter 6. But the significations remain in the world, in the public domain. It is the speed of a motorbike or a car that constitutes the topic of the chapter. The giant 'golf ball' on the cover of G8 is somewhere else in the world, Disneyland. The 'golf ball' is 'almost spherical', like some other buildings, two of which are shown in chapter 8, 'Cones, cylinders

and spheres'. The chapter also includes photographs of a football and a globe: the mundane made literal. The final cover in the Y series (Plate 10.13) returns to an ambiguity similar to that in the Y1 image. Has the world been mathematised: it's not a spherical but an icosahedral 'globe'? Or has the face of the world been superimposed on a mathematical index. The arcane and the mundane compete with the former coming out on top. The only practical thing about this 'globe' is that it won't roll away.

Clearly, more than one textual strategy is involved in the production of these covers. All involve projective recontextualisings, some (Y1, Y4, Y5, G4, G5) being (without consideration of the chapters that they index) mythical domain, and the rest public domain. Insofar as they involve narratives, the Y covers are far more open and the G covers relatively closed. Thus, the Y covers tend towards the general, the G covers towards the local. These are message (re)producing and distributing strategies. However, the covers also implicate positioning strategies. The enigmatising of the world by the Y covers constructs an intellectual reader, whilst what might be described as the factualising of the world by the G covers constructs a non-intellectual reader. Furthermore, the emphasis on the manual skills in some of the G covers (G1, G2, G3, G6 (by connotation)) constructs a manual reader. Through the respective attachment of the metonyms, intellectual and non-intellectual or manual, the two sets of covers achieve a *connotative mapping* of ability onto social class.

The manual connotations in the G texts are not restricted to the cover illustrations. The emphasis on 'practical work' (connoting the manual) in the G1 Teacher's Guide was discussed in Chapter 9. There is also a stress on measuring which recurs throughout the G series. For example, the development of decimal numbers is actually not dealt with at all. Instead, there are chapters dealing with the development of decimal measure¹, as illustrated in Plate 10.14. In the early G books there are a number of instances of the inclusion of drawings of hands, writing or drawing and stressing the manual aspects of the tasks (see, for example, Chapter 6).

The contrast between the G and Y approaches to manual work is particularly apparent in the comparison of two tasks discussed in Chapter 7. 'Detective dice' (Book G1) and 'Rolling a cuboctahedron' (Book Y1) are shown in Plates 7.7 and 7.5. These tasks both involve cutting out and making up dice from polyhedral nets.

¹ This is a crucial difference in mathematics which may be represented, grammatically, as a difference between nouns (number) and adjectives (measure): measuring always involves a number of something (see Pimm, 1987).

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 10.14
(G2: page 27)

As noted in Chapter 7, the G1 task makes this a very manual activity by the inclusion of images of the practical process of making the dice. By contrast, the Y1 task clearly minimises the manual. The contents as well as the covers of the G series map the subordinate voice onto a working class position via the metonym 'manual'. The Y books, by contrast, tend to downplay the manual, bringing the intellectual to the foreground and so mapping the dominant voice onto a middle class position.

10.1.4 Connotative mapping & form of presentation: style as a metaphor for class

The constitution of *SMP 11-16* as printed material facilitates an alternative metonymic route between voice and class. If we compare the physical construction of the G and Y series, the following differences emerge. The main textbooks of the G series, books G1 to G8, not only contains fewer pages overall than those of the 'Y' series, books Y1 to Y5, (512 pages as opposed to 832¹) but there are also more G books (8 compared with 5 Y books). This means that each G book has very much fewer pages than each Y books (G1 has 64 pages, Y1 has 160). The G books are, in fact, stapled as booklets, whereas the Y books are bound as books². Considered as a whole, the G series comprises 8 books of approximately 60 pages each together with G booklets, 3 'G resource packs', 'G supplementary booklets' and 'topic booklets' (shared with the 'B' books). This gives a total of 31 G items (apart from Teacher's Guides) listed in the price list as compared with 7 Y items (the five main books and two 'extension' books). The result is that the Y' materials, taken individually, are far more 'weighty' than individual G materials and this is enhanced by the difference in the binding of the main books in each series.

The analysis of signifying modes in Chapter 9 revealed that Book G1 contains three times as much iconic space per page as does the first one-hundred pages of Book Y1 and nearly nine times as much as the sample of one-hundred pages from Book Y5 (see Tables 9.2, 9.5 & 9.6). On the other hand, the Y1 sample contains approximately one-point-four times as much symbolic space per page as Book G1 (Table 9.5). Although there is a reduction in iconic space between G1 and G8, Book G8 still contains nearly six times as much iconic space per page as Book Y5 (Table 9.6). The ratio of symbolic space in G8 to that in Y5 is almost unity (Table 9.6), but

¹ These figures include the title and contents pages etc.

² Since 'extension book', YE1 is stapled (having only 64 pages), the difference in binding seems to be a consequence of the relative lengths of the books rather than (or, at least, in addition to) any deliberate decision to produce booklets for the 'less able' and 'books' for the more able. It is not, however, the authors' intentions which are at issue here.

this is accounted for by the substantial increase in graphs in Y5 (three times the page coverage by non-manuscript graphs in G8 (Table 9.6)). Furthermore, the character density of symbolic space in the Y series is significantly greater than that in the G books (Table 9.4), so that the symbolic text in the Y series is visually more dense than that in the G books.

This differentiation of the G and Y materials in terms of physical and textual 'weightiness' connotes similar differentiations in printed materials of other kinds. For example, the differentiation between books for young children in comparison with those intended for a more adult audience and, in particular, the differentiation between the 'popular' and the 'quality' press. 'Quality' newspapers are typically substantially more weighty. Thus, on 21st March 1990, *The Daily Telegraph* contained 48 broadsheet pages and *The Times*, 56 (in two sections). *The Sun* and the *Daily Star*, on the other hand, contained only 32 and 36 pages respectively. Since the tabloid format is only half the size of the broadsheets, the 'quality' papers cover approximately four times the page area of the 'populars'. Furthermore, as Jeremy Tunstall (1983) notes: 'typically about 60% of a tabloid's contents is in fact 'looked at' material—pictures, headlines, cartoons and display advertising ...' (p. 134).

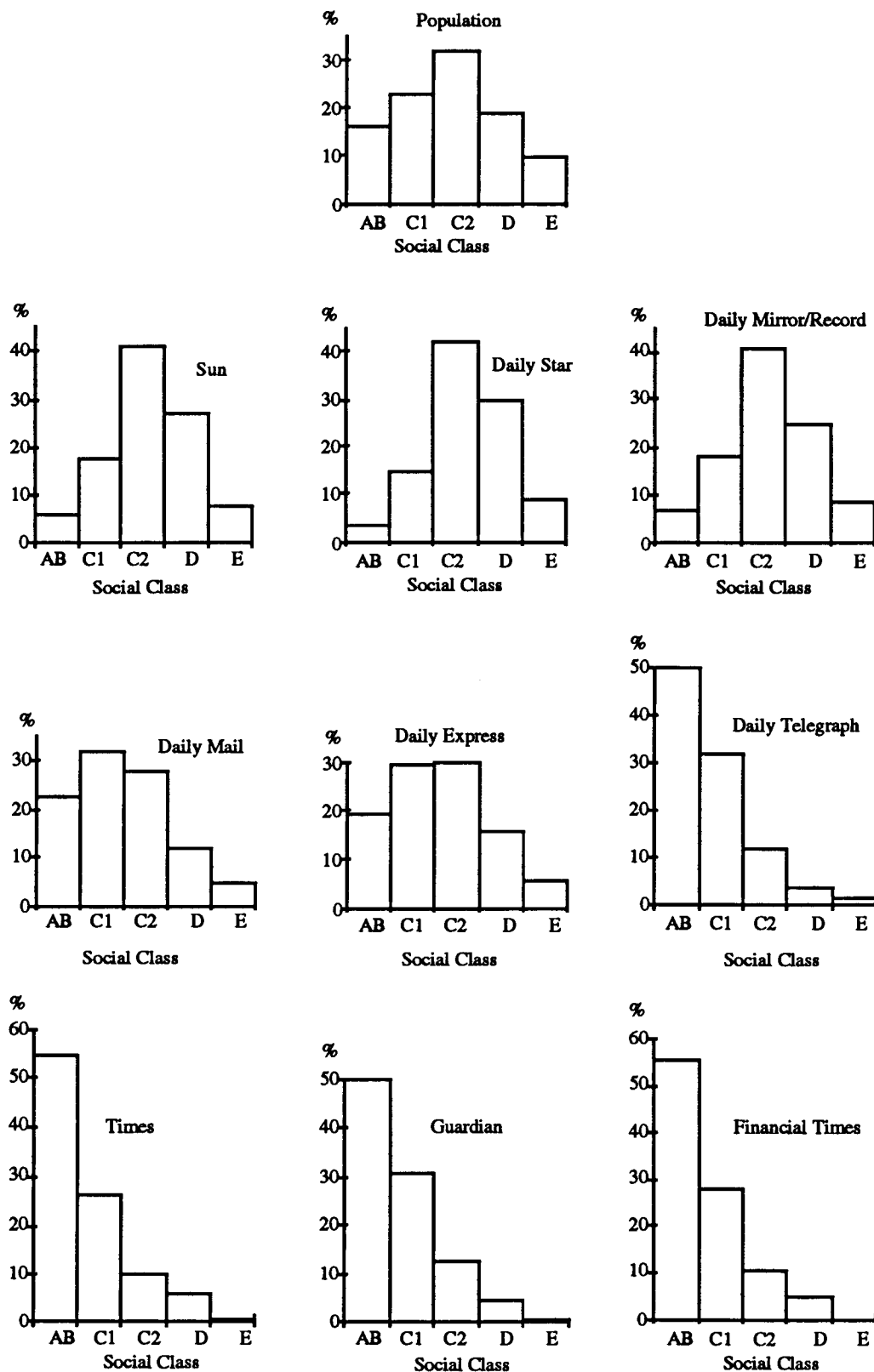
There is, in other words, a correspondence in form between the G books and the 'popular' press, on the one hand, and the Y books and the 'quality' press, on the other. However, there is a connotative differentiation between the 'populars' and the 'qualities' in terms of social class; Figure 10.1 illustrates the readership of the major 'quality' and 'popular' papers by social class. As Tunstall notes, this differentiation has a considerable history:

The 1947-49 Royal Commission referred to 'quality' and 'popular' national newspapers. Other nations have had similar distinctions—in France the *Grand* and *Petit* press—but in Britain this tradition is particularly long; it dates back to *The Times* and its radical rivals of the 1830s. To some extent the distinction between the large size prestige papers of the 1980s and the popular tabloids reflects real differences in education, reader interest and income. These real differences have become exaggerated because the two types of paper have not only, since the 1970s, acquired different physical sizes, but they rely on different prime sources of revenue. The 'prestige' papers operate primarily from an *advertising* revenue base; this forces them 'up market' more than a sales revenue base would require—because advertisers are willing to pay several times as much to reach readers who are several times as wealthy.

(Tunstall, 1983; p 77)

Economic targeting is also evidenced by the content of both reporting and advertising in different categories of newspaper. For example, in reporting income tax changes in the Budget in 1990, the *Daily Star* (21st March 1990) included tables

National Newspaper Readership



Adapted from Tunstall, 1983; p. 77

Figure 10.1

showing weekly wages up to £700 per week, whilst *The Daily Telegraph* of the same day presented incomes as annual salaries up to £70 000 per year. This constitutes a difference in both form and substantive income (£700 per week is £36 400 per annum). In respect of advertising, we find, for example, ‘SALES MANAGER Algarve, Portugal £45 - 70K + Benefits’, in *The Daily Telegraph*. In the *Daily Mirror*, a Department of Trade and Industry advertisement is a photograph showing hands, holding aloft hand tools of various kinds and bearing the slogan ‘HANDS UP FOR A JOB ... *REAL TRAINING • REAL SKILLS • REAL JOBS.*’

It may well be the case that, as Williams (1961) points out, we cannot find ‘quite the simple class affiliations used in popular discussion’ (p. 236), particularly if readership data are presented in terms of the proportion of each social class taking each newspaper, rather than, as in Figure 10.1, the proportion of the readership of each newspaper within each social class. However, there is no attempt, here, to unify the Registrar General’s sometime classifications in terms of cultural qualities. It is not a question of what social classes are like (insofar as such a question may have any meaningful answer). Nor is it entirely a question of what newspapers are like, although this is closer (and is, to a degree caught at by the social class profile of their respective readerships). It is more a question of what Williams describes as ‘popular discussion’ which enables connotative mappings, direct metonymic chains linking school textbooks to social classes via national newspapers.

10.1.5 Connotative mapping: summary

Connotative mapping is an interactivity positioning strategy which homomorphically maps the voice structure of a text (or a simplified form of the voice structure) onto a public domain voice structure. The principal public domain structure that has been discussed is social class, which is recruited and recontextualised from the field of commodity production and exchange. Gender—recontextualised from the domestic field—has also been mentioned. The mapping is achieved through the initiation of metonymic chains. Thus, ‘intellectual’ and ‘manual’ are attached respectively to dominant and subordinate voices within the school mathematics text and these attach to ‘intellectual’ and ‘manual’ labour, that is, to a recontextualised social class structure within the field of commodity production and exchange. Similarly, the differential physical forms of the G and Y books achieve their respective metonymic attachment to what might be referred to as ‘popular’ and ‘quality’ forms of printed material. These metonyms connote the ‘popular’ and ‘quality’ press which, in turn, connote ‘working class’ and ‘middle class’. Thus a connotative mapping is achieved

of the *SMP 11-16* voice structure onto social class, (re)producing the dominant and subordinate subject positions of school mathematics in social class terms.

10.1.6 Denotative mapping: income & social class

As I stated earlier, denotative mapping is achieved via the direct incorporation of recontextualised voice hierarchies into the school mathematics texts. For example, Books G8 and Y5, the terminal books in their respective series, each contain a short section relating to income tax. In G8, this section (section D of chapter 6, 'Percentages in use') also includes Value Added Tax; Y5 has a separate section dealing with VAT. The G8 section (Plate 10.15) opens with a photographic reproduction of a small collection of government information documents, including 'Income Tax and the Unemployed' (foregrounded, almost as a threat). The settings for the exposition and tasks on page 43 include the denotations of weekly wage earners and basic rate tax payers. In fact, nearly all of the references to income in the G series refer to weekly wages, even a doctor's income (G8, p. 37; Plate 9.4) is described as 'about £350 a week'.

However, the expression 'wages' is absent from 'Money matters: income tax' in Book Y5 (Plate 10.16), the term 'income' is used exclusively. Throughout the Y series, incomes are generally described as annual salaries and only very rarely as weekly wages. Furthermore, the Y5 exposition on income tax also denotes above-basic-rate income tax payers and the only calculation that the reader is to perform concerns an individual with a taxable income of £50 000. The Y5 section is distinguished from the rest of the book by its comprising almost entirely exposition. There are four tasks, grouped in pairs, under the heading 'Find out', but only the calculation of the tax paid by a £50 000 per annum earner can be completed on the basis of information given in the exposition; the others require further investigation. A substantial amount of the information given in the exposition is in fact historical information which has no utility value in respect of the calculation of income tax. In fact, there is, generally, no need to calculate the amount of tax paid on your salary, because this is done on behalf of the government by tax inspectors (illustrated by the cartoon shown in Plate 10.16) or by your employers (as is stated in the G8 section). In terms of mathematics, the Y5 section appears to be almost redundant, so that the non-mathematical denotations, including high income groups, are foregrounded.

Juxtaposed, the G and Y sections denote an income hierarchy within the economic field which now maps onto the ability hierarchy denoted by the G/Y

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 10.15
(G8: pages 42-3)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 10.16
(Y5: pages 18-9)

differentiation. This is a denotative resonance because high income earners and wage earners are incorporated directly into the Y and G texts. This was not the case with respect to different social classes described earlier, where social classes were connoted via the intellectual/manual dichotomy and via the opposition of 'quality'/'popular' newspapers. On the other hand, insofar as salaries and wages connote middle and working class positions, respectively, there is an additional connotative mapping of ability onto social class. The distinction between connotative and denotative mappings is made purely in terms of the directness of the link.

The mapping described above is achieved via a differentiation of denotations. However, resonances can also result from the differential modes of incorporation of public domain denotations into narratives. For example, although neither the G nor the Y series make many direct references to 'professionals' within what used to be Social Class 1 (OPCS, 1980), where they do, the reader is differentially placed in relation to them. A one-and-a-half page sequence of tasks in Y1 relating to a geologist, for example, involves calculations using formulae relating to the growth of stalactites. The fictional geologist has derived these formulae empirically, for example:

[...]

In a wetter cave nearby, the geologist studied two other stalactites.
the formula for the length of one of them was $l = 1231 + 1.8t$,
and for the other it was $l = 1384 + 2.1t$.

As before, l is in mm and t is the number of years since 1950.

- (a) Find the value of t when the two stalactites were of equal length.
- (b) In what year was this?

(Y1, p. 71)

There is a sense in which the reader is participating, as an apprentice, perhaps, in the work of the geologist¹, using her [sic] equations, etc. Another case concerns a 'biologist' who 'wanted to compare the lengths of worms living in two different kinds of soil' (Y3, p. 89). The final part of this task requires the reader to 'write a brief report comparing the two groups of worms' (ibid). The reader is again positioned alongside the professional.

By contrast, the doctor in G8 (mentioned above) is simply a wage earner. Dr Baxter, possibly an academic, is running a computer programme in G1 (p. 59). However, the G task involves calculations relating to the time that the programme takes and not anything to do with the professional activities of Dr Baxter. Another

¹ Although the effect of recontextualising is still visible, it not being entirely clear why the geologist might wish to know the answer to this particular question.

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 10.18
(Y5: page 181)

Plate 10.17
(G6: page 6)

medical doctor appears in G8, taking a girl's pulse (p. 48). However, the whole section (two pages) is about heart rates, more than half of it relating to a graph of the pulse rate of an athlete. Only in two tasks is the measurement of pulse rate placed within a medical setting (an earlier task has a nurse timing a patient's pulse rate) and the last task in the section involves the reader experimenting with her/his own pulse rate. Thus the timing of a pulse is not constituted as specialised medical practice in the way that the prediction of stalactite growth is constituted as specialised geological knowledge. The G reader is watching the doctor; the Y reader is, in a sense, a proto-apprentice of the geologist or of the biologist.

On the other hand, where non-professional occupations are denoted, the relationship between the Y and G readers and the occupational group is often reversed. Plates 10.17 and 10.18, for example, both show 'police' settings and both concern the estimation of the speed of a vehicle by measuring skid marks. On the face of it, the two texts produce similar tasks. However, the two texts implicate the police into their narratives in quite different ways. The photograph in the G6 section (Plate 10.17) situates the reader at the shoulder of the officer making the measurement. This viewpoint is minimally interrupted by the strong iconic code of presence of the photographic mode. The table to be completed by the reader is drawn in manuscript, as it might be in the policeman's notebook. The final task requires the reader to estimate the speed of a car from its skid marks, thus performing a simulation of the policeman's task. In addition, the formula is stated in words, which minimises the intrusion of specialised expression: the text barely penetrates the mythical domain. The reader is a proto-apprentice of the policeman.

The Y text (Plate 10.18) is actually very different. The code of presence is now interrupted by the drawing mode and the reader is positioned as more of an onlooker than a participant. The formula is now stated in what looks much more like mathematical language and the tasks move well away from the activity of the police officers. The reader must draw a graph and the final task inverts the relationship between the known and the unknown quantity so that it is quite irrelevant to the kind of question that the police are interested in¹. The Y text more clearly takes charge of the police activity as a resource to be incorporated in what looks much more like an esoteric domain mathematics task, although it remains mythical.

¹ F1-3(a) in the G text also invert this relationship, but apparently only to facilitate the more 'realistic' tasks in F3(b) and F4.

Other examples of the colonising of a non-professional work setting for resources are shown in these two examples:

Diana's car has a faulty speedometer. When it was checked these were the results.

I (indicated speed in m.p.h.)	20	30	40	50	60
T (true speed in m.p.h.)	19	29	44	54	67

- Draw axes with I from 0 to 60 and T from -10 to 70. Plot the values of (I, T) and draw a line of best fit.
- Find the equation of your line.
- According to the straight-line graph, when the speedometer indicates 0 m.p.h. the true speed is negative, so the car would be going backwards! This is not likely to be true! It is more likely that the pointer on the speedometer 'sticks' at a certain lowest value. Read from your graph what that value is.

(Y3, p. 21)

A plumber wishes to cut twelve pieces of copper pipe. They are to be cut from standard 3-metre lengths.

Piece	A	B	C	D	E	F	G	H	I	J	K	L
Length in metres	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	1	1	$1\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{3}{4}$

- What can you think of as the 'bins' in this problem?
- What is the smallest number of 3-metre lengths the plumber can use? Show how the pieces would be cut.

(Y5, p. 13)

In each of these tasks, the manual occupation is incorporated into the narrative such that the Y reader is again situated as mathematical observer and commentator. There is no pretence that mechanics draw graphs (and the point at which the speedometer sticks is, in any case, irrelevant: the meter needs replacing). Nor is it implied that plumbers make use of 'bin-packing' optimisation strategies which, in this chapter, are applied in a diversity of settings. Compare these examples with the 'bricklaying' tasks A6 and A7 in Plate 9.10 (discussed in Chapter 9). As was the case with the geologist in Y1, the G7 reader is using the builder's formula. In the G text, however, the reader is 'apprenticed' to a manual worker, and not to a member of the professional classes.

10.1.7 Denotative mapping: school settings

Both the G and Y series include denotations which are associated with school settings other than school mathematics. A number of cases in the Y series involve students carrying out science experiments:

A student was doing experiments with a pendulum [...]

(Y2, p. 51)

A student carrying out an experiment in electricity passed an electric current through a piece of copper wire. She varied the voltage across the ends of the wire and each time measured the current in the wire. (Voltage is measured in volts and current in amps.)

Here are her results. [...]

(Y2, p. 112)

Two students were looking over an old warehouse which was going to be demolished. They found a rope and pulleys which had been used for lifting sacks.

They decided to investigate the relationship between the load being lifted up and the amount of pull needed on the free end of the rope. [...]

(Y3, p. 19)

Marcus was doing some experiments to find the greatest load which various thicknesses of rope would hold without breaking.

These were the results he got. [...]

(Y4, p. 67)

Nadim measured the diameter, d mm, and the mass, m grams, of several ball bearings. Here are the measurements.

(Y4, p. 99)

A student studying electricity varied the voltage across a piece of wire and measured the current in the wire each time. Here are her results V stands for the voltage, in volts, and I for the current, in amps.

(Y5, p. 162)

In each of these cases, the reader is required to draw a graph or, in one case, to interpret a graph which has been drawn for them.

There is quite extensive use of non-mathematics school settings in the G series, but very rarely do these index curriculum content. The following extract from Book G4 is a very rare example:

Ajit is weighing this empty beaker.
The beaker weighs 0.096kg.
(a) How many grams does the beaker weigh?
(b) Ajit wants to put exactly 500g
of water in the beaker.
What will the scales read when there is
500g of water in the beaker?

(G4, p. 10; drawing omitted)

This looks like a science lesson setting, but it is not, in fact, about the knowledge content of a science lesson. The reader is given no indication as to why Ajit might be weighing water and, in any event, the beaker would quite possibly be marked at 500

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 10.20
(G1: page 37)

Plate 10.19
(G1: page 32)

ml level, making weighing redundant¹. Plate 10.19 incorporates a representation of science as bomb-making, an image which is confirmed in the bottom illustration in Plate 10.20. There are a number of other cases in which the reference is to the assessment of students in different lessons: History, in G6 (p. 7); Typing in G8² (p. 50). However, the knowledge contents of these disciplines are again absent. Very frequently, school settings in the G series concern non-curricular activities. The school timetable, in Plates 10.19 and 10.20 and also in G2; the school canteen, in G1, G4 and G6; and especially fundraising, usually sponsored events of one form or another, in G2, G3 and G4.

The G texts incorporate curriculum procedures (weighing the beaker, assessment) and stereotypical images (science as bomb-making), school organisation (the school timetable, the school canteen), and non-curricular activities (fundraising). All of these incorporations alienate specialist knowledges, which is to say, they alienate DS⁺ practices in the school. The examples from the Y texts, on the other hand, index precisely the DS⁺ quality of school science. Science, in the Y texts, is about the empirical discovery of relationships and not making mischievous explosions. The mapping, here, confirms the association of high ability with the DS⁺ practices and low ability with DS⁻ practices, but extends it beyond the mathematics classroom.

10.1.8 Interactivity positioning strategies: summary

Interactivity positioning strategies position school mathematics voices by differentially mapping school mathematics voices onto voices associated with the public domain, that is, with voices which are recontextualised from outside school mathematics. Connotative mappings metonymically ionise dominant and subordinate voices so that they readily bond with middle and working class positions, respectively. The metonymic attachments which have been discussed, here, are 'intellectual' and 'quality press', both of which connote middle class, and 'manual' and 'popular press', both of which connote 'working class'. Thus the mapping facilitates a semantic extension into the public domain: high ability is denoted by Y Book which denotes a particular style of presentation denoting the 'quality' press which, in turn, denotes a 'middle class' readership. In this way, high ability is connotatively linked to middle class. The metonymic chain, once established, can be elided by the metaphor: high ability = middle class³.

¹ The specific gravity of water being 1.

² The trajectory from 'academic' to vocational between G6 and G8 is also worthy of note.

³ The use of Recorde's symbol here is not, of course, mathematical; simply, there is a sense in which high ability can be taken to be the same thing as middle class.

In Eco's model (discussed in Chapter 3), the sequencing of metonym-metaphor has no syntactical rule as he illustrates, playfully, in his novel, *Foucault's Pendulum*:

It was a little like that game where you have to go from sausage to Plato in five steps, by association of ideas. Let's see: sausage, pig bristle, paintbrush, Mannerism, Idea, Plato. Easy ... I had a strict rule, which I think secret services follow, too: No piece of information is superior to any other. Power lies in having them all on file and then finding the connections. There are always connections; you only have to want to find them.

(Eco, 1989; p. 225)

That this can only be a game is revealed upon the realisation that 'idea' would serve as the single metonymic connection between any pair of signs: sausage, idea, Plato. There is no room, in this semiotic model for the social organising, or motivation of language. There is, in other words, no reason why the above metaphor should not be replaced by high ability = 'working class', indeed antonyms can just as easily be interpreted as metonyms, so that Eco's model is in this sense tantamount to social anarchy¹. In the present model, however, it is the recontextualising gaze that is recruiting a hierarchy and effecting a mapping onto its own structure of subject positions². The connections are always there, but only certain kinds of metaphorical outcome are possible and it is in this higher order grammar that the materiality of activity is revealed and whereby it is distinguished from decontextualised (which is to say procedural) descriptions of 'games'.

Denotative mappings simply make the metaphor explicit. Three examples have been given with respect to the textual strategies of the Y and G books. Firstly, the mapping is achieved through the exclusive (or near exclusive) implication of one pole of a hierarchy. Y books incorporate denotations of salaries, G books incorporate denotations of wages. This might be indexed as mapping by exclusion. Secondly, both series incorporate denotations of both poles of a hierarchy, but incorporate them differently into narratives in terms of their relationship to the reader. The Y reader observes and comments upon non-professional work and is a proto-apprentice to professionals: the G reader is an observer of professionals and an apprentice to non-professionals. This mapping concerns *reader association*. Thirdly, public domain voices are hierarchised in terms of their relationship to discourse/procedure. 'Students' in Y narratives carry out science experiments: G 'students' collect money

¹ This does not constitute a criticism of Eco who is clearly not engaging in sociology. Furthermore, he is fully aware of the existence of structuring in his use of 'codes', although he does not provide a strategy for writing specific codes. The essays in Eco, 1990, are also relevant in the bridling of semiosis.

² Although this is no claim that subject position structures are always simply hierarchical.

for charity. This can be referred to as *public domain elaboration*. There is no suggestion that this constitutes an exhaustive categorising, the fundamental category is 'denotative mapping' and not the specific and potentially forms of its realisation which are determined by stylistic resources.

10.2 Intervice positioning strategies

Interactivity positioning strategies constitute recontextualised hierarchies as metaphors for a given activity (school mathematics) voice structure. Intervice positioning strategies relate the voices directly to each other. There are, nominally, two categories of actor involved with pedagogic action: teacher and student. Intervice positioning strategies can therefore conveniently be discussed in terms of voice differentiation within each of these nominal categories and voice differentiation between them. It should be stressed that although intervice positioning strategies are defined as constituting direct relationships between voices, it is clearly not possible for this to be achieved in a manner which is textually entirely separate from message and distributing strategies, the issue is one of emphasis. In fact, most of the analysis which follows consists of a revisiting of text which has been discussed under other headings. I shall begin with a brief consideration of student-student differentiation, then I shall discuss teacher-student differentiation within each series of books. Finally, I shall look at differentiation within the nominal category teacher.

10.2.1 Student-student positioning

I have in fact already described the structuring of student voices in terms of age and ability. By age structuring, I mean that the mathematical competence of a student is in certain respects and to some degree positively correlated with their chronological age. This structuring is clearly not unique to school mathematics and, indeed, is almost certainly associated with the institutional siting of school mathematics within schooling (see Chapter 4). The relationship between age and competence in non-school activities (such as academic mathematics, athletics, politics) and in certain activities associated with schooling, such as teaching, is not likely to be the same. The strategies by which age structure is achieved clearly include the organisation of the SMP scheme on a cohort basis and, in particular, the break in format after the second year (see Chapter 5). Less explicit realisation is exemplified in, for example, the trajectory of signifying modes in the G series, that is, the transition from a high cartoon content in the early G books to a very low cartoon content and, in particular, a high photograph content in the final books in the series (see Chapter 9). The ability

structuring of student voices is, as has been mentioned, quite explicit in the SMP scheme, so that dominant and subordinate voices are signified, respectively, by 'able' and 'lower ability'. Alternatively, these voices may be differentiated (at the level of the signifier) by the absence or presence of the term 'lower ability' (or an associated term). It has also been illustrated that the theory of instruction, which is incorporated into the Teacher's Guides, constructs a relationship between age and ability such that ability can in some respects and to some degree compensate for age or, alternatively, that 'low ability' students are retarded in terms of their chronological age.

The previous discussion of these factors was in the context of the distribution of message. It was argued that the theory of instruction relating to the G reader inserted the reader her/himself into the public domain of school mathematics as a moment of the message of school mathematics (Chapter 9). For example, the G reader was described as being constructed as a resource within a form of pedagogic action that consists of a sharing of already acquired competences. The point being made in the current section is that the naming of student voices in terms of age and ability itself constitutes a textual strategy which positions them in relation to each other, irrespective of the semantics of such positioning in terms of school mathematics message.

10.2.2 Teacher-student positioning

With respect to teacher-student differentiation, these categories exhibit comparatively weak classification where the student is a Y reader. This is because the (implicit) theory of instruction assumes the successful acquisition even where this might be achieved with some difficulty. Thus, the acknowledgement that work on 'constructing formulas' (Y1, c. 3, section G) is both 'particularly important' and 'very difficult' (Y1TG, p. 18) clearly signifies optimism that student readers will, indeed, must overcome their difficulties. This is because, as was illustrated in Chapter 9, there is very little in the way of compromise in terms of the authority of the discourse in the Y series. In particular, there is comparatively little scope for students' individual methods in completing tasks and there is an almost universal adoption of a 'serious' attitude to the esoteric domain, although the public domain is occasionally satirised, as has been illustrated. There is, in other words, minimum objectification of the Y reader in the Y scheme and very little in the way of message that is made available to teachers but which is denied to students: the Y Teacher's Guides contain comparatively little apart from the answers. In terms of esoteric

domain subjectivity, there is clearly a potential career path from Y student to Y teacher.

By contrast, the classification between the G reader and the teacher is comparatively strong as it is constructed in the Teacher's Guides. This is because, as has been discussed earlier, the student reader is objectified by localising strategies via an explicit theory of instruction (Chapter 9). In G1 (the first student's book), however, there is a different kind of objectifying. The mathematics teacher, Mr Ling (Plate 6.7), is ironised in the narrative and cartoon: he wants to sell 200 pages of maths exercises; 'New 24-volume pocket edition'; 'Buy them today!!' Plate 10.19 satirises various aspects of school, in particular, science is about bomb-making (or some equivalently demonic activity) and mathematics is mystifying. The best time of the day is home time (or possibly school lunch) Another sequence is shown at the bottom of the page in Plate 10.21. Mathematics ('sums') is again presented as mystifying, lunchtime is good and hometime is what you have really been waiting for. In the cartoon sequence above this in Plate 10.21 we can see that history sends you to sleep. In Plate 10.20: comics and magazines are much more interesting than geography¹; a games lesson is a brawl, for which it is not even necessary to get changed; science is about making explosions. It is not the student that is being objectified, here, so much as school itself. There is an establishment of an apparently collusive relationship between the authorial voice and the student through the satirising of the lessons. The author appears to be siding with the reader against incomprehensible maths and boring history lessons, so that authorship itself is incorporated as a resource in the positioning strategy; we can refer to this resource as a *virtual author*.

The pervasive use of public domain settings in the G texts (see Chapter 9) recontextualises the reader's everyday life in such a way as to represent the reader as having a degree of autonomy within the public domain. The insistence on the invitation and sanctioning of students' own methods in the G Teacher's Guides also accords an autonomy. In both of these respects, the G texts contrast with the Y books which tend to alienate the reader from the public domain and insist on specific methods. The voice of authority in the G series is, apparently, far weaker than in the Y series.

¹ The gendered associations achieved in this image have implications for the whole scheme. The two boys in the front row are reading comics, the girl is reading 'True Story'. This association has a tendency to masculinise the kind of cartoons that are associated with comics and which also characterise much of the content of the earlier G books which are, thereby, masculinised.

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 10.22
(G6: page 30)

Plate 10.21
(G1: page 34)

IMAGE REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Plate 10.23
(G1: page 47)

However, the reader autonomy and the collusion between virtual author and reader are entirely spurious (which is why it is 'virtual'). As has been frequently argued, the public domain remains structured by the esoteric and not by domestic or other activities, thus Ruth, in Plate 10.22, may be just a little older than the reader and is already organising her own domestic space. However, the width of material required for making curtains is generally estimated by multiplying the width of the window by between two and three, so that the curtains will still have folds when drawn closed. Assuming that Ruth has used such a rule, it is certainly not 'silly' to go for the nearest to the amount required, even when this is slightly less than the estimate. As was argued in Chapter 9, the mythical cannot be generated from within the public domain, but only received as the voice of authority. In the G texts, the voice is heard, but cannot be understood and pedagogic action is 'invisible'.

Collusion in the form of the satirising of the school is also spurious, at least in relation to the satirising of school mathematics, which is certainly treated very seriously in the Teacher's Guides. The classification between the teacher and the student is in fact strengthened by the duping of the latter, the intentionality of which is, at times, almost explicit in the Teacher's Guide:

If pupils have covered part of level 3, they may meet some of the work again. We have tried to ensure that level 3 work is presented in a different way so that pupils will not complain that they have 'done all this before', but on the contrary will be provided with stimulating revision material.

(G1TG, p. 6)

The ironising of the school curriculum in G1 is, in fact, a satirising of the reader her/himself through the presentation of stereotypical reactions to the curriculum. A similarly double irony is illustrated in Plate 10.23. The waiters in the cartoon in the top righthand corner are in formal dress, but one of their number looks out of place. He has a broken nose, stubble on a lantern jaw, and a crew cut: a stereotypical image of a working class lag, perhaps. This waiter is also holding aloft a tray on which are placed a beer bottle and a foaming jug, establishing class connotations by comparison with the wine bottle and glasses carried by another waiter. The thuggish waiter strongly contrasts with the other, rather snooty waiters: perhaps a joke against pretentious restaurateurs. The joke is compounded by one of the illustrations accompanying task C10. What seems to be a composite bill for first course dishes served over a period of weeks, beginning with: 3625 soups, 1200 prawn cocktails, 2765 smoked salmon, 1490 whitebait. Patés are rather less popular, 169, but the really pretentious and, of course, foreign dishes are almost completely rejected, only 3 snails and 1 frog's leg. It is the five snooty waiters who are out of place in this

restaurant in which discriminating, middle class diners, are a fantasy. The clientele is actually less than sophisticated, that is, working class. The cartoon alienates middle class pretension through the satirical presentation and thus apparently allies a virtual author and the reader with each other and with the working class.

There is no necessary assertion, here, that working class individuals have lantern jaws and stubble, or that they refuse to eat exotic food and drink beer rather than wine. This image of the working class is constructed by the text which, through the use of satire, sides with this 'Andy Capp' image and eschews the middle class image presented. At the same time, we may suspect a double irony. Just as Andy Capp is a handicap to his wife and to the broader economy, the gastronomical incompetence of the working class, mirrored by its mathematical incompetence and ultimate unteachability, is an impediment to the free elaboration of middle class sophistication and of the erudition which characterises the authorial voice behind its virtual brutish mask. The cartoon images the middle class author looking at the working class reader looking at the author in a regression which is a truncated version of the photograph on the cover of Book Y3 (Plate 10.8).

Teacher-student differentiation in the G books, particularly in Book G1, is rather complicated. The classification of teacher and student categories is strong as it is constructed in the Teacher's Guides which incorporate an explicit theory of instruction which objectifies the student. The student's books, on the other hand, appear to collude with the student reader through the use of satire (mainly confined to Book G1) and through the apparent accordance to the student reader of public domain autonomy. The recontextualising effectivity of the gaze, however, abnegates the autonomy which is to be reinterpreted as the invisibility of pedagogic action. Similarly, the satirising of the school and of the middle classes (in the restaurant setting) is to be interpreted as ironic. The teacher-student relationship is one in which the former dupes the latter. There is no career path between G reader and teacher as there is between Y reader and teacher.

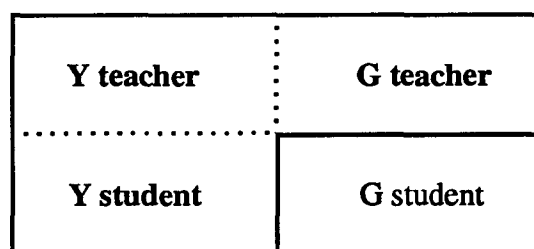
The extensive pedagogic advice offered to the teacher reader of the G series Teacher's Guides contrasts with the minimal pedagogic commentary in the Y Teacher's Guides. This contrast achieves a differentiation between the Y teacher and the G teacher which is in accordance with their respective forms of pedagogic action. The Y teacher is concerned with the apprenticing of the Y student reader, that is, in the (re)production of the dominant subject position, which is to say, the the (re)production of the Subject of the activity. Insofar as the Y student reader has been

appropriately selected (s/he is the correct social class, etc) there is minimal impediment to this apprenticing and no theory of instruction is required. The G teacher, on the other hand, is concerned with the (re)production of objectivity which is to be constituted in the already alienated (working class) G student reader. The G teacher therefore has to acquire knowledge about the inadequacies of the G student reader and must be instructed in the production of spurious autonomy: 'Work on timetables, map-reading, shopping and so on is far more motivating for pupils if it is seen to be "real". Blagdon can never substitute for your own town!' (G1TG, p. 8).

10.3 Positioning strategies: summary

It is now possible to represent the voice structure of school mathematics as constructed by intervoice positioning strategies in the *SMP 11-16* texts, this is shown in Figure 10.2.

Voice Structure in *SMP 11-16*



Bold type represents dominant voice. Normal type represents subordinate voice. Dotted line represents weak classification. Solid line represents strong classification. Careers are possible upwards and across, but are blocked by strong classification.

Figure 10.2

The diagram does not include the voice structure in relation to age which is illustrated in Table 9.14 (Chapter 9). However, it does show a number of other features that are derived from the discussion above. Firstly, there is comparatively weak classification within the category, dominant voice, which comprises Y and G teachers and Y student. There is a possible career path between Y student and Y reader. Y teacher and G teacher are more or less interchangeable given the availability of a theory of instruction with respect to the latter. The classification between dominant and subordinate voices, that is, between subjects and potential subjects, on

the one hand, and objects, on the other, is strong and there is no career path, because the classification is made in terms of the fixed attribute 'ability'. Additionally, the classification between G teacher and G reader is strengthened by the duping of the latter by the former.

The strong classification between dominant voices and subordinate voice is strengthened by mapping strategies which map, either denotatively or connotatively, the voices onto hierarchies relating to public domain voice structures. Thus, for example, a social class hierarchy is mapped onto high-ability/low-ability, that is, with dominant/subordinate voice. This is achieved either directly, through, for example, respective proto-apprenticeships, or indirectly, for example, through association with the 'quality' and 'popular' press. I have described these mappings as being achieved not by the hegemonising of school mathematics by fundamental subjectivities (gender, class, race, etc), but by the recruiting and recontextualising operation of the gaze of school mathematics itself.

This is not to argue that any kind of recontextualising is possible. The public domain is defined as being weakly classified with respect to the non-school-mathematical (although clearly not uniformly weakly classified with respect to all other social regions). This being the case, plausibility clearly delimits the degree of recontextualising violence that can be effected. In particular, the inversion of hierarchical voice relationships would introduce singularities that would need to be resolved through, for example, an interpretation of the text as ironic¹.

10.4 Subjectivity & fundamental organising principles

In this Chapter, I have been concerned with the recruiting of fundamental sociological and political categories (gender, class) which are often attributed ontological status. I want to indicate the possibility of a non-ontological conception of these categories. I shall attempt to do this by giving a brief consideration of subjectivity as (re)produced in empirical human subjects (introduced at the end of Chapter 4).

The patriarchal nature of society entails that most if not all activities construct subject positions on a gendered basis². Similarly, the stratification of society on the

¹ See Dowling (1991c) regarding the ironic defusing of potentially antisexist school mathematics texts.

² Although not all must maintain the patriarchal ordering of the hierarchy. Subordinated activities may invert it, patriarchy being maintained via the subordination of the activity as a whole. We might speculate that this is a feature of domestic subjects in school.

basis of social class, race and ablebodiedness is conceived of, within this model, as being achieved by the universalising of these principles within most or all activities. Thus, it is proposed that all activities bear the trace of gender, class, race and ablebodiedness and that this is the basis of the fundamental nature of these fundamental organising principles.

Subjectivity is conceived, here, as an articulation of subject positions. The gendered, classed, racial and physical state into which the human individual is born (or, in some cases, achieves) entails that subject positions relating to most if not all activities will tend to construct them as uniformly gendered, classed, raced, and ablebodied/disabled. The subject is thus substantially constrained to carry these 'qualities' with them on a pan-contextual basis.

The empirical human subject is, thus, not being denied their specificity in terms of physicality and life history. However, the meanings that these specificities can take are available only within the activities which inscribe them.

10.5 Textual strategies & resources: summary

The textual analysis in this Chapter and the five Chapters that have preceded it have constituted descriptions of the *SMP 11-16* texts which is the empirical achievement of this thesis. I shall defer further discussion of these 'findings' until the next and final Chapter of the thesis. In the context of the textual analysis, I have also been able to elaborate and expand upon the textual levels of the language of description that was introduced in Chapter 4. The language of description is the theoretical achievement of the thesis and will also be discussed in the concluding Chapter. However, it is appropriate, in concluding the present Chapter, to give a brief summary of the textual levels of the language which have been developed particularly in Chapters 8, 9 and 10. The components of the textual levels of the language, textual strategies and textual resources, are assembled in Figures 10.3 and 10.4.

Textual resources (Figure 10.4) are constituted as a 'reservoir' of possible resources by the specific form or genre of the pedagogic text. In this case, the text under analysis is a school textbook scheme. The reservoir of available resources thus includes the signifying modes that are possible or, at least, conventional within this format, that is, the various forms of icon, index and symbol which have been defined. The reservoir of resources also includes a range of stylistic possibilities relating to the form of presentation of textbook materials (size, number of pages,

Textual Strategies

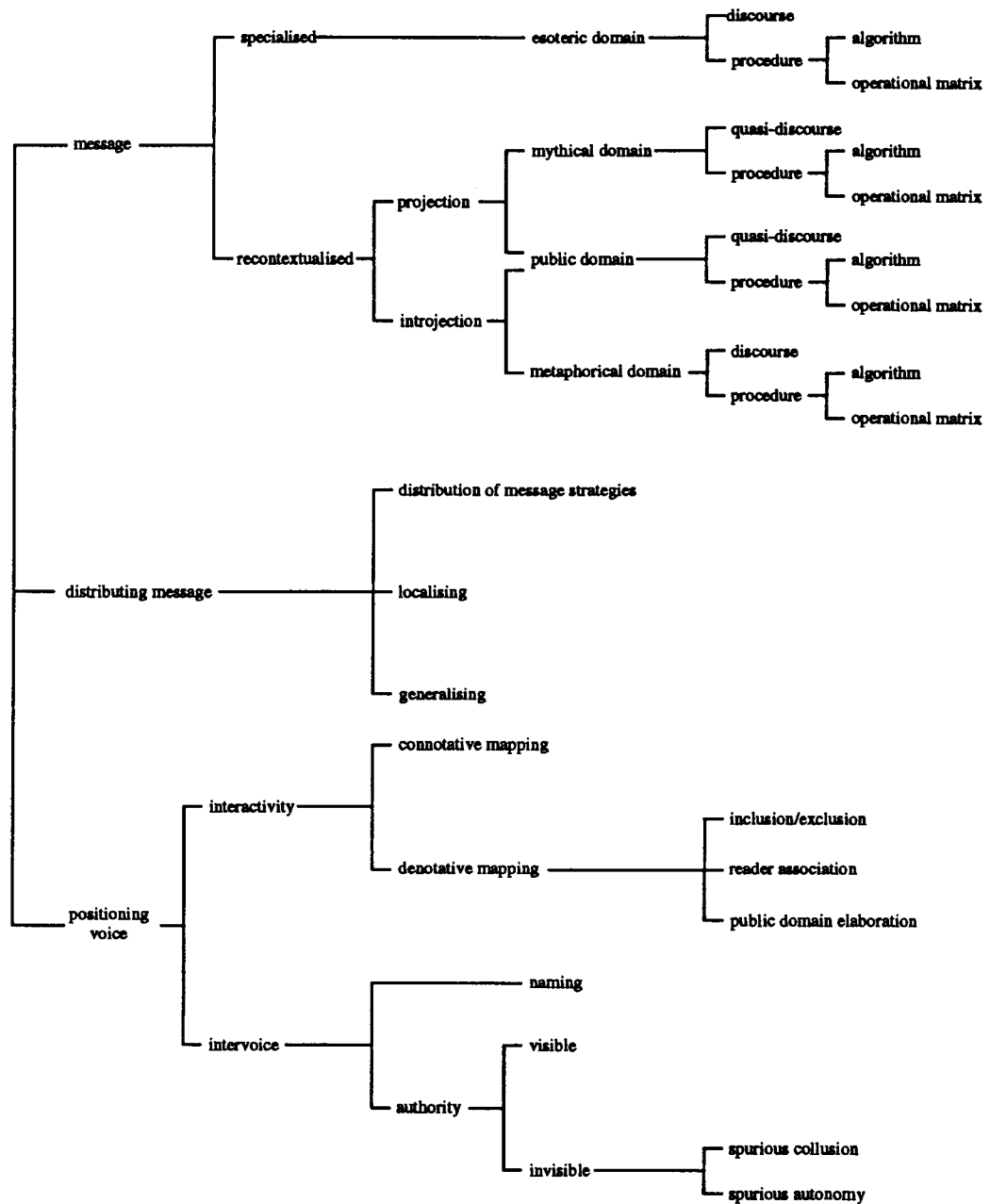


Figure 10.3

Textual Resources

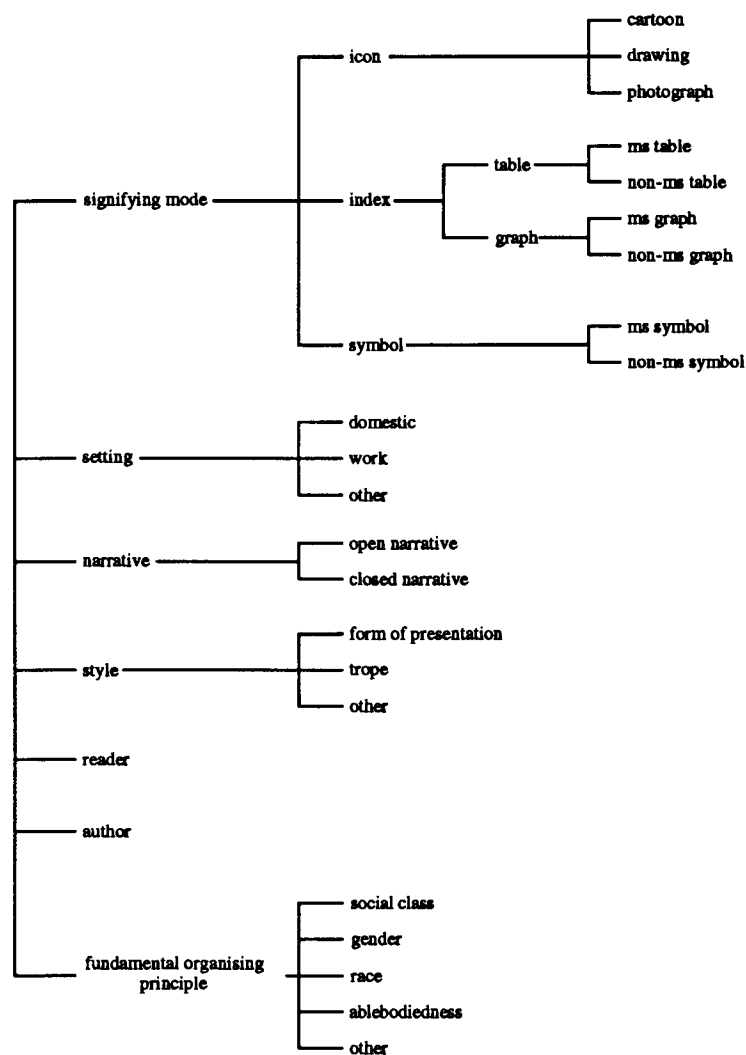


Figure 10.4

inclusion of worksheets, etc), the use of literal or tropical language, and so on. There are diverse settings that are available (domestic, work, school, etc) for incorporation into narratives, which may be relatively closed (highly detailed) or open (minimising detail). The text of necessity constructs a model reader and author. However, it must achieve this on an intertextual basis, so that the reservoir of resources also includes potential readers and authors. Finally, it has been argued that, within any given conjuncture, there are a number of fundamental organising principles which define and hierarchise all individuals and so must be recruited by strategies which articulate individuals. These fundamental organising principles certainly include social class, gender, race, and ablebodiedness.

I have delineated three categories of textual strategy. Message strategies include the construction of specialised esoteric domain text as either discourse or procedure (algorithm, operational matrix). Recontextualising (projection, introjection) constructs mythical, metaphorical and public domain text as either quasi-discourse or procedure, as shown in Figure 10.3. Distributing strategies include the distribution of message strategies and localising and generalising strategies. These strategies construct the textual voice structure via the distribution of message. Finally, positioning strategies, which have been explored in this Chapter, act directly on voice either by reference outside the activity (interactivity) or by establishing direct relationships between voices (intervoice).

The text, itself, is comprised of textual strategies which recruit from the reservoir of resources, constituting a repertoire or repertoires of resources, which contributes to the establishing of the specificity the specificity of the text. The construction of message and voice structure within a particular text constitutes an utterance of the activity to which it relates, in this case, school mathematics. The realisation of message and voice structure within a text thus (re)produces the practices and subject positions of the activity.

In the final Chapter, I shall discuss the achievements, limitations and possible developments of the theoretical and empirical work of the thesis. This will include further discussion of the language of description and of what it has revealed in the analysis of the *SMP 11-16* texts. I shall also discuss some of the limitations of the language and directions for its development. Finally, I shall consider possibilities for the relationship between, firstly, mathematics and the non-mathematical world and, secondly, the sociology of education and educational practice.

Chapter 11

Conclusion

In this concluding Chapter, I shall first give a brief overview of the thesis. The overview will include summaries of the language of description and of the principal findings of the analysis of the *SMP 11-16* texts. This will contextualise the second Section of the Chapter which will explore the constraints and limitations of the language and of the analysis. In this Section, I shall also offer some suggestions for further work and index further research that is currently under way. I shall conclude the thesis with a more speculative treatment of the language of description and of the analysis in order to generate a more productive conception of the relationships between mathematics and non-mathematical practices and between sociological research and the educational practice which this thesis objectifies. My intention, in this final Section, is to point to the potential educational use-value of this work beyond the sociological context of its production.

11.1 An overview of the thesis

11.1.1 Antecedents

The work of the thesis began, in Chapter 2, with a critical engagement with two epistemological positions which are prominent within mathematics education. ‘Utilitarianism’ is an established position which commonly informs governmental, professional and research writing. Elbert Fulkerson described the position some time ago:

One of the main purposes of mathematics is to provide the student with those mathematical abilities, meanings, and concepts which will assist him [sic] later in making satisfactory adjustments to the various problems of life.

(Fulkerson, 1939; p. 27)

Utilitarianism constructs a role for mathematics teaching in providing the mathematical tools which are necessary for the optimum participation in working and everyday practices. In this conception, the unschooled are represented pathologically, as incompetent even within their own domestic lives. Thus, Brigid Sewell (1981), reporting to the Cockcroft Committee, described shoppers as using an ‘avoidance strategy’ if they took a cheque book to the supermarket rather than keeping a running

total of what they were spending as they filled up their trolley. Even demonstrable success can be read, by the utilitarian mathematician, as evidence of incompetence, as is illustrated in an anecdote from Mike Cooley:

At one aircraft company they engaged a team of four mathematicians, all of PhD level, to attempt to define in a programme a method of drawing the afterburner of a large jet engine. This was an extremely complex shape, which they attempted to define by using Coon's Patch Surface Definitions. They spent some two years dealing with this problem and could not find a satisfactory solution. When, however, they went to the experimental workshop of the aircraft factory, they found that a skilled sheet metal worker, together with a draughtsman had actually succeeded in drawing and making one of these. One of the mathematicians observed: *'They may have succeeded in making it but they didn't understand how they did it.'*

(Cooley, M., 1985; 171, my emphasis)

'Mathematical anthropology' refers to a more recent position which is exhibited in the writings of the 'ethnomathematicians'. Writers such as D'Ambrosio, Bishop, Gerdes, and Mary Harris appear to take a view which is directly opposed to that of the utilitarians. For the mathematical anthropologists, mathematics is immanent in productive and domestic practices even where there has been no formal schooling; mathematics is a 'panhuman' phenomenon.

I have argued that, in fact, both of these positions construct an essentialist mythologising of mathematics. This is achieved by the elision of the social bases of the cultural practices that are described. It is appropriate to describe both of these positions as projecting mathematics onto a non-mathematical world, so that it can be recognised as such and attributed virtual origins within the world.

My initial response (Dowling, 1989) was to construct a 'map' of contexts within which 'mathematical' practices (including the practices objectified by the essentialists) might be conceived as being elaborated. This enabled me to situate my own area of empirical interest within the recontextualising field of school mathematics. Within this field, I targeted school textbooks as my empirical object. In the discussion of the research literature relating to school textbooks, it was concluded that there was an absence of work which had generated textbook analysis in sociological terms.

The map, however, was undertheorised. Although it provided an initial orientation, it could not offer the structure which would be needed in producing a sociological analysis of school textbooks. Essentially, this is because, although the map located contexts with respect to each other, it did not provide structure within contexts, neither did it seriously address the nature of relationships between contexts.

What would be needed was a more complete image of the social. This was the task undertaken in Chapter 3. Here, I considered three themes concerned with the theorising of the social space. The first theme involved a discussion of some of the ways in which practices have been construed as exhibiting a dual modality. Out of this discussion, I developed a concept of the level of discursive saturation. The second theme was concerned with the production of subjectivity. This discussion opened with a revisiting of Althusser's theory of ideology in general and resulted in the generation of theoretical resources for the conceptualising of pedagogic action as a particular form of apprenticeship. Finally, the third theme resulted in a more sophisticated concept of context and of the relationship between contexts than was offered by the original map. At the end of Chapter 3, I introduced ten Theoretical Propositions. These propositions, taken together, constituted an image of the social which was sufficiently developed to enable the derivation of a coherent theoretical framework.

11.1.2 The language of description

The Theoretical Propositions form an orientation to sociological analysis which is grounded in the work discussed in Chapter 3. However, not only is their coherence inadequately expressed in their formulation, but this formulation is at far too high a level of abstraction to enable the empirical analysis of textbooks. In Chapter 3, therefore, I derived a coherent theoretical framework which extended from the higher levels of the Theoretical Propositions to the level of the text which was to be described. This theoretical framework constitutes what Bernstein has described as a 'language of description'. It enables the translation of the textbooks, as information, into data which has meaning within the context of the general orientation provided by the Theoretical Propositions.

The language proposes that the fundamental context for practice is 'activity'. Activity regulates what can be said or done or meant by whom. In other words, it establishes practices and subject positions. Activity is conceptualised as a specific articulation of a notional Global Semantic Universe which specialises expression and content resources. The degree of specialisation of modes of expression and contents varies in terms of the strength of classification with respect to other activities. Thus, it is possible to conceptualise four domains of practice. The esoteric domain constitutes the region of strongest classification (maximum specialisation) which is the regulating domain for the activity. Activities are conceived as casting a gaze onto other activities in much the same way as the utilitarians and mathematical anthropologists cast a

mathematical gaze upon non-mathematical practices. The achievement of this gaze is the generation of a public domain (minimum specialisation) and of the mythical and metaphorical domains. Hence the practices of an activity constitute four domains, of which the esoteric is regulating.

In addition to varying with respect to specialisation, practices vary in terms of the level of discursive saturation. DS⁺ and DS⁻ activities maximise and minimise, respectively, the extent to which the regulating principles of the activity are discursively available.

An activity also establishes subject positions in a structure which, at least in respect of all pedagogic activities, is hierarchical; that is, there are relatively dominant and relatively subordinate subject positions. The most dominant subject position is the Subject of the activity.

Activity is the structural level of the language. The structural level regulates its concrete realisations in texts and subjectivities. Subjectivities are empirical human subjects who are conceived of as moving between activities in the elaboration of their life histories. Human subjectivity is, in this sense, multiple. The main focus of this dissertation, in both theoretical and empirical terms, however, has been on text. Thus, textual strategies recruit from a reservoir of textual resources to constitute a repertoire of resources which is the material and symbolic substance of the text. These strategies construct the message and voices of the text which (re)produce practices and subject positions, respectively. Message strategies construct the various categories of message corresponding to the structure of activity practices. Distributing strategies construct the voice structure via the distribution of message. Positioning strategies position voices directly with respect to each other either explicitly (interview strategies) or by constructing a connotative or denotative mapping with the voice structure of another activity or with the fundamental organising principles of class, gender, race and ablebodiedness.

The relationship between the categories of textual strategy needs a little more attention. Essentially, the distinction between the practices and subject positions of an activity is an analytic one. This is because subject positions are constituted by the distribution of practices and practices can only ever be interpreted in relation to one or more subject positions. The implications of this at the textual level are that the construction of voice is always achieved via the distribution of message and the construction of message is always achieved in the context of one or more voices.

There is a sense, then, in which there should be only a single main category of textual strategy, distributing strategies. Thus, for example, localising strategies construct subordinate voices. However, because localising generates context-dependency, these strategies are also involved in the construction of procedural message. Similarly, generalising strategies construct relatively dominant voices, but, via their generation of context-independency, they also construct discursive or quasi-discursive message. Positioning strategies, similarly, must always constitute voice positions in respect of specific message distributions, whilst message strategies must always address specific voice positions.

However, this is not to negate the theoretical work of Chapters 8, 9 and 10. The distinctions that are made between these three categories of textual strategy are, like the distinction between practices and subject positions, and like the distinction between activity and text, analytic. The distinction enables different aspects of an analysis to be foregrounded. An analysis in terms of message strategies foregrounds the (re)production of the domains and discursive level of the practices of an activity, whilst backgrounding differentiation in terms of subject positions. This category of strategy would be of particular value, for example, in the analysis of a text which constructs a peer relation between its author and its reader (such as the research paper analysed by Myers (1992; discussed in Chapter 2)). Positioning strategies, on the other hand, focus sharp attention on the voice structure, backgrounding the details of the message. Elsewhere (Brown & Dowling, 1992, 1993) the cover illustration of a primary school communication to parents was described in terms of positioning strategies, revealing the structure of the relationship between teacher, parent and student. Finally, the discussion of distributing strategies enables the analysis of voice construction in terms of message construction. In particular, an analysis in terms of these strategies enables consideration to be given to pedagogic action in apprenticeship and alienation.

The review of literature in relation to the analysis of textbooks has revealed nothing which bears any direct comparison with the language of description which has been developed in this thesis. Clearly, there is a considerable body of work in the general field of linguistics which facilitates and conducts the systematic analyses of texts (for example, Halliday, 1978; Hodge & Kress, 1988, 1993). In addition to Bernstein's own work, there are also instances of the production of highly detailed principles of description within the context of sociology (see, for example, Cook-Gumpertz, 1973). Hassan (1990, 1991, 1992a, 1992b) has established an empirical link between Bernstein's sociology and her own systematic analysis of maternal

discourse, but the theoretical establishing of the link has not been achieved. The claim is made, here, that this is the first attempt to construct a coherent theoretical framework for the empirical analysis of pedagogic texts in sociological terms. This is the principal achievement of this thesis. However, the elaboration has also generated an analysis of a mathematics textbook scheme. I shall now turn to a summary of the main findings of this analysis.

11.1.3 The analysis of the SMP 11-16 materials

It must be emphasised that the analysis of the texts was conducted in parallel with the development of the language of description. Had this not been the case, in other words, had the language been in place in advance of coming to the texts, it may be that the analysis would have developed in a somewhat different way. Some possibilities, in this respect, are discussed in the next Section. One feature, however, should be restated at this point. This is that the analysis treats the texts as closed. By this I mean that the voices that are constructed by the text are not being generalised to empirical teachers and students; the use of these terms below refers to voices and not to individuals. Thus, there is, in this analysis, no consideration of the possibilities of the negotiation of the text within interactional situations. The reasons and implications of this are considered in Section 11.2.

The analysis of the school mathematics texts has revealed a number of features. The fundamental difference between the two series of textbooks, Y and G, is signalled by the front covers of the first book of each (Plates 10.1 and 10.2). The enigmatic cover of Y1 is an invitation into the unknown, whilst the pictures of a clock and two watches, on the cover of G1 signifies the mundane, the already known. Throughout the Y series the esoteric domain is foregrounded, especially by the extensive use of algebra. As the analysis in Chapter 5 illustrated, this foregrounding is progressively strengthened as the series develops. The trajectories in terms of signifying mode (Chapter 9) indicate a progression in terms of increasing generalisation as the iconic (which cannot constitute esoteric domain text) is replaced by indexical text. The discussion of positioning strategies in Chapter 10 also pointed to another progression, this time in terms of a potential career path from student to teacher.

Within the G series, the progressions are, generally, in the other direction. The analysis in Chapter 5 actually indicated a progressive reduction in the quantity of esoteric domain time within the series from 14.2% of the G1 text to 5.2% in G8

(Table 5.6). This compares with the increase in the Y series from 36.6% in Y1 to 54.4% in Y5 (Table 5.5). The analysis of signifying modes in Chapter 9 showed an increase in localising strategies within the G series. This increase was most apparent in the 'exchange' of cartoons, in G1, for photographs, in G8, giving rise to an increase in the strength of the visual code of presence. Alternatively, this exchange was described as a substitution of the childish world of comics in G1 for the adult world of the supermarket, in G8.

This construction of maturation is entirely in accord with the finding, in Chapter 10, that the career path between the G student and the G teacher was blocked via the construction of a collusive virtual author. This collusion essentially locates the G voice outside of mathematics.

The treatment of the public domain in the two series was equally divergent. The Y series alienates the public domain through the use of humour and satire. Open narratives are frequently employed, minimising the valency of settings in terms of possible reader identification; the reader is to recognise, not her/himself, but mathematics in the text. There is a tendency, also, for the Y texts to switch quickly between settings, again minimising the importance of any one. The early G texts also employ humour and satire. However, the use of relatively closed and extended narratives salt the familiar settings with metonyms which address the reader, drawing her/him into the public domain; the G reader must recognise her/himself in the text and so becomes its object. Thus, the satirising of the public domain is a satirising of the reader her/himself. In the later G books, humour is almost entirely excluded, so that there is minimal disruption of the public domain and of the reader's location within it.

The Y books exhibit an apprenticing of the reader, who is interpellated by the public domain and subsequently drawn into the esoteric via mathematically-oriented metonymic chains. From the location of the esoteric domain, the neophyte mathematician can survey the public domain with the mathematical gaze. This omniscient position establishes the mythologising of mathematics as universal description. Esoteric domain message is generally discursive, (re)producing the DS⁺ of mathematical practices. There are, generally, no concessions to be made in respect of esoteric domain authority either in terms of individual methods or pedagogic 'needs'. The individuality of the reader is almost entirely excluded by the text which establishes, via apprenticing, a new identity in the form of the dominant voice of school mathematics. With respect to school mathematics, therefore, the Y reader is

constituted as in deficit. The orientation of the Y reader is towards their future positioning within school mathematics.

The G books alienate the reader with respect to the esoteric domain. Essentially, the text rarely enters the esoteric. When it does, however, the message is procedural, so that the DS^+ of mathematics is (re)produced as DS^- . The procedures constitute an incomplete message by incorporating semantic vacancies which must be filled by the reader her/himself. This is evident in the reader-oriented settings. It is also evident in the pedagogic objectification and individualisation of the G reader in the Teacher's Guides. Mathematics is mythologised as the precondition for optimum public domain participation. However, mathematics is, at the same time, predicated upon the preexisting public domain competence of the reader. There is a genuine sense in which the duality of the message is equivalent to the combining of utilitarianism and mathematical anthropology: mathematics education is a requirement for the participation in everyday and working practices which nevertheless already incorporate mathematics in advance of schooling. Within the literature, the positions are generally (but not always) kept apart. Within the G series of textbooks, their unification constitutes the deconstruction of the myth of participation.

The public domain, in both series, comprises recontextualised settings. The message is abstracted from any social context outside of school mathematics. It is the transformation contingent upon recontextualising that establishes the myth of participation and the myth of universal description as mythical. Thus, the G reader is doubly duped: the mathematical tools will not work outside of mathematics; the competence attributed to the reader can have no basis within any non-mathematical practice. The Y reader, however, is duped only in respect of a false celebration of omniscience as omnipotence. The public domain, to which the G reader is confined, is an imaginary message (re)producing an imaginary set of practices. The DS^+ esoteric domain, on the other hand, constitutes a real message, (re)producing a real practice.

Thus the G books have, effectively, no esoteric domain instruction. However, they do incorporate a theory of instruction via the Teacher's Guides (Chapter 9). This theory of instruction localises its voice via the construction of indices of individual needs and pathologies. Furthermore, the localising of the subordinate voice within the public domain frequently enmeshes the ideal reader within a network of ideal relationships: peer, familial, workplace. The G texts thus construct the subordinate voice within the interactional in a way that is rare in the construction of the relatively

dominant voice by the Y texts. There is a consonance between the interactional public domain relating to the G voice and the emphasis, within the theory of instruction, upon 'discussion' and the sharing of techniques which apparently originate within the G reader. Thus, the subordinate voice is alienated from the principles of mathematical practice, for which is substituted a reservoir of popular techniques which is constituted by the G readers themselves. The sharing of these techniques is to be achieved within public domain interaction, rather than by visible pedagogic action. There is, thus no future-orientation of the G readers, rather they are to elaborate and share a corporate repertoire of techniques within real time.

The Y and G texts thus construct their ideal readers in opposite forms. The G reader—the 'lower ability' voice—is pathologically disabled with respect to the esoteric domain. However, this subordinate voice also embodies a public domain competence. So, the (ideal) reader her/himself is constituted as a resource within a pedagogic practice that thereby focuses on the present, on that which is already known. The Y reader, on the other hand, is a potential initiate into the esoteric domain, but this potential is necessarily constructed as a current deficit to be compensated by a future mathematical career. It is precisely this chiasmus that is effected by the juxtaposition of the covers of G1 and Y1: G1 resides within the real time of the already known public domain; Y1 interpellates the deficient reader, drawing them into an esoteric future which is indexed, but never grasped, by the enigmatic contour map/face.

The SMP scheme employs various instances of connotative and denotative mapping strategies to position its readers as dominant and subordinate voices (Chapter 10). The differentiation of the G and Y books in terms of their connotation of manual and intellectual labour, respectively, is achieved via a resonance with the 'quality' and 'popular' press as well as by the foregrounding and backgrounding of the practical and the intellectual. The association of the G and Y books with a social class hierarchy is also established, denotatively, via differential treatment of the public domain. Thus, the Y reader is likely to be positioned as a potential apprentice to a professional worker or as a potential salary earner; the G reader is potentially apprenticed to non-professionals and wage earners. Where manual workers are incorporated into the Y texts, the relationship of the reader to them is likely to be one in which they objectify and rationalise the work. The inclusion of professional workers within the G texts is, on the other hand, not associated with any form of potential apprenticeship or rationalising.

These interactivity positioning strategies, which associate the Y student with the professional middle class and the G student with the working class are important insofar as they construct stereotypical 'high ability' and 'low ability' students. There is no intention of any determinism, here and, in any event, any comment about actual Y and G students and their substantive relationships to the texts would require a different empirical base. However, it seems likely that a reversal of these positioning strategies would be unacceptable. In other words, it is difficult to conceive of a 'high ability' textbook which constructs its reader as working class whilst its corresponding 'low ability' book constructs its reader as professional middle class. There is a sense, then, in which the insertion of class-specific stereotypes into school assessment practices is plausible. Evidence that this has been the case in the past is supplied by Rist's (1970) study of teacher expectations in a ghetto elementary school. Evidence that gender stereotypes are implicated in mathematics assessment practices is offered by Walkerdine (1989; Walden & Walkerdine, 1985).

In any event, keeping to textual subjects: the Y and G series clearly 'recognise' their readers in terms of social class and this is in sharp contrast to a clear attempt (not universally successful) at 'political correctness' in relation to gender and race. Following recognition, mathematical message is then distributed in the construction of dominant and subordinate voices on a social class basis. The result is the construction of esoteric domain discourse and its associated myth of universal description as exclusively dominant. Correspondingly, public domain procedures and the myth of participation is constructed as dominated class. As Patrick Colquhoun wrote in 1806:

It is not proposed that the children of the poor should be educated in a manner to elevate their minds above the rank they are destined to fill in society Utopian schemes for an extensive diffusion of knowledge would be injurious and absurd.

(quoted by Lawton, 1975; p. 2)

More recently (1867), Robert Lowe stated:

The lower classes ought to be educated to discharge the duties cast upon them. They should also be educated that they appreciate and defer to a higher cultivation when they meet it: and the higher classes ought to be educated in a very different manner, in order that they may exhibit to the lower classes that higher education to which, if it were shown to them, [they] would bow and defer.

(quoted by Gordon, 1978; p. 121)

The apparent finding that not much has changed may be of little surprise in the light of the statement, quoted in Chapter 5, made by the first editor of the 'numbered' SMP books, Geoffrey Howson:

With all types of pupil, the final teaching language may have to take account of their *social* language: it is no good using the language of mandarins to the children of factory workers, as studies by teachers of English have shown. For example, the early SMP texts T and T4 were written in the language of mathematical specialists intent on getting the mathematics right. These were rewritten in the language of grammar school boys, and the resulting books 1-5 were again rewritten (with modifications) in the language of 'CSE' children, as books A-H.

(Griffiths & Howson, 1974; pp. 340-1)

As I have argued, however, the violence of recontextualising has rendered the public domain of the G texts inappropriate even for the 'duties cast upon' their working class readers. The public domain competences attributed to the subordinate voice are only imaginary substitutes for what they cannot have, which their 'low ability' denies them—esoteric domain subjectivity.

The subordinate, working class voice of the G books is constructed as highly individual in its artisanal condition. The message that is distributed to the subordinate voice is localised in the extreme—the localising to continue even beyond that which is possible in the books themselves as the teacher collects real tins of food (or labels, as second best) to play shopping with. This is, textually, student-centred education. The dominant, middle class voice of the Y book is granted no individuality, because they are to be apprenticed to a mathematical heritage that predates them. This is curriculum-centred education. The middle class is to carry the culture, the working class would only spoil things, as a Justice of the Peace realised in 1807:

It is doubtless desirable that the poor should be generally instructed in *reading*, if it were only for the best of purposes—that they may read the Scriptures. As to *writing* and *arithmetic*, it may be apprehended that such a degree of knowledge would produce in them a disrelish for the laborious occupations of life.

(quoted by Williams, 1961; p. 156)

For those who have no trajectory, it might be reasonable to optimise where they are. Give them the Scriptures and allow them to read. But don't tell them how we read them.

11.2 *Limitations & potential*

In this Section, I want to consider some limitations of the language of description and of the analysis as far as they have been developed in the thesis. Limitations arise in respect of four areas: the fundamental assumptions of the language itself; the application of the language; the specificities of the activity under consideration; and the specificities of the text under analysis. Following discussion under each of these headings, I shall consider possibilities for further research.

11.2.1 Fundamental assumptions

Any language of description is subject to constraints which are imposed upon it by its fundamental assumptions and core concepts. It is certainly the case, for example, that the object of analysis has been explicitly described as a 'pedagogic text'. This object has been defined as a text which constructs a pedagogic voice and at least one other voice (to-be-apprenticed, to-be-alienated). It has not been established that the language can describe any other categories of text, for example, research papers. Nor, of course, has it been established that the language cannot describe such texts. However, any movement away from pedagogic texts in the terms described here would minimally require some theoretical development. To this extent, the openness of the language, as it has thus far been developed, is limited. At a more fundamental level, however, the general methodological position which has been adopted has tended, in two ways, to limit the analysis to the description of a closed text. I shall explore this limitation at greater length.

Firstly, the methodological position adopted here has been referred to as (post)structuralist. My use of this expression is meant to invoke both a denial and an assertion. I want to deny that my analysis is intended to reveal a supposed 'true' meaning of the text. Rather, my reading is an explicitly biased reading. Fundamental to my methodological position is that the interpretation is necessarily a function of the interpreter. However, this is not necessarily to accept an untrammelled relativism whereby all interpretations are of equal value. My assertion, therefore, is that, within the realm of discourse, there is value in aiming towards coherence and explicitness, which is to say, the production of structure. The structure which is produced, the language of description, explicates the text in its own terms, thus closing it. Structure, in this sense, constructs its readers as subjects rather than as objects in laying itself open both to appropriation and to critique.

The production of structure, however, of necessity delimits freedom of interpretation by prescribing recognition and realisation rules relating to that which can be described and how it can be described. For example, my language of description must describe the text as an utterance of a specific activity. The utterer is of necessity a subject position of the activity and, if the text is a pedagogic text, the utterer is the dominant subject position. The SMP text, then, is interpreted as an utterance of the dominant subject position of school mathematics. Through its utterance, the dominant subject position establishes its own voice and the voices of relatively subordinate subject positions. That is to say, the relatively subordinate

subject positions do not establish their own voices. The text is thereby closed in a second sense, that is, there is an exclusion of any consideration of interaction between empirical subjects.

An alternative project could have included a range of other data-gathering strategies, such as, classroom observation, interviews with the SMP authors, publishers, teachers, students, parents, and so on. Such triangulation procedures are clearly of great importance where an assumption is made that there is a particular or optimum or true reading of the text. However, within the methodological position being adopted here, the interactional situations within which the text is negotiated by empirical subjects would have to be treated as interactions between subject positions relating to different activities. In the classroom, for example, neither students nor teachers are not objectively positioned by school mathematics. Rather, they occupy ambiguous positions which only are partially penetrated by school mathematics. However, students' (and teachers') behaviours are also regulated by what might be described as the polythetic practices of the classroom. The negotiation of the text, therefore, entails a confrontation of activities. These activities would have to be delineated within the course of the analysis, presenting a problem of far greater complexity than that which is addressed in this thesis.

Thus, the general methodological position which is adopted in the thesis constitutes the closed text as the simplest case for analysis. Since the thesis constitutes an origination of a language of description, it is appropriate to start with a comparatively simple analytical problem. Furthermore, the results of the analysis have established a starting position for the analysis of the negotiation of mathematical texts within the contexts indicated above.

Furthermore, although interaction is excluded as a component within the data, it is not excluded within the context of the analysis more generally. This is because the analysis itself constitutes a negotiation of the text. The analysis is a reading of a mathematical text from a sociological position and is, therefore, of necessity a critical reading, as was noted in Chapter 3. However, to assert that the reading is critical is not to admit to descriptive imperialism. This is because the reading is produced within the context of the developing of the theoretical language which enables it. The research represented by this thesis entailed, firstly, an inductive 'immersion' in the empirical texts in much the same way as is described by advocates of 'grounded theory' or 'analytic induction' (for example, Bulmer, 1984): the form in which Chapters 6 and 7 have been written is intended to hint at this process. Secondly, the

research involved the deductive development of concepts at a theoretical level as is illustrated in Chapters 3 and 4. The development in respect of both induction and deduction can never be said to have reached a definitive conclusion. The requirement that the thesis be, in a sense, a closed text rather conceals the two-way and incomplete nature of this process. That is to say, the thesis represents a tactical conclusion, or cadence, in an ongoing deductive-inductive dialogue. The site of this dialogue is in the application of the language of description, to which I shall now turn.

11.2.2 Application of the language of description

I have asserted that a language of description should aim to make explicit the recognition and realisation principles relating to that which it can describe. This has been done at the conceptual level in Chapter 4. However, there are clearly variations in the precision with which these principles have been realised at the operational level. In terms of their recognition rules: 'Icon', for example, is defined as a mode of signification which combines a code of presence, physically locating the reader, with a visual code; 'index' employs a visual code (even if reduced to tabulation), but no code of presence; symbol is uni- or zero-dimensional in terms of its visual code (it may or may not be constituted as a linear sequence). These definitions, it is asserted, offer a high degree of reliability in their use (although this has not been formally tested). In terms of their realisation as data, this involves their quantification via the use of a centimetre grid and, again, a high degree of reliability is asserted. A high degree of reliability is a common feature of quantitative analysis. However, where the analysis is more qualitative, it is clear that the operationalising of recognition and realisation principles is less reliable.

A clear example is in the recognition of esoteric and public domain text. 'Esoteric domain' text is defined as specialised in terms of both expression and content. However, there are no explicit principles which define, either intensionally or extensionally, the operational recognition of specialised expression or content. My contention is that the definition of the esoteric domain is one of the achievements of the analysis and is not something that is prefigured. Nevertheless, it also has to be said that the development of an explicit definition and, indeed, the description of the structuring of the esoteric domain is not something which has been given a high priority in the analysis. The reason for this concerns the specificity of mathematics as the object of analysis and will be discussed below. However, the analysis cannot be said to be exhaustive without this development.

The recognition of the esoteric domain is tied up with the recognition of the public domain which opposes it conceptually. It is clear, therefore, that the analysis has not prioritised the development of explicit operational recognition principles for the public domain. Indeed, since the public domain includes the recontextualising of everything that is not mathematics, its recognition depends upon a prior recognition of the esoteric. To this extent, the recognition principles for the public domain must be the negatives of those relating to the esoteric domain, the latter having already been announced as programmatic. Recognition of the mythical and metaphorical domains and, hence, of the gaze as projective or introjective, are, similarly, contingent upon the recognition of the esoteric and public. The operational distinction between them is, as was discussed in Chapter 5, dependent upon a decision as to whether it is the public domain or the esoteric domain which defines the setting: the text being mythical domain, in the former case, and metaphorical domain, in the latter.

More attention has been given to the structuring of the public domain, particularly in the discussion of localising and generalising strategies (Chapter 9) and positioning strategies (Chapter 10). Thus, for example, the public domain is structured in terms of quasi-discourse/procedure and in terms of social class-specific denotation and connotation. This is not to assert that the analysis of the public domain is exhaustive. For example, little attention has been paid to the recontextualising of the interactional within the public domain. Thus, whilst it is clear that the G series, far more than Y, places emphasis on the construction of voices within ideal peer and hierarchical relationship (see, for example, Chapter 6), there has been no systematic exploration of these relationships in the present analysis.

A second area of limitation in respect of the application of the language of description arises in relation to the unit of analysis. Apart from the work on signifying modes and the trajectory analysis in Chapter 5, there has been no attempt to define the specific unit of analysis. This is, essentially, because the text is being understood as a weaving together of textual strategies. In other words, any segment of the empirical text can be read in more than one way within the language of description. The particular analytic question that is being addressed to the text will influence the punctuation of the text. However, this cannot be a determinate influence insofar as strategies ramify, cotextually, throughout the text. For example, the description of the Y1 book's introduction of π (Chapter 8) or of algebra (Chapter 6) as discursive message relies, to a certain extent, upon the incidences of references throughout book Y1 (and throughout the Y series). Similarly, the description of the

G2 polygon drawing task (Chapter 8) as procedural relies on the absence of such references within the G series.

The difficulty of operational punctuation is compounded by the fact that the language of description does not prioritise textual strategies, as was discussed in Section 11.1.2. Positioning and distributing strategies might be described, respectively, as recognition and realisation rules for voices/subject positions. This might suggest a procedure in which the former were addressed first—the opposite to what has, in fact, been done. However, the recognition that the relationship between recognition and realisation is, within my perspective, dialectical, reinstates the uncertainty.

Thus, there are unresolved difficulties in applying the language relating to the operationalising of its concepts. I have mentioned the domains of message, but the recognition of discourse and procedure is similarly underdeveloped. The problem of the unit of analysis is enhanced by the lack of prioritising of textual strategies by the language of description. However, the problem would remain unresolved even if such a priority were to be established. Clearly, then, the principal, qualitative part of the analysis begs the question of its own reliability. This is, to a certain degree, offset by the inclusion of the quantitative component. However, this relates only to one category of textual strategy and, in any event, does not overcome the general difficulty with reliability in respect of the further use of the language.

In concord with the general methodological position adopted in this thesis, I contend that any language which is developed beyond mere triviality is incapable of being reduced to algorithmic formulations. Furthermore, algorithms are, themselves, of necessity context dependent. This is not to argue that there is no point in attempting to be more explicit and precise regarding the operational procedures that are used. It is, however, to assert that the generation of highly explicit operational principles will tend to increase reliability, but to reduce the validity of the description at a theoretical level. This trade-off is a compelling reason for routinely combining qualitative and quantitative methods of analysis. Ultimately, the reliability of the application of a language of description is a function of socialisation into the language. I have attempted to respond to this through explicit formulation at a conceptual level and the presentation of a large number of exemplars. However, it is acknowledged that the adequacy of the socialisation provided by this thesis has not been formally tested, so that the reliability of the description which has been produced remains, to a degree, an open question.

Having discussed limitations in the formulation and application of the language of description, I shall now move to a consideration of limitations relating to the specificity of the activity that has been described and, subsequently, to the specificities of the particular empirical text that has been analysed in this thesis.

11.2.3 Specificities of the activity

In the discussion of general methodological issues (Chapter 3) there was very little that was in any way specific to school mathematics. As has been mentioned above, however, the object of analysis is explicitly described as a pedagogic text. Thus, the fundamental assumptions of the language indicate that it is concerned with pedagogic activities, but do not point to one in particular. In the development of the language itself, however, and particularly with respect to the definition of the domains of practice/message, use was made of the SMP texts. This is entirely in keeping with the point, made earlier, that the language had been developed within the context of a deductive/inductive dialogue between conceptual development and textual reading. However, this also entails that the specificities of school mathematics have entered into the formulation of the language of its description and, to an even greater extent, into its procedures of application (discussed above). It is, therefore, important that the implications of the specificities of school mathematics as an activity be examined.

There are at least three aspects of mathematics which do not appear to be shared by all other school disciplines. Firstly, school mathematics exhibits a highly explicit grammar in respect of what can count as a mathematical utterance and what can count as a true mathematical utterance. This is evidenced by the fact that very few of the answers in the Teacher's Guides include variations. This is the case even in respect of the chapters on 'investigations' in the Y series—where the emphasis is explicitly on process rather than on product. It is also the case in the G series, where stress is laid on students developing their own strategies. Recently, there has been, within mathematics education, a degree of advocacy of a 'fallibilist' epistemology of mathematics, the view that 'mathematical truth is fallible and corrigible, and can never be regarded as beyond revision and correction' (Ernest, 1991; p. 18). It is not at all clear, however, that such arguments concerning the metaphysical qualities of mathematics have resulted in, or reflect, any weakening of the recognition and realisation principles of school mathematics. Certainly, *SMP 11-16* does not present such an image of mathematics, neither do the documents relating to the UK National Curriculum in mathematics (DES, 1988, 1989, 1991; NCC, 1988, 1989; see, also,

discussions in Dowling & Noss (eds), 1990). Writing, prospectively, with Bryan Wilson about 'school mathematics in the 1990s', Geoffrey Howson, noted that:

Only in mathematics is there verifiable certainty. Tell a primary child that World War 2 lasted for 10 years, and he [sic] will believe it; tell him that two fours are ten, and there will be an argument. Children know what is wrong at their own level of competence in mathematics and can verify it themselves, even if they may not always be encouraged to do so.

(Howson & Wilson, 1986; p. 12)

The Cockcroft Committee acknowledged this view in their quoting of Carl Sagan in their Report:

So far as we can tell, mathematical relationships should be valid for all planets, biologies, cultures, philosophies. We can imagine a planet with uranium hexafluoride in the atmosphere of a life form that lives mostly off interstellar dust, even if these are extremely unlikely contingencies. But we cannot imagine a civilization for which one and one does not equal two or for which there is an integer interposed between eight and nine.

(Sagan, quoted in Cockcroft et al, 1982; p. 3)

My point is not to concur with such platonism, but rather to demonstrate that it is an explicit feature of school mathematics. However it may be wrapped up as investigational or constructivist, school mathematics remains, empirically, a discipline with an explicit grammar. The consequences of this feature of mathematics in relation to my language of description and the procedures of its application are less clear. Certainly, it has entailed that the evaluative criteria of the SMP texts have been readily accessible, because the 'answers' in the Teacher's Guides and worked examples are unambiguous. This has facilitated the discussion of, for example, the effects of recontextualisation. Clearly, however, a comparative analysis is called for if this issue is to be adequately explored.

The second distinguishing feature of school mathematics is the recognisability of certain of its forms of expression and contents. Halliday (1978) and, following him, Pimm (1987) have made reference to a 'mathematical register' incorporating specialised and reinterpreted terms. The recognisability of elements of this register is substantially a function of the high degree of explicitness of the grammar of mathematics and this has certainly penetrated secondary school mathematics, so that mathematical texts can, generally, instantly be classified as such. Thus, it can be asserted that school mathematics constructs its esoteric domain in a very explicit and visible way.

One of the consequences of this has been the lack of attention given, in this thesis, to the description of the esoteric domain. This was referred to earlier. It is

acknowledged, however, that this is an inadequate justification, because the result has been a certain deferral to mathematical discourse in defining that which is mathematical. In this sense, a limit has been placed on the sociological and critical power of the analysis, although not necessarily on the power of the language itself. Again, comparative analysis is called for. It is possible that the analysis of an activity exhibiting a lesser degree of recognisability—say, school social science—would produce more generalisable procedures for the demarcation of esoteric domains. The choice of school mathematics as an area for analysis has, therefore, both facilitated and delimited the research.

The third specificity of school mathematics is its mythologising quality. By this I mean that the public domain in school mathematics is much more than a route into the discipline. It is, in addition, celebrated as the justification for the high priority that is placed upon school mathematics. This is apparent in the utilitarian and mathematical anthropological myths described in Chapter 2 and in the myths of participation and universal description revealed in the analysis of the SMP texts. One of the consequences of this feature of school mathematics is the constitution of an extensive public domain. This has been important in developing the concept of recontextualising and in describing localising and positioning strategies.

Another consequence is the possible interpretation of mathematics itself as masculinised in its celebration of rational control over the world (see Walkerdine, 1988, 1989). Such an interpretation would clearly off-set the finding that the SMP books have a tendency towards 'political correctness' in respect of gender. Indeed, such an interpretation appears to question the value of the highly rational esoteric domain itself. This interpretation is not challenged, here, in respect of the mythologising aspect of mathematics. However, it is considered that the development of an alternative to the mythologising of 'reason's dream' is possible without abandoning the rationality of the esoteric. This is discussed in the final Section of this Chapter.

In relation to mythologising, a comparison between mathematics and a discipline which, on the face of it, is more closed, may be revealing. Surveying the school curriculum in order to isolate such a discipline may, however, be rather more of a task than it appears. Certainly, the other subject which dominates the school curriculum (at least in England) is English. This discipline is mythologising via the notion of 'English across the curriculum' which was established in the Bullock Report (DES, 1975). An assessment in relation to other disciplines, such as science,

history, and so on, is contingent upon at least a preliminary demarcation of their respective esoteric domains¹. It may be that physical education and music would qualify as generally introspective disciplines. However, as is the case with the level of explicitness of the grammar of an activity, a mythologising quality is not an essential property, but must be described empirically.

I have described constraints on the language which have resulted from the specificities of school mathematics in terms of its explicit grammar, its highly distinctive 'register' and its mythologising qualities. I want to conclude the discussion of limitations with a consideration of the specificities of the particular empirical text that was chosen for analysis.

11.2.4 Specificities of the SMP 11-16 text

The reasons for the selection of the SMP texts are given in Chapter 5. The principal reason is concerned with the widespread use of the scheme, so that it has a representative status. It was felt that this is important, even though the analysis treats the text as closed and so is not emphasising generalisability. However, a second and important feature of *SMP 11-16* is that it is explicitly differentiated in terms of age and 'ability'. Indeed, the choice of the Y and G series in sampling the scheme was motivated by the fact that they are constituted by the highest level of 'ability' differentiation within the main part of the course.

As is the case with the explicit grammar of school mathematics, this explicitness in the textbook scheme has clearly facilitated the analysis. The earlier 'numbered' SMP books (see Chapter 5) were differentiated in terms of age, but not in terms of 'ability'. Thus, any given 'numbered' book—say, *School Mathematics Project, Book 1*—constructs a single student voice which is a 'to-be-apprenticed' voice. An analysis of this kind of textbook would not allow inferences to be made about the more general voice structure of the activity. In other words, the analysis represented in Chapters 9 and 10 would have been severely impoverished. Clearly, some compensation may have been possible via a more detailed description of differentiation in terms of age, or a greater delicacy of message strategies might have been generated. Nevertheless, the construction of 'ability' is clearly of sociological interest, so that its omission would be a serious loss.

¹ None of these disciplines exhibit the particular quality of mathematics which Walkerdine (1988) describes, that is (in her terms) the virtual exclusion of metaphor, resulting in the potential to refer to anything.

The original SMP 'numbered' scheme was used in classes that had already been selected for 'ability'. It would have been possible to retrieve 'ability' as a variable via a comparative analysis between the 'numbered' books and a series intended for 'lower ability' students, such as the SMP 'lettered' series. However, there are empirical cases of the use of a single mathematics textbook or series of textbooks for the whole of a cohort; this is generally the case in Greek secondary schools, for example. Under such circumstances, the construction of ability could only be recoverable from the pedagogic practices in which the textbooks are situated, for example, in the classroom. As was discussed earlier, however, moving into the classroom significantly increases the problem of analysis in that the text can no longer be considered to be closed. Thus, the particular choice of text for analysis has enabled the theoretically motivated choice to treat the text as closed. It has thereby facilitated the avoidance of the more complex problem which is programmatic.

An additional feature of *SMP 11-16* has already been mentioned. This is its apparent attempt at 'political correctness' with respect to gender and race. This does not exclude the possibility of orienting the analysis in these directions. As has been suggested above, for example, the 'mythologising' of mathematics is interpretable as a masculine fantasy of rationalised control. The inclusion of token Black, Asian and Jewish characters in the texts is countered by the almost total exclusion of any context which is not stereotypically 'mainstream British', to which imaginary culture the various 'immigrant' groups are thereby assimilated. The emphasis on social class, however, has been motivated by a perceived lack of attention to this dimension in the analysis of pedagogic texts and a recent decline in its profile within sociology generally.

11.2.5 Further research

In the preceding discussion, I have indicated a number of constraints and limitations which suggest a need for further research. In this Sub-section, I shall draw these areas together and give mention to work which is currently being carried out.

Firstly, I have suggested that there exists the possibility to extend the language to include the description of texts which are not, strictly pedagogic. This may, in fact, be necessary if the second possibility is to be taken up, that is, the opening of the empirical text within the sites of its negotiation in order to access the ways in which empirical readers interpret or negotiate the text. The extension of the language is

necessary because the kinds of data that would be collected might include, for example, interview and observation transcripts and fieldnotes, which are not pedagogic texts. Moving into the principal site of the negotiation of the text—the classroom—would also require that further theoretical attention be given to the area of polythetic practices which, we might expect, substantially characterise classroom activity. There is clearly a cosiness about restricting oneself to a closed, discursive text which is, of necessity, disrupted upon entering a multiply authored context in which the non-discursive can neither be grasped nor ignored.

A major theoretical and empirical problem would be to mark out and structure the activities which regulate the space. An initial theorising of the teacher's positioning might be to consider her/him as ambiguously positioned by an instructional activity (school mathematics) and a regulative activity relating to control. It would then be of interest to discover the extent to which the instructional activity becomes reduced to the level of a reservoir of resources by the exigencies of control. We might speculate that a measure of the subordination of instruction to control would be an indicator in respect of the negotiation of 'ability'.

Opening the text, in this way would also enable an adequate description of non-differentiating textbooks, or of schemes which do not make explicit their differentiating principles. One such programme which is used widely in London schools is the SMILE scheme. This scheme consists, centrally, of some eighteen-hundred cards. The cards are arranged into levels and topics. The routes taken by individuals through the scheme, however, are highly individualised. The construction of 'ability' is facilitated by the scheme and may, indeed, be describable as implicit within it. However, it can only be recovered by an analysis of its use. Jayne Johnston, of the Institute of Education, is currently engaged in developing the language of description within an empirical analysis of the use of the SMILE materials.

Another site of negotiation is in communications between school and home. Preliminary work has already been published on the description of the colonising of domestic space by the primary school mathematics project IMPACT (Brown, 1993; Brown & Dowling, 1992, 1993). This very widespread project attempts to involve parents in the mathematical education of their children. However, parents are given no access to the regulative principles of the mathematical practices that they and their children engage in, so that they are constructed, by the IMPACT texts, as subordinate voices. The early publications do not involve any theoretical development of the

language. However, this is an important part of Brown's current project. He also hopes to be able to develop the empirical description of the public domain, in this case, the domestic space and its relationships, which is constituted by the IMPACT practices.

A third region of sites of negotiation is within the area of initial and in-service teacher education. I specify sites, in the plural, because there are clearly distinct contexts within which teacher education takes place. These include, at least, the higher education institution and the school classroom. Work in this area might consider the relationship between, for example, school mathematics and educational research, again requiring the demarcating of the specific activities involved. I should acknowledge the projects involving the application and development of the language of description in mathematics teacher education in South Africa, which have recently been initiated by Paula Ensor and Zain Davis, both of the University of Cape Town.

The programmatic work referred to above relates to the extension of language of description via the opening of the text and by the inclusion of non-pedagogic texts. However, the limitations discussed earlier reveal that there remains scope for the development of the language and its applications within the context of the closed text. It was suggested, for example, that there are limitations in respect of the operationalising of the principles of the application of the language. As was noted in Chapter 1, the relationships between the concepts of the language of description and the textual indicators are established through the elaboration of exemplars, rather than via the use of markers. It was acknowledged that there is often a trade-off between the reliability gained by formal operational procedures, on the one hand, and the loss of theoretical validity that this can entail. This was certainly the case in respect of the quantitative analysis of signifying modes as was illustrated by the finding (Chapter 9) that some of the photographs in books R3 and B4 were incorporated into the texts such that they signified themselves as photographs rather than photographic significations of their contents. However, this does not abnegate the value of quantitative analysis of this kind. Rather, it is suggested that further work on the formalising of the principles of application of the language of description may facilitate a wider range of quantitative analysis which could complement the qualitative analysis favoured in this thesis.

There are, clearly, resources that are already available for the development of the application of the language. I am thinking, particularly, of the field of linguistics and social semiotics (Fairclough, 1989; Halliday, 1978; Hassan, 1991, 1992a, 1992b;

Hodge & Kress, 1988, 1993). These areas offer well-developed and highly reliable modes of describing texts, and can usefully guide development on this project. Such work can also increase the breadth of the reading to include, for example, modes of address, the use or exclusion of hedging, and other resources which have not been considered in the analysis. The quantitative analysis which focused attention of signifying modes, on the other hand, has not been highly developed in terms of validity, as was mentioned above. Again, there exist resources which can be of assistance in generating a more delicate analysis of images and diagrams (for example, Kress & Van Leeuwen, 1990).

The development of the operational and conceptual levels of the language would also be encouraged by comparative studies which moved beyond the specificities of mathematics. That is, there is clearly scope for the application and development of the language in activities that exhibit a less explicit grammar and/or which have less distinctive esoteric domain 'registers' and/or which do not exhibit the same degree of mythologising as school mathematics.

Finally, there is scope for the theoretical development of the language. There is certainly scope for the development of the concept 'discursive saturation'. The discussion in Chapter 3, although representative of the work which has been influential in the development of this thesis, is very far from exhaustive in addressing the range of theorising of the dual modality of the discursive. Further research in this area is clearly called for. Theoretical development, here, would also be assisted by empirical research addressing activities which exhibit low discursive saturation; the classroom-based studies mentioned earlier may prove fruitful in this respect.

It is also clear that the language of description has no temporal dimension. As a result, the public domain is represented as having no history. The particular forms of mythologising are presented as essentially arbitrary. This suggests a need for genealogical studies which would, for example, chart the historical development of the public domain. A genealogical analysis of the development of mathematics texts, for example, would probably necessitate the development of the Foucauldian concept of 'strategy' in order to theorise the history.

The concepts 'institutional level' and 'subjectivity', introduced in Chapter 4, are, as yet, undeveloped and suggest a further range of empirical work. The development of 'institutional level' points to the analysis of the school or, perhaps, of the state as higher order activities which are, themselves, (re)produced by activities such as

school mathematics. An analysis at the level of the state might facilitate the reintroduction of the concept 'ideology', as the specific form of the division of labour within given conjuncture.

The development of 'subjectivity' indexes the analysis of empirical human subjects in their movement between activities and in their negotiation of subjectivities within pedagogic contexts. Essentially, it is the multiple nature of subjectivity which leaves scope for conscious political action insofar as the multiplicity does not comprise hermetically sealed compartments. It is this assumption that grounds the discussion in the final Section of this Chapter, which concerns a re-conceptualising of the relationship between activities.

11.3 From myth to metaphor: a language for research & practice

My intention, in this final Section, is to work more freely with the fundamental assumptions of the language of description and with some of the findings of the analysis in order to generate a theoretical speculation. The speculation concerns a possible alternative conception of the relationship between different activities. In particular, I am concerned, firstly, to explore the possibility of a non-mythologising relationship between the mathematical and the non-mathematical. Secondly, I shall generalise the form of relationship that I shall describe in order to establish a productive, non-mythologising role, within the context of educational practice, for the sociology of education, in general, and my own work, in particular.

11.3.1 Towards a non-mythologising mathematics

The theoretical and empirical work of this thesis has produced a description of mathematics as a mythologising activity. It constitutes itself as a universally generalisable rationality which can be applied to any other activity or which is immanent within any optimised practice; these are the myths of universal description and participation. These myths are established by the projective gaze of which the highly organised esoteric domain is the subject. I have argued and demonstrated, however, that the mathematical gaze achieves a recontextualising of non-mathematical practices which subordinates them to mathematical regulating principles. In this respect, the world is constituted as a reservoir of resources. Viewed in this light, school mathematics cannot, within the context of its own elaboration, generate use-value outside of itself. A general principle seems to be: if you want to teach children

shopping, take them to the supermarket; if you want to teach them mathematics, then don't pretend it's shopping.

This contention that, within its public domain, school mathematics can have no use-value will almost certainly run counter to appearances. Isn't it the case, after all, that work in mathematics has facilitated, even if not entirely alone, the information technological revolution which has characterised the northern hemisphere in much of the second half of the twentieth century? However, none of this has been achieved without the subordination of the grammar of analytic mathematics to the exigencies of an empirical enterprise. The 'regime of truth' of the empirical is, of necessity, quite fundamentally different from that of mathematics. To refer to a more mundane setting. Whilst I might import mathematical knowledge into my supermarket shopping, my decisions are ultimately contingent upon and achieved by something other than mathematical truth (as Lave has illustrated). Furthermore, my ability to implicate mathematics as a resource, in this way, is contingent upon my prior apprenticeship into its esoteric domain. Without that, (my) mathematics would lack the systematic or discursive nature that enables me to use it to organise my domestic activities. Foucault gives an example of such an ill-conceived application:

A patient who had consulted Brulley wanted to be operated upon for a stone; there were two 'favourable probabilities' in favour of intervention: the good condition of the bladder and the small size of the stone; but there were four unfavourable probabilities against them: 'the patient is in his sixties; he is of the male sex; he is of a bilious temperament; he has a skin disease'. The subject would not hear of this simple arithmetic, and did not survive the operation.

(Foucault, 1973, p. 104)

Mathematical jargon is here being used (unsuccessfully, as it happens) as a rhetoric in a situation in which clinical knowledge had already achieved an organising of the situation. Mathematics can achieve nothing within the clinical discourse, although it might have been expected to fool the patient. My proposition, by contrast, is that mathematics might, by virtue of its systematic nature, be implicated as a resource for the productive creation of order out of comparative chaos. However, this is not tantamount to an assertion of the myth of universal description, because mathematics is, at least in some degree, arbitrary as an organising system. Lévi-Strauss (1964, 1972) has proposed a similar role for, so-called, 'totemic' systems as is summarised, below, by Sturrock:

It might seem that the system of differences between animal species is too powerful to match the much weaker differences between human groups. Members of the same society look alike and live in similar ways and conditions; social groups do not differ in the ways natural species do. But the point precisely is that human groups are trying through 'totemic' institutions not to match two pre-existing systems of differences, but rather to build one system with the help of the other. They are trying not so much to

express social differences as to create or strengthen them. In this respect the force of the animal system is never excessive; whatever aspects of it can be mapped on to the social system are welcome'

(Sturrock, 1979, p. 32)

Lévi-Strauss himself provides a particular illustration:

The real question is not whether the touch of a woodpecker's beak does in fact cure toothache. It is rather whether there is a point of view from which the woodpecker's beak and a man's tooth can be seen as 'going together' (the use of this congruity for therapeutic purposes being only one of its possible uses), and whether some initial order can be introduced into the universe by means of these groupings. Classifying, as opposed to not classifying, has a value of its own, whatever form the classification may take.

(Lévi-Strauss, 1972, p. 9)

Mathematics and 'totemic' systems may be conceived of such that they have this in common, that they both constitute (more or less) systematic languages of description for the productive organising of the social. School mathematics can, indeed, have a use-value outside of itself, but, within my conception, two conditions must be satisfied. Firstly, its regulating structure must be made available. Without this, mathematics can only mystify. Secondly, the mythologising, projective gaze of school mathematics must not be trusted. The rationalising of the public domain by the esoteric of necessity imposes upon the non-mathematical world relationships and priorities which are not of the non-mathematical world. Rather, the direction of the gaze must be introjective, which entails that its subject is the subject of the activity within which a use-value can be realised.

In other words, mathematics is to be constituted as a reservoir of pre-structured, metaphorical resources which can be used to help to build structures within the recontextualising activity. This is, precisely what happens within the context of other academic disciplines. Mathematics is appropriated by the natural and social sciences and by the engineering disciplines, for example. In these recontextualisings, it is always clear that it is scientific or engineering problems that are being addressed and not mathematical ones. The key to this relationship is that, whilst a division of labour is maintained between academic mathematics and the other academic disciplines, there also exists the possibility of traversing the division: there are applied mathematicians and there are theoretical physicists, as well as pure mathematicians and empirical physicists. The disciplines can learn from each other.

In the field of the primary production of mathematical knowledge, it is possible (at least, for the time being) to adopt the position of G.H. Hardy:

I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world. I have helped to train other mathematicians, but mathematicians of the same kind as myself, and their work has been, so far at any rate as I have helped them to it, as useless as my own. Judged by all practical standards, the value of my mathematical life is nil; and outside mathematics it is trivial anyhow. I have just one chance of escaping a verdict of complete triviality, that I may be judged to have created something worth creating. And that I have created something is undeniable: the question is about its value.

(quoted by Davis & Hersh, 1981; 85-6)

In the academy, pure mathematics has an institutionalised security (which is not dependent upon Hardy's romanticism) and is rarely, if ever, called upon to justify its practices beyond its comparatively small number of followers. Higher mathematics can avoid mythologising and yet still be available for recontextualising.

School mathematics, however, must continually re-establish its right to be a compulsory activity for all people of school age and hence the value of mythologising. Furthermore, it is within those activities which are without their own institutionalised language, the polythetic practices of low discursive saturation, that the myth is most credible. The avoidance of such mathematical imperialism can be achieved from within these non-mathematical practices, but not from within mathematics. My point is that if the use-value of school mathematics is to be realised within school, then it can be realised only within a space which is not mathematics, but which is in need of organisational resources. I have discussed such an approach to the consideration of political issues in the curriculum elsewhere (Brown & Dowling, 1989; Dowling, 1993b, 1993c; see also Appendix 4).

This point enables me to move on to a consideration of the use-value of the sociology of education and of my own work within the sphere of educational and political practice.

11.3.2 Educational practice & the sociology of education

The *Teacher's Guide to Book G1* emphasises the importance of discussion in teaching the 'lower ability' student. However, it is not at all clear precisely what might constitute 'discussion', nor whether all of its forms might be equally appropriate. 'Discussion' is being used in the production of an exposition on mathematics education, but it is being used in an everyday and unspecialised sense. A similar example occurs in the extracts from an interview transcript that were introduced in Chapter 4. The interview was conducted by Jayne Johnston, of the Institute of Education, with a mathematics teacher:

After having Jason who was on his own, working with Elaine, they seemed to get on OK, so I tried them together again and that seems to be working reasonably. Jason is quite capable but wanders all the time. Elaine struggles a bit and Robyn and Ann are quite strong so I put them altogether because they also seem quite considerate of the others' needs.

[...]

Over here I felt that Cyril he's reminded me a lot of kids I see, particularly black boys, they come in and they are very capable but within a few months they start drifting off for some reason. There is something wrong there, so I've put him in with two very strong boys.

I want to point, in these extracts, to the use of a particular organising principle that reminds me of horticulture. In each case, a capable but potentially wayward individual, Jason and Cyril, respectively (the latter is, presumably recognised as such because he is black) is 'staked' to two other individuals whose essential characteristic is 'strength'. Whether or not this teacher was thinking of the vegetable garden when making this utterance is not crucial. What these extracts illustrate is the use of available semiotic resources to explain decisions that have almost certainly been taken in advance of their rhetorical exposition, rather like Brulley's attempted management of his patient, there is a sense of *post hoc* rationalisation. In neither case does the rhetoric do any work in organising the practice any more than 'discussion' organises teaching. We might assume that Brulley had recourse to medical discourse and was implicating mathematics to add arcane weight to such a low status activity. The teacher, however, has no such discipline, because the expertise of the secondary mathematics teacher resides in mathematical discourse and not in didactics, as is suggested by the need to render explicit a theory of instruction in the G texts.

There is an extent, in other words, to which the educational practices of teachers constitute precisely the kind of space that I am proposing in order to make practical use of school mathematics. This space, however, already has its own institutional basis, its own practices and subject positions. What it might lack is a language. The conditions under which teachers are called to account for their actions and decisions, in discussions with pupils and their parents even in interviews with researchers, are generally such as to discourage theoretically informed explanations in favour of a kind of *bricolage* with everyday language. 'Everyone' has been to school, so everyone is an expert and they are not prepared to be mystified or, rather, bamboozled by educationalese. The basis for the teacher's practices is precisely polythetic.

The demands that are placed upon sociology, on the other hand, are precisely those that will encourage systematic discourse: cases must be argued with a consistency of language and rationality; terms must be clearly defined; evidence must be methodically collected, structured and presented. Sociology is an activity which exhibits relatively high discursive saturation (although certainly not as high as mathematics). Furthermore, the gaze of the sociology of education is directed at educational practice, so that the metonymic links between the esoteric and public domain are, at least potentially, visible. The sociology of education is 'ionised' with respect to the classroom and this is achieved within empirical work: the sociologist must move into the space of the curriculum.

The products of sociological labour can thus be made available for appropriation by the educational practitioner. However, if research is treated in a superficial manner, then it can provide no more than a rhetoric for the *post hoc* rationalisation of existing practices. This is analogous to Brulley's appropriation of mathematics, described earlier. This is the projective gaze which can discover nothing surprising or transformative. Rather, the gaze must be introjective, which is to say, the gaze must construct a metaphorical domain. There must be a recognition of the possibility of the other in oneself. This recognition, however, is contingent upon prior apprenticeship into sociology. The subject of educational practice must also be the subject of sociology.

This is made (theoretically) possible via the conception of the human subject as comprised of multiple subject positions. These subject positions are not roles, nor is the human subject conceived as possessing an existential function of selection. Rather, the human subject is inscribed as specific subject positions within respective material contexts: the teacher is a teacher within the classroom and cannot simply choose to be a sociologist. This is not to say, however, that inscription within a particular subject position effects a total amnesia with respect to all others. Rather, it is proposed that the practices associated with non-activated subject positions remain available as resources for recontextualising. Thus, the educational practitioner who is also a sociologist can galvanise dead sociological labour within their educational practice, not to prescribe, but to systematise and to interrogate.

The language and the analysis of this thesis are generated within a social space which is other than that of the educational practice. This being the case, it is available for appropriation by the practitioner, for whom it may serve as a resource in the productive organising and interrogation of their practice. The appropriation of the

language is facilitated by its educational ionising which arises out of its empirical basis. Effective appropriation is enabled to the extent that the practitioner is prepared to learn the language. Only in this way can it do more than rationalise, at the level of rhetoric, that which already happens. Language acquisition is subjugation and, to this extent, pupillage entails a puparial shedding of individuality. But, although this metamorphosis constitutes an imaginal rebirth, it does not signal the death of the pupa. On the contrary, a new subjectivity has been established which lies, in some sense, adjacent to that which preceded it. Like adding an extension onto a house: the occupant can move between rooms.

The projective gaze enables the sociologist to describe educational practices in such a way that sociological discourse is ionised for the practitioner. But the sociologist must not attempt to prescribe the practice of the practitioner: this is a moral as well as a theoretical position and is the condition which underwrites the democratic nature of the position. The practitioner, whose practice is constrained in different ways and by different forces than that of the sociologist, may, nevertheless, introject sociological theory. Sociological and educational practices thus occupy separate spaces. As with 'totemic' systems of classification, one world may be used to construct and reconstruct another, but that world must never become the other. The productivity of an analysis which employs a language of description is, as was argued in Chapter 4, contingent upon the maintenance of a discursive gap between that which is describing and that which is to be described. Mathematics has internal beauty and external organisational power, both make it worthwhile. But it is not as mathematician that I can use mathematics to reconstruct my domestic life, far less should the mathematician attempt to organise someone else's domestic life. Sociology has internal beauty and organisational power. But sociological procedures will not reconstruct educational practice. Mathematics and sociology must recruit other mathematicians and sociologists who are also practitioners in other contexts. But the context for the development of domestic activity is the home and the context for the development of educational practice is the classroom.

This is the value that I perceive for my own, developing, language of description within the context of educational practice. That is, as a system which is approaching coherence and which can be made available as resources for recontextualisation, as a language for practice. In Appendix 4, I have given a brief presentation of a pedagogic interpretation of a part of the language of description. But the language is also a language for research within its original activity, and this is its primary purpose.

I opened this thesis with an interest in describing the obvious. The covers of Book Y1 and Book G1 are very obviously very different, as are their contents. My intention has not been simply to reassert, at very great length, this difference. Rather, I have been concerned to generate a theoretically and empirically grounded system within which the difference emerges as a strategy within a larger game. And so it is. The reader (albeit ideal) of the Y text confronts the text as an apprentice and is progressively defined by the contours of mathematical practices. This is neither a humanising of mathematics nor a mathematising of humanity, it is an initiation into an esoteric language of description. Mathematical culture is inscribed where it is meant to be inscribed, within the purview of the dominant class. There is no initiation, no apprenticeship, no mathematical career for the reader of the G text. There is, it would seem, no need to disturb the individual intellectual slumber of the subordinate classes. This reader is simply whiling away the time of their own maturation from comics and sweets to the supermarket and DIY.

Appendices

Appendix 1

Supplement to Literature Review

In conducting a search of the research literature on the analysis of textbooks, it was found that very little work had been carried out which is of direct relevance to the project of this thesis. The literature which is most closely associated with my project is discussed in Chapter 2. Other work that was discovered in the course of the literature search is referenced, with brief comments, here.

A1.1 Ideological analysis

In addition to those discussed in Chapter 2, a number of 'ideological' analyses of textbooks were discovered. Beyer (1983), for example, describes the commodification of the aesthetic in the materials of the Aesthetic Education Program¹ and points to points of contradiction which militate against a simplistic reproduction model. Carlson (1989) reveals a general legitimization of the Cold War in US history textbooks and draws on the work of Habermas to suggest directions for teachers' responses. Mulcahy (1988) classifies Irish history textbooks into 'purist' and 'moderate' categories according to their pro-Irish/anti-English and neutral positions, respectively. Gilbert (1986) includes a consideration of textbooks in an ideological study of geography education, but he does not conduct any systematic analysis of textbooks. May et al (1990) look at the reflection of the interests of dominant social groups in elementary art and music textbooks. Straka & Bos (1989) conduct a content analysis of the German translations of six Chinese elementary school textbooks in assessing what they refer to as 'socialisation objectives'. Vitz (1985) analyses the representation of religious and 'traditional' values in sixty social studies texts. Seddon (1983) proposes the replacement of the ideology of war, evident in Australian textbooks, by an ideology of peace. In a study oriented more towards the discipline, Agger (1989) provides a critique of sociology textbooks from a marxist-feminist perspective. Some studies focus on ideology specifically in terms of gender or ethnicity: Clark (1980) conducts a historical study which looks at the representation of female roles in French elementary textbooks during the Third Republic; Dean et al (1983) looks at the representation of different ethnic groups in South African

¹ Developed by the Central Midwestern Regional Education Laboratory.

secondary school history textbooks; Tietze & Davis (1981) discover sexist ideology in educational administration textbooks; and Vaughn-Roverson et al (1989) consider the gendered ideology of elementary reading textbooks.

In a comparative study of Indian and Canadian reading textbooks, Kumar (1982) constructs a matrix for the quantitative analysis of texts in terms of five dimensions: agents, acts, scenes, agency and purpose. He suggests that:

Mass reading of literary texts creates common spheres of individual participation, and thus generates a system of symbolically represented behaviours. The task of the investigator is to discover this system of symbols in order to explain the function of literature in a society.

(Kumar, 1982; p. 301)

However, Kumar offers no model of society which would enable him to determine function. He has a classificatory framework, but lacks a language into which to translate the texts.

A PhD thesis by Anuar (1990) considers the extent to which Malaysian textbooks are likely to generate or threaten national unity in Malaysia. Ahier's (1987) thesis suggests that history and geography textbooks in England, 1880-1960, can be described as struggling to produce a national identity during a period of imperialism via an idealising of a rural past. A thesis by Hassan (1990) conducts a content analysis of officially designed social studies textbooks in Kuwait. Hassan concludes that these texts are likely to lead to a passive and deferring citizenry. Al-Abduljadir (1987) looks at Islamic values in officially edited Kuwaiti biology textbooks. Bos (1990) conducts a content analysis evaluation of the German translations of Chinese language textbooks as 'mother tongue teaching materials for ethnic minorities'. He concludes that these texts are more effective at transmitting knowledge about Chinese culture than about the ethnic minority situation. Again, these studies lack any kind of model of the social or of the cultural.

A1.2 The representation of gender and/or ethnicity

The representation of gender and/or ethnicity in textbooks has been widely investigated. Focussing, for example, on US elementary reading textbooks Gonzalez-Suarez & Ekstrom (1989), Heathcote (1981, 1982) and Hitchcock & Tompkins (1987) look specifically at sex-stereotyping; Garcia & Sadowski (1986) considers the representation of ethnic minorities; and Britton (1984; also, Britton & Lumpkin, 1983) addresses both of these dimensions. Penman (1986) reports on a project, in the UK, in which secondary school pupils re-wrote primary reading books in a non-

gender-stereotyped way, thus serving the dual purpose of raising these pupils' awareness of gender issues and providing the primary school with more suitable materials. Etim (1988) finds evidence of stereotyping and the underrepresentation of females in secondary school reading textbooks in Nigeria. Rice points to the underrepresentation and stereotyping of black people (1987) which is even more extreme in respect of Chinese and Japanese people (1988). Fillmer & Meadows (1984) produce a rare analysis which looks at the representation of older characters in elementary reading textbooks. The representation of elderly (and also disabled) people and of one-parent families also gets some attention in Britton et al (1984).

Other curriculum areas are also represented in this kind of research. In economics, the representation of women is investigated by Hahn & Blankenship (1983) and a lack of attention to the the position of Blacks and Hispanics in relation to the issues of poverty, unemployment and labour is found by Ellington (1986). The incidence of sexist language in English handbooks is studied by Patten (1988) and a quantitative analysis of gender representation in English language textbooks used in Singapore is reported by Gupta & Lee Su Yin (1989). History textbooks are interrogated in respect of gender representation by Cairns (1988), Cairns & Inglis (1989), Light (1989), Selke (1983) and Tetreault (1984), and in respect of various categories including gender and ethnicity by the Michigan State Board of Education (1984). Koza (1992) investigates the representation of gender in music textbooks. Browne (1990) reports bias towards males in physical education textbooks. In respect of science textbooks, Ives (1984) finds evidence of a masculine bias, Potter & Rosser (1992) address sexist language and Walford (1983) locates the representation of gender within a two-dimensional space scaled by strong and weak values for gender differentiation and gender consolidation. Holt (1990) looks at the treatment of women in social studies textbooks and suggests some strategies and resources for teachers. Hall (1988) considers the treatment of gender issues in introductory sociology textbooks. Kirk (1985a, 1985b) considers sex, race and cultural bias in school textbooks generally.

Textbooks for adults have also received some attention. Peterson & Kroner (1992) study gender bias in psychology and human development texts. Carrell (1991) considers gender stereotypes in professional writing textbooks. Connor-Greene et al (1988) analyses the treatment of gender and its differentiation from sex in introductory psychology textbooks. Nusbaumer (1989) focuses on the treatment of war in college social problems textbooks to argue the role of sociologists in defining social problems.

A1.3 Other textbook analysis

Outside of a focus on mathematics as a curriculum area or broadly sociological analyses of textbooks, there is a considerable body of research which concentrates on the subject content of textbooks. This corpus lacks direct relevance to the present study, but it is worth mentioning a brief selection for illustrative purposes. Saad (1984) evaluated Iraqi civics education textbooks in respect educational aims. Benson (1990) evaluates English textbooks used in Japanese schools in terms of the extent to which their treatment of interactional skills are adequate for Japanese learners of English and Rao (1989) English coursebooks used in the Punjab in the light of current trends in English language teaching and the specific language situation in Pakistan. Eichinger & Roth (1981), O'Hear & Ashton (1987) and O'Hear et al (1987) evaluate textbooks in the areas of science, sociology and English composition, respectively, in terms of their facilitating of conceptual development in the relevant discipline. Reusser (1988) evaluates science textbooks in respect of the contextual effects on the solution of word problems. Tull (1991) compares concepts in biology texts with those displayed by students. Bagchi (1985) and Ventimiglia & DiRenzo (1982) evaluate the representation of particular themes: Bagchi looks at the treatment of sex education in Indian biology texts; Ventimiglia & DiRenzo find evidence of the transference of the trait conception of 'personality' from psychology to sociology. Perrucci (1982) finds that introductory sociology textbooks fail to represent the diversity of views obtaining in the contemporary discipline and relates this to the exigencies of publishing; Vogel (1987) finds that biology textbooks fail to represent the discipline in a manner likely to encourage the students to consider careers in biology. Crismore (1989) presents a framework for the evaluation of social studies textbooks for the purposes of selection and use. Selander (1990) offers a framework for the analysis of pedagogic texts as a genre with an introduction to an analysis of a history textbook. This framework, however, is neither theoretically nor empirically developed and the expression 'pedagogic text' seems to be used synonymously with 'textbook'. Regester (1991) finds no significant differences in the relationship between visual and verbal content in US history textbooks of the 1950s and 1980s.

Appendix 2

Domain Trajectories in *SMP 11-16* 'Y' and 'G' Series

The tables below give the results of the trajectory analysis of the Y and G series of books which is the basis for the discussion in Chapter 5. In fact, not a great deal of use has been made of the trajectory analysis in the thesis, itself. However, the tables were used as guides in the quantification of esoteric domain textual time (Chapter 5), so there inclusion serves to make this process a little more visible. The tables also include the titles and domain trajectories of all of the chapters in the Y and G series, which may prove useful for reference purposes.

Book Y1

Chapter Y1.01: 'Multiplication and division'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	B	My	B								
B	Me	B									
C	B	My									
D	Me	B	My								

Chapter Y1.02: 'Locl'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My										
B	My	B	My								
C	B	Me	My/Me	My	B	My					
D	My	B	My	B	My	B					
E	P	My	My/Me	My							
F	My										

Chapter Y1.03: 'The Language of algebra'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	E/P	B									
B	B										
C	B										
D	My										
E	B										
F	B	My	B	My							
G	My	B	My								

Chapter Y1.04: 'Using a calculator'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	P	My	P								
B	P	B	My	B							
C	B										
D	B	P	My								
E	P	My	P	My	P	My	P	My	Me	My	P
F	P	My	P								

Chapter Y1.05: 'Negative numbers and equations'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	B	My	B								
B	B										
C	B										
D	B										
E	B										
F	B										
G	P	My	P	My							

Chapter Y1.06: 'Ratio'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My	E	My	E	My	E	My	E			
B	My	E	My								
C	My	E	My								
D	My										
E	E	My	E								
F	E	My	E	My	E						

Chapter Y1.07: 'Polygons and circles'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	E	My	E								
B	E	My	E	My							
C	P	My									
D	My										
E	P	E									

Chapter Y1.08: 'Investigations'											
Section	a	b	c	d	e	f	g	h	i	j	k
1	P	My									
2	My										
3	My										
4	P										
5	E										

Chapter Y1.09: 'Brackets'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	E										
B	E										
C	E										
D	E										
E	E										
F	E	P	My								

Chapter Y1.10: 'Percentage'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My	E									
B	My	P	My	P	My	P					
C	E	My	E								
D	My	P									
E	P	My	P	My	P						
F	P	My	P								
G	P	My	P								

Chapter Y1.11: 'Gradient'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	P	My	E	My							
B	E	My									
C	E										
D	E	My	E								
E	My										

Chapter Y1.12: 'Probability'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	P	My									
B	P	My	Me								
C	Me	E	Me	My							
D	My										
E	Me										

Book Y2

Chapter Y2.01: 'Relationships'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	P	My	E	My	E	My	E				
B	P	My	E								
C	E	My	E	My	E						
D	My	E	My	E	My						

Chapter Y2.02: 'Accuracy'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My	E	My								
B	My	E	My	E							
C	My	E	My	E	My	E	My				
D	E	My	E	My	E	My					

Chapter Y2.03: 'Trigonometry (I)'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	E										
B	E	My									
C	E	My									
D	E	My									
E	E	My	E	My	E						

Chapter Y2.04: 'Rates'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My	P	My	P							
B	My										
C	My										
D	My	P									

Chapter Y2.05: 'Algebraic expressions'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	B										
B	My	E	My	E							
C	B										
D	B										
E	B										

Chapter Y2.06: "Trigonometry (2)"											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My	B									
B	B	My									
C	B	My	E	My							
D	B	My									
E	B	My	B								

Chapter Y2.07: 'Investigations (1)'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	P	My									
B	P										
C	P										
D	My										

Chapter Y2.08: 'Distributions'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My										
B	My										
C	My										
D	P	My									
E	R										

Chapter Y2.09: 'Re-arranging formulas (1)'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My	E	My	E							
B	E										
C	E										

Chapter Y2.10: 'Points, lines and planes'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	P	My									
B	B	Me									
C	B	Me	B	Me							
D	B	Me									
E	B	Me	B	Me	B	Me					
F	Mv										

Chapter Y2.11: 'Re-arranging formulas (2)'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	B										
B	B										
C	B										
D	B										

Chapter Y2.12: 'Proportionality'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My	B	My	E							
B	B	My	B								
C	My	B	My								
D	B	My	B	My							
E	My										

Chapter Y2.13: 'Area'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	B	My	B								
B	B										
C	B	My									
D	B										
E	B										

Chapter Y2.14: 'Linear equations and inequalities'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My	E									
B	My	E	Me								
C	My										
D	P	Me	E								

Chapter Y2.15: 'Investigations (2)'											
Section	a	b	c	d	e	f	g	h	i	j	k
Mc											

Chapter Y2.16: 'Periodic graphs'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My										
B	My										
C	My										
D	My										

Chapter Y2.17: 'Probability'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	P	My									
B	Me	E	Me	My							
C	Me	My									
D	Me										
E	Mv										

Book Y3

Chapter Y3.01: 'Stretching and enlargement'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	Me	B									
B	B										
C	B	My									
D	B	Mv	B	Mv							

Chapter Y3.02: 'Linear relationships'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	E										
B	E										
C	My	E									
D	E										
E	My										
F	E										

Chapter Y3.03: 'Vectors'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My	E	My	E							
B	E										
C	E										
D	My	E									
E	E										

Chapter Y3.04: 'Percentage (1)'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My										
B	My										
C	My										

Chapter Y3.05: 'Mappings'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	E	Me	E								
B	Me	E									
C	E	Me									
D	E	Me									
E	E										
F	Me	E	Mv	E	Mv						

Chapter Y3.06: 'Investigations (1)'											
Section	a	b	c	d	e	f	g	h	i	j	k
Me											

Chapter Y3.07: 'TV programmes survey'											
Section	a	b	c	d	e	f	g	h	i	j	k
	P	Mv									

Chapter Y3.08: 'Direct and inverse proportionality'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My	E	My	E							
B	My	E	My	E	My	E	My	E			

Chapter Y3.09: 'Representing information'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My										
B	My										
C	E										

Chapter Y3.10: 'Looking at data'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	E	My	E	My	E	My					
B	My										
C	P	E	My								

Chapter Y3.11: 'Percentage (2)'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My										
B	My	P	My								

Chapter Y3.12: 'Investigations (2)'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	P	My									
B	My										
C	Mv										

Chapter Y3.13: 'Right-angled triangles'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	E	My	E								
B	E										
C	E										
D	E										
E	E										
F	E	My	E								
G	E	My									

Chapter Y3.14: 'Volume'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	B	My	E	My							
B	E	My	E								
C	B										

Chapter Y3.15: 'Problems in planning'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	P										
B	P										
C	P										

Chapter Y3.16: 'Linear equations'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My	P									
B	My	E	My								
C	E										
D	E										
E	E										

Chapter Y3.17: 'Distributions'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My										
B	My										
C	My										
D	My										

Book Y4

Chapter Y4.01: 'Selections and arrangements'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	P	My									
B	My										
C	My										
D	My	Me	My								

Chapter Y4.02: 'Enlargement and volume'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My										
B	B	My									
C	B	My									
D	Me	My									
E	B	My	P	My							

Chapter Y4.03: 'Linear equations'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	E										
B	E										
C	E										

Chapter Y4.04: 'Angle relationships'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	B										
B	My	E									
C	E										
D	My	E		My	E						

Chapter Y4.05: 'Algebraic fractions'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	E	My	E	My	E						
B	E										
C	E	My									
D	E	Me	E								

Chapter Y4.06: 'Vectors'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	B										
B	E										
C	E										
D	E	My									

Chapter Y4.07: 'Sequences (1)'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	Me	E	My								
B	E										
C	E	My									
D	E										
E	E										

Chapter Y4.08: 'Types of proportionality'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My	E	My								
B	My	E									
C	My										

Chapter Y4.09: 'Manipulating formulas'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My	E									
B	E										
C	E	My	E								

Chapter Y4.10: 'Probability'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	Me										
B	Me	E	Me								
C	Me	My									
D	My	Me									
E	E										
F	My	Me	My								

Chapter Y4.11: 'Sequences (2)'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	E										
B	E										
C	E										

Chapter Y4.12: 'Exponential growth and decay'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My										
B	E	My									
C	My	E	My								

Chapter Y4.13: 'Optimisation'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My										
B	My										
C	P	My									

Chapter Y4.14: 'Sampling'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	Me	My									
B	My										
C	My	E	My								
D	My										
E	P	My									
F	My										
G	My										
H	My										

Chapter Y4.15: 'Functions'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	Me	E									
B	Me	E									
C	E	My									
D	Me	E									
E	E										

Chapter Y4.16: 'Three dimensions'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	E										
B	E	My	E								
C	E										
D	My	E	My								

Chapter Y5.12: 'Vector geometry'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	B	Me	E								
B	B										
C	B										
D	B										
E	B										

Book G1

Chapter G1.01: 'Estimating and scales'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	P	B	My	P							
B	P										
C	P										
D	My	P									

Chapter G1.02: 'Money'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	P										
B	My										
C	P	My	Me								
D	My	P									

Chapter G1.03: 'Formulas'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	P	My									
B	P	My									
C	My										

Chapter G1.04: 'Time'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	P	My									
B	P	My	P								
C	P										

Chapter G1.05: 'Chains'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	Me	E	Me								
B	P	My	P	My	P						
C	P										

Chapter G1.06: 'Calculate ...'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	Me										
B	P	My									
C	E										
D	E	Me									
E	E										

Book G2

Chapter G2.01: 'Raising money'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	P										
B	P										
C	P										
D	P										

Chapter G2.02: 'Reading scales'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	P	My	P								
B	P										

Chapter G2.03: 'Working backwards'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	Me	B									
B	Me										

Chapter G2.04: 'Polygon patterns'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	B	P									
B	Me	E	P								
C	Me	E	P								
D	B	P	B	P							

Chapter G2.05: 'Further beyond the point'											
Section	a	b	c	d	e	f	g	h	i	j	k
A	My	P									
B	My										
C	My										
D	My	E		Mv	E						

Appendix 3

Operational Decisions Made in the Quantification of Signifying Modes

The following operational decisions were taken during the conduct of the quantifying of signifying modes discussed in Chapter 9.

1. Icons and indices are taken to extend to their own 'natural' boundaries rather than to any indexical border, unless the latter is clearly a part of the icon/index. Some icons and indices do not have regular boundaries, in these cases, it was necessary to construct imaginary boundaries which preserved the integrity of the text. In a small number of instances (eg some cartesian graphs) this resulted in squares which did not contain any print being counted as containing iconic or indexical significations.
2. Where symbolic text is an integral part of an icon or index, or where an icon or index is labelled symbolically (including speech/thought bubbles), the symbolic text is counted as iconic/indexical; where symbolic text is appended as a commentary (a supplement rather than a complement), it is not counted as a part of the icon/index except where it may overlap the latter.
3. The superimposition of an iconic mode on an indexical or symbolic mode (or vice versa) is counted as iconic.
4. Any non-iconic text which includes any manuscript is counted as manuscript.
5. A drawing which is completely reduced to a geometric figure (including diagrams of boxes, sheets of metal, etc if unshaded, evenly shaded, or differentiated by face) is counted as a graph. Where diagrams are iconically shaded or include additional detail, they are icons.
6. All plans, schematic drawings, and maps are indices unless iconically embellished in which case they are icons.
7. Page numbers are not counted.

8. Calculator button keys are symbols, but reproductions of displays are icons.
9. Brackets are counted as whatever they are associated with.
10. Manuscript symbols and indices drawn on 'paper' or on 'exercise books' are symbols/icons unless a hand or pen/pencil or another iconic object is included in which case it is an icon. In the former case, the 'paper'/'exercise book' is treated as an indexical border, ie it is not counted.
11. A supermarket checkout slip is a drawing unless it is a photograph (there were no instances of the latter).
12. A flowchart is a table unless it is an in-line sequence for calculator operation, in which case it is symbolic.
13. Photographic reproductions of documents are photographs. Photographic reproductions of drawings and paintings are not photographs.

Appendix 4

A Pedagogic Interpretation of the Language of Description

In this appendix, I shall offer an example of an interpretation of a part of the language of description¹. My purpose is to illustrate the practical value of my conception of the relationship between sociological research and educational practice, as outlined in Chapter 11. I want to focus specifically on the contents of a mathematics curriculum or a mathematics lesson.

Drawing on a part of my language of description, I want to suggest that these contents can be divided, approximately, into two *domains*. Firstly, there are those contents which are unambiguously mathematical and which do not explicitly concern anything beyond mathematics. This is the *esoteric domain*. Secondly, there are those contents which are or which appear to be about something other than mathematics, whether it be another curriculum area, such as science, or a working or domestic practice, such as bricklaying or shopping. These contents lie within the *public domain*.

However, the contents might also be classified in a different way. Thus, certain of the contents may be considered, for whatever reason, to be particularly important. Other contents will be of lesser importance. The contents in the second category, but not the first, may be substituted by other contents. Thus contents may have *restricted* or *extended substitutability*. This will be the case whether they are esoteric or public domain. The product of these two scales gives rise to four categories of mathematical content. These are represented in Figure A4.1.

The value of this schema lies in its organising and interrogative power. It is, for example, relatively easy to assign a given content to one of the four categories. Having done so, the features of the category suggest certain questions and/or strategies. 'Canonical mathematics' (box A in Figure A4.1), for example, comprises esoteric domain mathematics of restricted substitutability. The introduction of π , in Book Y, is an example of content within this category. My discussion of this text, in

¹ An expansion of this interpretation is available on video, Dowling, 1993b, 1993c.

Chapter 8, describes it as discursive in making explicit the regulating principles of mathematics and in linking the topic with other topics. The topic is, therefore, treated in accordance with its restricted substitutability. The G text that was compared with this—‘Polygon patterns’ in Book G2—was described differently. Here, there were no definitions, no principles, no linking with other topics. The mathematical discourse was disarticulated by the procedural text. The appearance of this topic in the G book could thus be described as tokenism. It therefore serves no pedagogic purpose.

Categories of Mathematical Content

		DOMAIN	
		ESOTERIC	PUBLIC
SUBSTITUTABILITY	RESTRICTED	A Canonised Mathematics (the mathematics syllabus)	D The 'Real World' (utilitarian or political interests)
	EXTENDED	B Imaginary Mathematics (investigations)	C The 'Imaginary World' (metaphors for mathematics)

Figure A4.1

Moving to box B in Figure A4.1: here, substitutability is extended. The question, then, is why is it included as content. Examples of ‘imaginary mathematics’ are to be found in a great deal of ‘investigational work’, such as the text described in Chapter 9. Here, it was suggested that school mathematics tended to prioritise heuristics, rather than the particular system proposed in the task, the Y1 Teacher’s Guide noted that:

Obviously, the investigations in this chapter are not important pieces of mathematics in their own right. So the teacher is not trying to teach, or the pupils to learn, any important mathematical ‘facts’.

(Y1TG, p. 33)

Thus, potentially any system can be substituted for the particular ones on offer. What is important is the methods to be used in elaborating the system. This, however, is prioritising means over ends. The formal system which is set up (with more or less formality) by the task is reduced to the level of a resource to be implicated in the practising of heuristic strategies. Insofar as any attempt is being made to apprentice the student to mathematical subjectivity, this is an inappropriate inversion. Insofar as mathematics may be described as the exploring of formal systems, then heuristics should be understood as resources in such exploration. The fundamental question is not, how can we proceed to play around with this system, but, what is this system like, how does it work. This is a more genuine acceptance of the proposition that imaginary mathematics is an important source of new mathematics. The imaginary system must shed its arbitrariness once it has been selected.

Box C comprises those contents which, in the language of description itself, would be referred to as metaphorical domain. Examples are the Y5 text introducing orthographic projection and the G4 text involving the 1938 Mercedes Benz; these texts were discussed in Chapter 8. In each case, there is a public domain element which is arbitrary: a table; a car. In each case, there is a metaphor, a 'view', which substitutes for a mathematical sign, 'projection'. The content is in the 'imaginary world'. There is no attempt, in either text, to mythologise the world, mathematically. Rather, the imaginary world has been organised so as to offer a way in to canonised mathematics. The texts differ, however, in that only the Y text punctuates the metaphor and establishes the mathematics:

The orthographic projection is not a view you could ever actually see.
But if you are a long way back from the table, the front legs can almost
hide the back legs.

(Y5, p. 93)

The G text, on the other hand, never actually mentions the term 'projection', although it imagines the car being 'put into a glass box' for no reason that is apparent in the text.

Finally, box D, concerns restricted substitutability public domain content. The substitutability is restricted, because the public domain setting is important. It may concern an important aspect of everyday or working life or an engineering or science problem. Or it may be concerned with a politically sensitive issue, such as physical disability. It has been established, via the analysis in this thesis, that public domain settings in school mathematics are always structured according to mathematical

grammar and are, thereby, distorted; this is the nature of recontextualising. However, where a the restriction of substitutability of a content derives from its public domain setting, such mathematical restructuring and reprioritising must be unacceptable. Elsewhere, I have described this situation with reference to a SMILE card concerned with disability (Brown & Dowling, 1989; Dowling, 1993b, 1993c). A similar argument has been mounted regarding the tendency of the mathematical anthropologists to see only mathematics in the cultural practices that they describe.

The implication of this point is that there must be an inversion of the conventional relationship between the public domain setting and the esoteric domain, such that it is the former, rather than the latter, which is prioritised. It seems clear that this inversion cannot be achieved within the context of school mathematics. Thus an additional curriculum space must be established for the elaboration and exploration of 'real world' content. Within this space, it may be that mathematical resources as well as resources relating to other disciplines are employed. However, they must be subordinated to the project and, indeed, there can be no certainty that the project will involve any mathematics at all. In Brown & Dowling (1989), this has been referred to as a 'research-based curriculum'.

These examples are intended to illustrate the possibility of re-interpreting the language of description so that an alternative set of criteria are prioritised. In this case, the alternative criteria are concerned with the development of materials. Resources, drawn fairly freely from the language of description, have been used to produce a structure for the interrogation of materials and for their possible recasting. The dimension 'substitutability' has been inducted from educational practice as the voice of the syllabus and of utilitarian and political interests. It is important to note that the sociological priorities have, essentially, been lost in the transition. Although the interrogation makes reference to a politically sensitive area—physical disability, ethnomathematics—it does not explicitly address a social structure of relationships. It is, in this sense, idealist. However, the transition has also generated an optimism insofar as it presupposes that a transformation in educational practice is possible and comparatively easy.

References & Bibliography

References & Bibliography

- ABRAHAM, J. & BIBBY, N., 1992, 'Mathematics and Society: ethnomathematics and the public educator curriculum', in Nickson, M. & Lerman, S. (eds), *The Social Context of Mathematics Education: theory and practice*, London: Southbank Press
- AGGER, B., 1989, 'Do Books Write Authors? A Study of Disciplinary Hegemony', *Teaching Sociology*, 17, 3, pp. 365-9
- AHIER, J.G., 1987, Representations of Town and Country in the Production of a National Identity: an analysis of school textbooks for English children 1880-1960, unpublished PhD, Kings College, London University
- AL-ABDULJADIR, F.F., 1987, Islamic Values and Biology Texts in Kuwait, unpublished MEd, University of Wales, Cardiff
- ALTHUSSER, L., 1971, *Lenin and Philosophy*, London: New Left Books
- ANUAR, M.K., 1990, The Construction of a 'National Identity': a study of selected secondary school textbooks in Malasia's education system with particular reference to peninsular Malaysia, unpublished PhD thesis, City University
- ANYON, J., 1979, 'Ideology and United States History Textbooks', *Harvard Education Review*, 49, 3, pp. 361-386
- ANYON, J., 1981a, 'Schools as Agencies of Social Legitimation', *Journal of Curriculum Theorizing*, 3, 2, pp. 86-103
- ANYON, J., 1981b, 'Ideology and United States History Textbooks' (abridged), in Dale, R. et al (eds), *Education and the State Volume II: politics, patriarchy and practice*, Lewes: Falmer
- ANYON, J., 1983, 'Workers, Labor and Economic History, and Textbook Content', in Apple, M.W. & Weis, L. (eds), *Ideology and Practice in Schooling*, Philadelphia: Temple University Press
- APPLE, M., 1986, *Teachers and Texts: a political economy of class and gender relations in education*, New York: Routledge
- APPLE, M., 1989, 'The Political Economy of Text Publishing', in Castell, S. de, Luke, A.L. & Luke, C. (eds), *Language, Authority and Criticism: readings on the school textbook*, London: Falmer
- ATKINSON, P., 1985, *Language, Structure and Reproduction: an introduction to the sociology of Basil Bernstein*, London: Methuen
- ATKINSON, P., 1990, *The Ethnographic Imagination: textual constructions of reality*, London: Routledge

- BAGCHI, J.P., 1985, 'Indian Biology Textbooks in Sex Education, A Comparative Study', *Journal of Science and Mathematics Education in Southeast Asia*, **8**, 2, pp. 180-23
- BAILEY, D.E., et al, 1981, *Mathematics in Employment (16-19)*, Bath: University of Bath
- BAKHTIN, M.M., 1981, *The Dialogic Imagination*, ed. Michael Holquist, Austin: University of Texas Press
- BAKHTIN, M.M., 1986, *Speech Genres and Other Late Essays*, ed. Caryl Emerson & Michael Holquist, Austin: University of Texas Press
- BANNET, E.T., 1989, *Structuralism and the Logic of Dissent*, Basingstoke: MacMillan
- BARTHES, R., 1972, *Mythologies*, London: Granada
- BARTHES, R., 1974, *S/Z*, London: Jonathan Cape
- BARTHES, R., 1981a, 'Theory of the Text' in Young, R. (ed), *Untying the Text: a poststructuralist reader*, London: RKP
- BARTHES, R., 1981b, 'Textual Analysis of Poe's "Valdemar"' in Young, R. (ed), *Untying the Text: a poststructuralist reader*, London: RKP
- BAUDRILLARD, J., 1988, 'Simulacra and Simulations' in *Jean Baudrillard: selected writings*, Cambridge: Polity
- BENACERRAF, P. & PUTNAM, H. (eds), 1983, *Philosophy of Mathematics: selected readings, second edition*, Cambridge: CUP
- BENSON, H., 1990, Dialogues and Interactional Skills—a study of English textbooks use in Japanese senior high schools, unpublished MA dissertation, Exeter University
- BERNSTEIN, B., 1971a, 'On the Classification and Framing of Educational Knowledge', in Young, M.F.D., (ed) *Knowledge and Control: new directions for the sociology of education*, London: Collier-MacMillan
- BERNSTEIN, B., 1971b, *Class, Codes and Control Volume 1: theoretical studies towards a sociology of language*, London: RKP.
- BERNSTEIN, B., 1977, *Class, Codes and Control Volume 3: towards a theory of educational transmissions, second edition*, London: RKP
- BERNSTEIN, B., 1982, 'Codes, Modalities and the Process of Cultural Reproduction' in Apple, M.W. (ed) *Cultural and Economic Reproduction in Education*, London: RKP
- BERNSTEIN, B., 1985, 'On Pedagogic Discourse' in Richards, J. (ed) *Handbook of Theory and Research for the Sociology of Education*, Greenwood Press
- BERNSTEIN, B., 1988, 'On Pedagogic Discourse: revised', *Collected Original Resources in Education*, **12**, 1
- BERNSTEIN, B., 1990, *Class, Codes and Control, volume 4*, London: RKP

- BERNSTEIN, B. et al, in press, *Class, Codes and Control*, volume 5
- BEYER, L.E., 1983, 'Aesthetic Curriculum and Cultural Reproduction', in Apple, M.W. & Weis, L. (eds), *Ideology and Practice in Schooling*, Philadelphia: Temple University Press
- BIBBY, N., 1988, 'Social History of Mathematical Controversies: some implications for the curriculum', mimeo, London: KQC
- BISHOP, A., 1977, 'Is a Picture Worth a Thousand Words', *Mathematics Teaching*, 81
- BISHOP, A., 1988a, 'Mathematics Education in its Cultural Context', *Educational Studies in Mathematics*, 19, 2, pp. 179-91
- BISHOP, A., 1988b, *Mathematical Enculturation: a cultural perspective on mathematics education*, Dordrecht: Kluwer
- BLOOR, D., 1976, *Knowledge and Social Imagery*, London: RKP
- BOS, W., 1990, 'On Mother Tongue Teaching Material of Ethnic Minorities: a comparative content analysis', *Research in Education*, 43, pp. 45-62
- BOURDIEU, P., 1977, *Outline of a Theory of Practice*, Cambridge: CUP
- BOURDIEU, P., 1984, *Distinction: a social critique of the judgement of taste*, London: RKP
- BOURDIEU, P., 1990, *The Logic of Practice*, Cambridge: Polity
- BOURDIEU, P. & PASSERON, J-C., 1977, *Reproduction in Education, Society and Culture*, London: Sage
- BOWLES, S. & GINTIS, H., 1976, *Schooling in Capitalist America*, London: RKP
- BOWLES, S. & GINTIS, H., 1986, *Democracy and Capitalism: property, community, and the contradictions of modern social thought*, London: RKP
- BOWLES, S. & GINTIS, H., 1988, 'The Correspondence Principle', in Cole, M. (ed), *Bowles and Gintis Revisited: correspondence and contradiction in educational theory*, London: Falmer
- BRAVERMAN, H., 1974, *Labor and Monopoly Capital: the degradation of work in the twentieth century*, New York: Monthly Review Press
- BRITTON, G. et al, 1984, 'The Battle to Imprint Citizens for the 21st Century', *Reading Teacher*, 37, 8, pp. 724-33
- BRITTON, G. & LUMKIN, M., 1983, 'Basal Readers: Paltry Progress Pervades', *Interracial Books for Children Bulletin*, 14, 6, pp. 4-7
- BROWN, A.J. & DOWLING, P.C., 1989, *A Critical Alternative to Internationalism and Monoculturalism in Mathematics Education*, Centre for Multicultural Education, Occasional Paper Number 10, London: Institute of Education, University of London, Centre for Multicultural Education, also in Gardener, R. (ed), 1992, *An International Dimension in the National Curriculum: an imperative for Britain for 1992 and beyond*, London: Department of

International & Comparative Education, Institute of Education, University of London

- BROWN, A.J. & DOWLING, P.C., 1992, "'Who's Been Restructuring *My* Primary Socialisation?'" the impact of school mathematics on domestic space', *Ruling the Margins: problematising parental involvement*, Institute of Education, University of London, September 1992, conference proceedings obtainable from IMPACT project, University of North London
- BROWN, A.J. & DOWLING, P.C., 1993, 'The Bearing of School Mathematics on Domestic Space' in Merttens, R., Mayers, D., Brown, A.J. & Vass, J. (eds), *Ruling the Margins: problematising parental involvement*, London: IMPACT, University of North London
- BROWN, S.I., 1981, 'Ye Shall be Known by Your Generations', *For the Learning of Mathematics*, 1, 3, pp. 27-36
- BROWN, S.I., 1984, 'The Logic of Problem Generation: from morality and solving to de-posing and rebellion', *For the Learning of Mathematics*, 4, 1, pp. 9-20, also in Burton, L. (ed), 1986, *Girls Into Maths Can Go*, London: Holt, Rinehart & Winston
- BROWNE, J., 1990, 'Gender Bias in Physical Education Textbooks', *ACHOER National Journal*, 127, pp. 4-7
- BUERK, D., 1982, 'An Experience with some Able Women who Avoid Mathematics', *For the Learning of Mathematics*, 3, 2, pp. 19-24
- BULMER, M. (ed), 1984, *Sociological Research Methods: an introduction*, second edition, London: MacMillan
- BURTON, L. (ed), 1986, *Girls Into Maths Can Go*, London: Holt, Rinehart & Winston
- BURTON, L., 1988, 'Images of Mathematics', in Ernest, P. (ed), *The Social Context of Mathematics Teaching: Perspectives 37*, Exeter: School of Education, University of Exeter
- BURTON, L. et al, 1986, *Girls Into Mathematics*, Cambridge: CUP
- CAIRNS, J.E., 1988, A Cultural Analysis of Primary Level History Textbooks with a Special Focus on the History of Women, unpublished MEd, Stirling University
- CAIRNS, J. & INGLIS, B., 1989, 'A Content Analysis of Ten Popular History Textbooks for Primary Schools with Particular Emphasis on the Role of Women', *Educational Review*, 41, 3, pp. 221-6
- CAIRNS, M.E., 1984, Schools Mathematics Project 7-13: comparative content analysis and a study of implementation in primary and junior schools, unpublished MPhil, University of Lancaster
- CARLSON, D.L., 1989, 'Legitimation and Delegitimation: American history textbooks and the Cold War', in Castell, S. de, Luke, A.L. & Luke, C. (eds), *Language, Authority and Criticism: readings on the school textbook*, London: Falmer

- CARRAHER, D.W., 1991, 'Mathematics in and out of School: a selective review of studies from Brazil', in Harris, M. (ed), *Schools, Mathematics and Work*, London: Falmer
- CARRAHER, T.N., CARRAHER, D.W. & SCHLIEMANN, A.D., 1987, 'Written and Oral Mathematics', *Journal for Research in Mathematics Education*, **18**, 2, pp. 83-97
- CARRAHER, T.N. et al, 1985, 'Mathematics in the Streets and in the Schools', *British Journal of Developmental Psychology*, **3**, 1, pp. 21-9
- CARRAHER, T.N., SCHLIEMANN, A.D. & CARRAHER, D.W., 1988, 'Mathematical Concepts in Everyday Life', in Saxe, G.B. & Gearhart, M. (eds), *Children's Mathematics: new directions for child development*, San Francisco: Jossey-Bass
- CARRELL, D., 1991, 'Gender Scripts in Professional Writing Textbooks', *Journal of Business and Technical Communication*, **5**, 4, pp. 463-8
- CARROLL, H., 1991, 'Educational Levels', *The Aboriginal Child at School*, **19**, 1, pp. 3-5
- CHIMOMBO, M., 1989, 'Readability of Subject Texts: implications for ESL teaching in Africa', *English for Specific Purposes*, **8**, 3, pp. 255-64
- CLARK, L.L., 1980, 'The Education of French Schoolgirls: Pedagogical Prescriptions and Social and Economic Realities during the Third Republic', working paper, Mary Bunting Institute, Radcliffe College, Cambridge, Mass.
- COCKBURN, C., 1983, *Brothers: male dominance and technological change*, London: Pluto
- COCKBURN, C., 1985a, 'Caught in the Wheels: the High Cost of Being a Female Cog in the Male Machinery of Engineering', in D. MacKenzie & J. Wajcman (eds), *The Social Shaping of Technology*, Milton Keynes: Open University Press
- COCKBURN, C., 1985b, 'The Material of Male Power', in D. MacKenzie & J. Wajcman (eds), *The Social Shaping of Technology*, Milton Keynes: Open University Press
- COCKBURN, C., 1985c, *Machinery of Dominance*, London: Pluto
- COCKCROFT, W. et al, 1982, *Mathematics Counts*, London: HMSO
- COHEN, I.J., 1987, 'Structuration Theory and Social Praxis', in Giddens, A. & Turner, J.H. (eds), *Social Theory Today*, Cambridge: Polity
- CONNOR-GREENE, P. et al, 1988, 'Gender Issues in Psychology: A Content Analysis of Introductory Psychology Textbooks', paper presented at the Annual Meeting of the Southeastern Psychological Association, New Orleans, March 1988
- COOK-GUMPERTZ, J. (ed), 1973, *Social Control and Socialization: a study of class differences in the language of maternal control*, London: Routledge

- COOLEY, M., 1985, 'Drawing Up the Corporate Plan at Lucas Aerospace', in MacKenzie, D.A. & WAJCMAN, J. (eds), *The Social Shaping of Technology*, Milton Keynes: Open University Press
- COOPER, B., 1983, 'On Explaining Change in School Subjects', *British Journal of Sociology of Education*, 4, 3, pp. 207-22
- COOPER, B., 1985, *Renegotiating Secondary School Mathematics*, Lewes: Falmer
- CORRAN, G. & WALKERDINE, V., 1981, *The Practice of Reason, Volume 1: reading the signs*, mimeo, London: University of London, Institute of Education
- CRESSWELL, M. & GUBB, J., 1987, *The Second International Mathematics Study in England and Wales*, Windsor: NFER-Nelson
- CRISMORE, A., 1989, 'Rhetorical Form, Selection and Use of Textbooks', in Castell, S. de, Luke, A.L. & Luke, C. (eds), *Language, Authority and Criticism: readings on the school textbook*, London: Falmer
- CURRIE, J., KISSANE, B. & PEARS, H., 1992, 'An Enriched Mathematical Program for Young Aboriginal Children', *The Aboriginal Child at School*, 20, 1, pp. 15-37
- DAHRENDORF, R., 1959, *Class & Class Conflict in Industrial Society*, London: RKP
- D'AMBROSIO, U., 1985, 'Ethnomathematics and its Place in the History and Pedagogy of Mathematics', *For the Learning of Mathematics*, 5, 1, pp. 44-8
- D'AMBROSIO, U., 1990, 'The Role of Mathematics in Building up a Democratic Society and the Civilizatory Mission of European Powers Since the Discoveries' in Noss, R. et al (eds) *Political Dimensions of Mathematics Education: action and critique, proceedings of the first international conference, revised edition*, London: Department of Mathematics, Statistics and Computing, Institute of Education, University of London
- DAVIS, P.J. & HERSH, R., 1981, *The Mathematical Experience*, Brighton: Harvester
- DEAN, E. et al, 1983, *History in Black and White: An Analysis of South African School History Textbooks*, Paris: UNESCO
- DEPARTMENT OF EDUCATION AND SCIENCE, 1975, *A Language for Life*, (The Bullock Report), London: HMSO
- DEPARTMENT OF EDUCATION AND SCIENCE, 1985, *Mathematics From 5 to 16*, London: HMSO
- DEPARTMENT OF EDUCATION AND SCIENCE, 1988, *Mathematics for Ages 5 to 16*, London: HMSO
- DEPARTMENT OF EDUCATION AND SCIENCE, 1989, *Mathematics in the National Curriculum*, London: HMSO
- DEPARTMENT OF EDUCATION AND SCIENCE, 1991, *Mathematics in the National Curriculum*, London: HMSO

- DERRIDA, J., 1981, 'Semiology and Grammatology' in *Positions*, London: Athlone Press
- DESFORGES, C. & COCKBURN, A., 1987, *Understanding the Mathematics Teacher: a study of practice in first schools*, London: Falmer
- DONALDSON, M., 1978, *Children's Minds*, Glasgow: Fontana/Collins
- DOWLING, 1985, 'Does Cuba Educate for Freedom?', discussion paper available from Department of Mathematics, Statistics & Computing, Institute of Education, University of London
- DOWLING, P.C., 1986, *Mathematics Makes the Difference: A Sociological Approach to School Mathematics*, Unpublished MA dissertation, London: University of London, Institute of Education
- DOWLING, P.C., 1989, 'The Contextualising of Mathematics: towards a theoretical map', in *Collected Original Resources in Education*, 13, 2 (also in Harris, M. (ed), 1990, *Schools, Mathematics and Work*, Basingstoke: Falmer)
- DOWLING, P.C., 1990a, 'Some Notes Towards a Theoretical Model for Reproduction, Action and Critique', in Noss, R. et al (eds), *Political Dimensions of Mathematics Education: action and critique, proceedings of the first international conference*, London: Department of Mathematics, Statistics and Computing, Institute of Education, University of London
- DOWLING, P.C., 1990b, 'The Shogun's and Other Curricular Voices', in Dowling, P.C. & Noss, R. (eds), *Mathematics versus the National Curriculum*, Basingstoke: Falmer
- DOWLING, P.C., 1990c, 'Mathematics in the Marketplace: the National Curriculum and Numerical Control', *Teaching London Kids*, 26, pp. 28-31
- DOWLING, P.C., 1991a, 'A Touch of Class: ability, social class and intertext in SMP 11-16', in Pimm, D. & Love, E. (eds), *Teaching and Learning School Mathematics*, London: Hodder & Stoughton
- DOWLING, P.C., 1991b, 'A Dialectics of Determinism: deconstructing information technology', in McKay, H., Young, M.F.D. & Beynon, J. (eds), *Understanding Technology in Education*, London: Falmer
- DOWLING, P.C., 1991c, 'Gender, Class and Subjectivity in Mathematics: a critique of Humpty Dumpty', *For the Learning of Mathematics*, 11, 1, pp 2-8
- DOWLING, P.C., 1992a, 'Textual Production and Social Activity: a language of description', *Collected Original Resources in Education*, 16, 1
- DOWLING, P.C., 1992b, 'Pedagogic Voices, Pedagogic Messages: a sociological algebra', presented at 'Research into Social Perspectives in Mathematics Education', South Bank University, May 1992, (available from Department of Mathematics, Statistics & Computing, Institute of Education, University of London)
- DOWLING, P.C., 1993a, 'Mathematics, Discourse and 'Totemism': a language for practice', in The NECC Mathematics Commission, PDME.2: Curriculum Reconstruction for Society in Transition, 2nd-5th April, 1993, Broederstroom, Pre-conference papers

- DOWLING, P.C., 1993b, 'Mathematical Texts: sociological critiques and implications', video of lecture presented at the Mathematics Education Project, University of Cape Town, 8th April 1993, video obtainable from Mathematics Education Project, University of Cape Town
- DOWLING, P.C., 1993c, 'Domain and Substitutability: a language for the development of mathematical educational materials', video of public lecture presented at University of Cape Town, 17th April 1993, video obtainable from Mathematics Education Project, University of Cape Town
- DOWLING, P.C., in press, 'Theoretical "Totems": a sociological language for educational practice', in Angelis, D. et al (eds), *Political Dimensions in Mathematics Education: curriculum reconstruction for society in transition*, Cape Town, to be published October 1993
- DOWLING, P.C. & NOSS, R., 1990, 'Multiplying by Zero', in Dowling, P.C. & Noss, R. (eds), *Mathematics versus the National Curriculum*, London: Falmer
- DOWLING, P.C. & NOSS, R. (eds), 1990, *Mathematics versus the National Curriculum*, London: Falmer
- DREYFUS, H.L. & RABINOW, P., 1982, *Michel Foucault: beyond structuralism and hermeneutics*, Brighton: Harvester
- DURKHEIM, É., 1951, *Suicide: a study in sociology*, London: RKP
- DURKHEIM, É., 1984, *The Division of Labour in Society*, Basingstoke: MacMillan
- ECO, U., 1973, 'Social Life as a Sign System', in Robey, D. (ed) *Structuralism: an introduction*, Oxford: Clarendon
- ECO, U., 1976, *A Theory of Semiotics*, Bloomington: Indiana University Press
- ECO, U., 1979, 'The Semantics of Metaphor', in *The Role of the Reader*, London: Hutchinson
- ECO, U., 1984, *Semiotics and the Philosophy of Language*, Basingstoke: MacMillan
- ECO, U., 1989, *Foucault's Pendulum*, London: QPD
- ECO, U., 1990, *The Limits of Interpretation*, Bloomington: Indiana University Press
- EICHINGER, D. & ROTH, K.J., 1981, *Critical Analysis of an Elementary Science Curriculum: Bouncing around or Connectedness?*, Elementary Subjects Center Series No. 32, E. Lansing: Center for the Learning & Teaching of Elementary Subjects
- ELLINGTON, L., 1986, 'Blacks and Hispanics in High School Economics Texts', *Social Education*, 50, 1, pp. 64-7
- ENSOR, P., 1991, *Upsetting the Balance: Black girls and mathematics*, unpublished MSc dissertation, Institute of Education, University of London
- ENSOR, P., 1993, 'Boundaries at the Centre—differentiating pupils in mathematics classrooms', in The NECC Mathematics Commission, PDME.2: Curriculum Reconstruction for Society in Transition, 2nd-5th April, 1993, Broederstroom, Pre-conference papers

- ERNEST, P., 1988, 'Editor's Introduction', in Ernest, P. (ed), *The Social Context of Mathematics Teaching: Perspectives 37*, Exeter: School of Education, University of Exeter
- ERNEST, P., 1991, *The Philosophy of Mathematics Education*, London: Falmer
- ETIM, J.S., 1988, 'Sex Role Portrayal in 15 YA Books Used in Nigerian Secondary Schools', *Journal of Reading*, **31**, 5, pp. 452-7
- FAIRCLOUGH, N., 1989, *Language and Power*, London: Longman
- FARWELL, R. & KNEE, C., 1990, 'Demystifying Mathematics: popular mathematics as education', in Noss, R. et al (eds), *Political Dimensions of Mathematics Education: action and critique, proceedings of the first international conference*, London: Department of Mathematics, Statistics and Computing, Institute of Education, University of London
- FASHEH, M., 1991, 'Mathematics in a Social Context: math within education as praxis versus math within education as hegemony', in Harris, M. (ed), *Schools, Mathematics and Work*, London: Falmer
- FASHEH, M., 1993, 'From a Dogmatic, Ready-Answer Approach of Teaching Maths Towards a Community-Building, Process-Oriented Approach', in The NECC Mathematics Commission, PDME.2: Curriculum Reconstruction for Society in Transition, 2nd-5th April, 1993, Broederstroom, Pre-conference papers
- FENNEMA, E. (ed), 1985, 'Explaining Sex-related Differences in Mathematics: theoretical models', *Educational Studies in Mathematics*, **16**, 3, pp. 303-20
- FEYERABEND, P., 1975, *Against Method: outline of an anarchistic theory of knowledge*, London: Verso
- FILLMER, H.T. & MEADOWS, R.E., 1984, 'The Portrayal of Older Characters in Five Sets of Basal Readers', paper presented at the Annual Meeting of the International Reading Association, Atlanta, May, 1984
- FISKE, J., 1989, *Reading the Popular*, London: Routledge
- FOUCAULT, M., 1965, *Madness and Civilization: a history of insanity in the age of reason*, London: Tavistock
- FOUCAULT, M., 1970, *The Order of Things: an archaeology of the human sciences*, London: Tavistock
- FOUCAULT, M., 1972, *The Archaeology of Knowledge*, London: Tavistock
- FOUCAULT, M., 1973, *The Birth of the Clinic*, London: Tavistock
- FOUCAULT, M., 1977a, 'The Political Function of the Intellectual', *Radical Philosophy*, **17**, pp. 12-14
- FOUCAULT, M., 1977b, *Discipline and Punish: the birth of the prison*, London: Penguin
- FOUCAULT, M., 1978, *The History of Sexuality: volume 1, an introduction*, Harmondsworth: Penguin

- FOUCAULT, M., 1980, *Power/Knowledge*, Brighton: Harvester
- FOUCAULT, M., 1981, 'The Order of Discourse', in Young, R. (ed), *Untying the Text: a poststructuralist reader*, London: RKP
- FOUCAULT, M., 1982, 'The Subject and Power', in Dreyfus, H.L. & Rabinow, P., *Michel Foucault: beyond structuralism and hermeneutics*, Brighton: Harvester
- FOUCAULT, M., 1984, *The Use of Pleasure: volume 2, the history of sexuality*, Harmondsworth: Penguin
- FOUCAULT, M., 1986, *The Care of the Self: volume 3, the history of sexuality*, New York: Vintage
- FRANKENSTEIN, M., 1989, *Relearning Mathematics: a different third R—radical mathematics, volume 1*, London: Free Association Books
- FREEMAN, D.J. et al, 1983a, 'Consequences of Different Styles of Textbook Use in Preparing Students for Standardized Tests', research report, E. Lansing: Institute for Research on Teaching, Michigan State University
- FREEMAN, D.J. et al, 1983b, 'Do Textbooks and Tests Define a National Curriculum in Elementary School Mathematics?', *Elementary School Journal*, **83**, 5, pp. 501-13
- FREEMAN, D.J. et al, 1983c, 'The Influence of Different Styles of Textbook Use on Instructional Validity of Standardized Tests', *Journal of Educational Measurement*, **20**, 3, pp. 259-70
- FREEMAN, D.J. & PORTER, A.C., 1989, 'Do Textbooks Dictate the Content of Mathematics Instruction in Elementary Schools?', *American Educational Research Journal*, **26**, 3, pp. 403-21
- FREUD, S., 1973a, *Introductory Lectures on Psychoanalysis*, London: Penguin
- FREUD, S., 1973b, *New Introductory Lectures on Psychoanalysis*, London: Penguin
- FREUD, S., 1976, *The Interpretation of Dreams*, London: Penguin
- FULKERSON, E., 1939, 'Teaching the Law of Signs in Multiplication', *The Mathematics Teacher*, **32**, 1, pp. 27-9
- GADAMER, H-G., 1976, *Philosophical Hermeneutics*, Berkeley: University of California Press
- GARCIA, J. et al, 1990, 'The Portrayal of Females and Minorities in Selected Elementary Mathematics Series', *School Science and Mathematics*, **90**, 1, pp. 2-12
- GARCIA, J. & SADOWSKI, M., 1986, 'The Treatment of Minorities in Nine Recently Published Basal Series', paper presented at the Annual Meeting of the American Educational Research Association, April 1986
- GARFINKEL, H., 1967, *Studies in Ethnomethodology*, Englewood Cliffs: Prentice-Hall

- GERDES, P., 1985, 'Conditions and Strategies for Emancipatory Mathematics Education in Undeveloped Countries', *For the Learning of Mathematics*, 5, 1, pp. 15-20
- GERDES, P., 1986, 'How to Recognize Hidden Geometrical Thinking: a contribution to the development of anthropological mathematics', *For the Learning of Mathematics*, 6, 2, pp. 10-12
- GERDES, P., 1988a, 'On Culture, Geometrical Thinking and Mathematics Education', *Educational Studies in Mathematics*, 19, 2, pp. 137-62
- GERDES, P., 1988b, 'On Possible Uses of Traditional Angolan Sand Drawings in the Mathematics Classroom', *Educational Studies in Mathematics*, 19, 1, pp. 3-22
- GERTH, H.H. & MILLS, C.W., (eds) 1948, *From Max Weber*, London: RKP
- GIDDENS, A., 1984, *The Constitution of Society: outline of the theory of structuration*, Cambridge: Polity
- GILBERT, R., 1986, "'That's Where They Have To Go" The Challenge of Ideology in Geography', *Geographical Education*, 5, 2, pp. 43-6
- GILBERT, R., 1989, 'Text Analysis and Ideology Critique of Curriculum Content', in Castell, S. de, Luke, A.L. & Luke, C. (eds), *Language, Authority and Criticism: readings on the school textbook*, London: Falmer
- GINTIS, H. & BOWLES, S., 1988, 'Contradiction and Reproduction in Educational Theory', in Cole, M. (ed), *Bowles and Gintis Revisited: correspondence and contradiction in educational theory*, London: Falmer
- GLEICK, J., 1988, *Chaos: making a new science*, London: Heinemann
- GONZALEZ-SUAREZ, M. & EKSTROM, R.B., 1989, 'Are U.S. Elementary School Reading Textbooks Sex Stereotyped?', paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, March, 1989
- GORDON, C., 1980, 'Afterword' in Foucault, M., *Power/Knowledge: selected interviews and other writings, 1972-1977*, Brighton: Harvester
- GORDON, P., 1978, 'Tradition and Change in the Curriculum', in Lawton, D. et al (eds), *Theory and Practice of Curriculum Studies*, London: RKP
- GRIFFITHS, H.B. & HOWSON, A.G., 1974, *Mathematics, Society and Curricula*, London: CUP
- GUIDER, J., 1991, 'Curriculum, Classroom Management and Discipline for the Aboriginal Student', *The Aboriginal Child at School*, 19, 4, pp. 21-32
- GUPTA, A.F. & LEE SU YIN, A., 1989, 'Gender Representation in English Language Textbooks used in the Singapore Primary Schools', *Language and Education*, 4, 1, pp. 29-50
- HAHN, C.L. & BLAKENSHIP, G., 1983, 'Women and Economics Textbooks', *Theory and Research in Social Education*, 11, 3, pp. 67-76

- HALES, M., 1980, *Living Thinkwork: Where Do Labour Processes Come From?* London: CSE Books
- HALL, E.J., 1988, 'One Week for Women? The Structure of Inclusion of Gender Issues in Introductory Textbooks', *Teaching Sociology*, **16**, 4, pp. 431-42
- HALLIDAY, M.A.K., 1978, *Language as Social Semiotic: the social interpretation of language and meaning*, London: Arnold
- HARRIS, M., 1985a, 'Wrapping up Mathematics in the World of Work', *Contact*, London: ILEA, March 8th 1985
- HARRIS, M., 1985b, 'Wrapping it Up', *Mathematics Teaching*, **113**, pp. 44-6
- HARRIS, M., 1987, 'An Example of Traditional Women's Work as a Mathematics Resource', *For the Learning of Mathematics*, **7**, 3, pp. 26-8
- HARRIS, M., 1988a, 'Common Threads: mathematics and textiles', *Mathematics in Schools*, **17**, 4, pp. 24-8
- HARRIS, M., 1988b, 'Common Threads', *Mathematics Teaching*, **123**, pp. 15-17
- HARRIS, M., 1993, 'Mathematics in the Experience of School and Work', in The NECC Mathematics Commission, PDME.2: Curriculum Reconstruction for Society in Transition, 2nd-5th April, 1993, Broederstroom, Pre-conference papers
- HARRIS, P., 1991, *Mathematics in a Cultural Context: Aboriginal perspectives on space, time and money*, Geelong: Deakin University Press
- HARRIS, S., 1988, 'Culture Boundaries, Culture Maintenance-in-change, and Two-way Aboriginal Schools', *Curriculum Perspectives*, **8**, 2, pp. 76-83
- HART, K.M. (ed), 1981, *Children's Understanding of Mathematics: 11-16*, London: John Murray
- HASSAN, M.M., 1990, Political Socialization in Kuwait, unpublished PhD thesis, Exeter University
- HASSAN, R., 1991, 'Questions as a Mode of Learning in Everyday Talk' in Lê, T. & McCausland, M. (eds), *Language Education: interaction and development*, Proceedings of the International Conference, Ho Chi Minh City, Vietnam, 30th March-1st April 1991
- HASSAN, R., 1992a, 'Contexts for Meaning', paper presented at the Georgetown University Round Table, GURT'92, Georgetown University, April 20th-23rd 1992
- HASSAN, R., 1992b, 'Meaning in Sociolinguistic Theory' in Bolton, K. & Kwok, H. (eds), *Sociolinguistics Today: international perspectives*, London: Routledge
- HEATH, S.B., 1986, 'Questioning at Home and at School: a comparative study' in Hammersley, M. (ed) *Case Studies in Classroom Research*, Milton Keynes: Open University Press

- HEATHCOTE, O.D., 1981, 'Sex Stereotyping in Mexican Reading Primers', paper presented at the Annual Meeting of the International Reading Association, New Orleans, April-May, 1981
- HEATHCOTE, O.D., 1982, 'Sex Stereotyping in Mexican Reading Primers', *Reading Teacher*, **36**, 2, pp. 158-65
- HEIDEGGER, M., 1962, *Being and Time*, London: SCM Press
- HENRIQUES, J. et al, 1984, *Changing the Subject: psychology, social regulation and subjectivity*, London: Methuen
- HIRSCHKOP, K., 1989, 'Bibliographical Essay' in Hirschkop, K. & Shepherd, D., (eds) *Bakhtin and Cultural Theory*, Manchester: Manchester University Press
- HITCHCOCK, M.E. & TOMPKINS, G.E., 1987, 'Basal Readers: Are They Still Sexist?', *Reading Teacher*, **41**, 3, pp. 288-92
- HODGE, R. & KRESS, G., 1988, *Social Semiotics*, Cambridge: Polity
- HODGE, R. & KRESS, G., 1993, *Language as Ideology, second edition*, London: Routledge
- HOFSTADTER, D.R., 1979, *Gödel, Escher, Bach: an eternal golden braid*, Harmondsworth: Penguin
- HOLT, E.R., 1990, "'Remember the Ladies"—Women in the Curriculum. ERIC Digest', Bloomington: ERIC Clearinghouse for Social Studies/Social Science Education
- HOWSON, A.G., 1987, 'Challenges and Responses' in Howson, G. (ed), *Challenges and Responses: essays to celebrate the twenty-fifth anniversary of the School Mathematics Project*, Cambridge: CUP
- HOWSON, A.G. (ed), 1987, *Challenges and Responses: essays to celebrate the twenty-fifth anniversary of the School Mathematics Project*, Cambridge: CUP
- HOWSON, A.G., KEITEL, C. & KILPATRICK, J., 1981, *Curriculum Development in Mathematics*, Cambridge: CUP
- HOWSON, A.G. & WILSON, B., 1986, *School Mathematics in the 1990s*, Cambridge: CUP
- HUDSON, B., 1985, 'Social Division of Adding Up to Equality? militarist, sexist and ethnocentric bias in mathematics textbooks and computer software', *World Studies Journal*, **5**, 4, pp. 24-9
- ILYATJAR, N., 1991, 'Traditional Aboriginal Learning: how I learned as a Pitjantjatjara Child', *The Aboriginal Child at School*, **19**, 1, pp. 6-12
- INNER LONDON EDUCATION AUTHORITY CENTRE FOR LEARNING RESOURCES, 1985, *Everyone Counts: looking for bias and insensitivity in primary mathematics materials*, London: ILEA Learning Resources Branch
- ISAACSON, Z., 1988, 'The Marginalisation of Girls in Mathematics: some causes', in Pimm, D. (ed), *Mathematics, Teachers and Children*, London: Hodder & Stoughton

- IVES, R., 1984, 'The Maleness of Science', *Australian Science Teachers' Journal*, 30, 1, pp. 15-19
- JAKOBSON, R., 1956, 'Two Aspects of Language and Two Types of Aphasic Disturbances' in Jakobson, R. & Halle, M., *Fundamentals of Language*, The Hague: Mouton & Co.
- JANVIER, C. (ed), 1987, *Problems of Representation in the Teaching and Learning of Mathematics*, Hillsdale: Lawrence Erlbaum Associates
- JOSEPH, G.G., 1987, 'Foundations of Ethnocentrism in Mathematics' *Race & Class*, 28, 3, pp. 13-28
- JOSEPH, G.G., 1990, 'The Politics of Anti-racist Mathematics' in Noss, R. et al (eds) *Political Dimensions of Mathematics Education: action and critique, proceedings of the first international conference, revised edition*, London: Department of Mathematics, Statistics and Computing, Institute of Education, University of London
- JOSEPH, G.G., 1991, *The Crest of a Peacock: non-European roots of mathematics*, London: Penguin
- KEARINS, J., 1991, 'Number Experience and Performance in Australian Aboriginal and Western Children', in Durkin, K. & Shire, B. (eds), *Language in Mathematics and Education: research and practice*, Philadelphia: Open University Press
- KESSLER, S.J. & McKENNA, W., 1978, *Gender: an ethnomethodological approach*, New York: John Wiley
- KIM, H., 1993, 'A Comparative Study Between an American and a Republic of Korean Textbook Series' Coverage of Measurement and Geometry in First Through Eighth Grades', *School Science and Mathematics*, 93, 3, pp. 123-6
- KIRK, L. et al, 1985a, *Confronting the Stereotypes: Kindergarten to Grade 4. Volume I*, Winnipeg: Manitoba Department of Education, Curriculum Development Branch
- KIRK, L. et al, 1985b, *Confronting the Stereotypes: Grades 5-8. Volume II*, Winnipeg: Manitoba Department of Education, Curriculum Development Branch
- KLEIN, M., 1975, *Envy and Gratitude: and other works 1946-1963*, London: Virago
- KNEE, C., 1983, *The Incarnation of Reason: geometry as a case for a sociology of mathematics*, unpublished PhD thesis, University of London, Institute of Education
- KNIJNIK, G., 1993, 'Pedagogy of the Oppressed in Practice: ethnomathematics and the landless of Southern Brazil', in The NECC Mathematics Commission, PDME.2: Curriculum Reconstruction for Society in Transition, 2nd-5th April, 1993, Broederstroom, Pre-conference papers
- KOGAN, M., 1978, *The Politics of Educational Change*, Glasgow: Fontana/Collins

- KOZA, J.E., 1992, 'Picture This: Sex Equity in Textbook Illustrations', *Music Educators Journal*, **78**, 7, pp. 28-33
- KRESS, G. & LEEUWEN, T. van, 1990, *Reading Images*, Geelong: Deakin University Press
- KUHN, T., 1970, *The Structure of Scientific Revolutions, second edition*, Chicago: University of Chicago Press
- KUMAR, K., 1982, 'Literature in the Reading Textbook: A Comparative Study from a Sociological Perspective', *Research in the Teaching of English*, **16**, 4, pp. 301-19
- KUO, C-H., 1993, 'Problematic Issues in EST Materials Development', *English for Specific Purposes*, **12**, 2, pp. 171-81
- LACLAU, E., 1984, 'Transformations of Advanced Industrial Societies and the Theory of the Subject' in Hanninen, S. & Paldan, L. (eds) *Re-thinking Ideology*, Berlin: Argument-Verlag
- LACLAU, E. & MOUFFE, C., 1985, *Hegemony and Socialist Strategy: towards a radical democratic politics*, London: Verso
- LAVE, J., 1988, *Cognition in Practice: mind, mathematics and culture in everyday life*, Cambridge: CUP.
- LAVE, J., MURTAUGH, M. & de la ROCHA, O., 1984, 'The Dialectic of Arithmetic in Grocery Shopping' in Rogoff, B. & Lave, J. (eds) *Everyday Cognition: its development in social context*, Cambridge, Mass.: Harvard University Press.
- LAWTON, D., 1975, *Class, Culture and the Curriculum*, London: RKP
- LAWTON, D., 1980, *The Politics of the School Curriculum*, London: RKP
- LEDER, G.C., 1987, 'Teacher Student Interaction: a case study', *Educational Studies in Mathematics*, **18**, 3, pp. 255-71
- LEE, L., 1992, 'Gender Fictions', *For the Learning of Mathematics*, **12**, 1, pp. 28-37
- LEONT'EV, A.N., 1978, *Activity, Consciousness, and Personality*, Englewood Cliffs: Prentice-Hall
- LEONT'EV, A.N., 1979, 'The Problem of Activity in Psychology' in Wertsch, J.V. (ed), *The Concept of Activity in Soviet Psychology*, New York: M.E. Sharpe
- LÉVI-STRAUSS, C., 1964, *Totemism*, London: Merlin Press.
- LÉVI-STRAUSS, C., 1972, *The Savage Mind (La Pensée Sauvage)*, London: Weidenfeld & Nicholson.
- LIEBECK, P., 1986, 'In Defence of Experience', *Mathematics Teaching*, **114**, pp. 36-8
- LIGHT, P., 1986, 'Context, Conservation and Conversation' in Richards, M. & Light, P., (eds) *Children of Social Worlds: development in a social context*, Cambridge: Polity

- LIGHT, B. et al, 1989, 'Sex Equity Content in History Textbooks', *History and Social Science Teacher*, 25, 1, pp. 18-20
- LING, J., 1987, 'SMP Activity in the 11-16 Sector: 1961-86', in Howson, G. (ed), *Challenges and Responses: essays to celebrate the twenty-fifth anniversary of the School Mathematics Project*, Cambridge: CUP
- LIVINGSTON, E., 1986, *The Ethnomethodological Foundations of Mathematics*, London: RKP
- LORIMER, R. & KEENEY, P., 1989, 'Defining the Curriculum: the role of the multinational textbook in Canada', in Castell, S. de, Luke, A.L. & Luke, C. (eds), *Language, Authority and Criticism: readings on the school textbook*, London: Falmer
- LOVE, A.M., 1991, 'Process and Product in Geology: an investigation of some discourse features of two introductory textbooks', *English for Specific Purposes*, 10, 2, pp. 89-109
- LUKES, S., 1982, 'Introduction' in Durkheim, É., *The Rules of Sociological Method (and selected texts on sociology and its method)*, London: MacMillan
- LURIA, A.R., 1976, *Cognitive Development: its cultural and social foundations*, Cambridge: Harvard University Press.
- McBRIDE, M., 1989, 'A Foucauldian Analysis of Mathematical Discourse', *For the Learning of Mathematics*, 9, 1, pp. 40-6
- MACDONALD, B. & WALKER, R., 1976, *Changing the Curriculum*, Shepton Mallet: Open Books
- MACKENZIE, D. & WAJCMAN, J. (eds), 1985, *The Social Shaping of Technology*, Milton Keynes: Open University Press
- McTAGGART, R., 1989, 'Aboriginalisation Implies Empowerment and Disempowerment', *The Aboriginal Child at School*, 17, 2, pp. 37-44
- MAIER, E., 1980, 'Folk Mathematics', *Mathematics Teaching*, 93, pp. 21-3
- MARSHALL, S. & GILMOUR, M., 1993, 'Lexical Knowledge and Reading Comprehension in Papua New Guinea', *English for Specific Purposes*, 12, 1, pp. 69-81
- MARX, K., 1968, 'Preface to a Contribution to the Critique of Political Economy', in Marx, K. & Engels, F., *Selected Works*, London: Lawrence & Wishart
- MARX, K., 1973, *Grundrisse: foundations of the critique of political economy (rough draft)*, Harmondsworth: Penguin
- MARX, K., 1976, *Capital, volume 1*, Harmondsworth: Penguin
- MARX, K., 1978, *Capital, volume 2*, Harmondsworth: Penguin
- MARX, K., 1981, *Capital, volume 3*, Harmondsworth: Penguin
- MATTHEWS, J., 1989, *Tools of Change: new technology and the democratisation of work*, Sydney: Pluto

- MAUSS, M., 1979, *Sociology and Psychology: essays*, London: RKP
- MAY, W.T. et al, 1990, "Whose" Content, Context, and Culture in Elementary Art and Music Textbooks? Elementary Subjects Center Series No. 23', paper presented at the Annual Meeting of the American Educational Research Association, Boston, Mass., April 1990, E. Lansing: East Lansing Institute for Research on Teaching, Michigan State University
- MICHIGAN STATE BOARD OF EDUCATION, 1984, *A Study of Selected Eighth Grade United States History Textbooks. 1982-83 Michigan Social Studies Textbook Study. Volume III. Selected Sections from the Complete Report*, Lansing: Michigan State Board of Education
- MILIBAND, R., 1969, *The State in Capitalist Society*, London: Quartet Books
- MOON, B., 1986, *The 'New Maths' Controversy: an international story*, Lewes: Falmer.
- MOREHEAD, G., 1984, 'Nice Girls Don't Do Maths', *Mathematics in School*, 13, 5, pp. 16-7
- MULCAHY, B.J., 1988, A Study of the Relationship between Ireland and England in Irish Post-primary School History Textbooks, published since 1922 and dealing with the period 1800 to the present, unpublished PhD, Hull University
- MULKEY, L.M., 1987, 'The Use of a Sociological Perspective in the Development of a Science Textbook Evaluation Instrument', *Science Education*, 71, 4, pp. 511-22
- MYERS, G., 1992, 'Textbooks and the Sociology of Scientific Knowledge', *English for Specific Purposes*, 11, 1, pp. 3-17
- NATIONAL CURRICULUM COUNCIL, 1988, *National Curriculum Consultation Report: mathematics*, London: NCC.
- NATIONAL CURRICULUM COUNCIL, 1989, *Mathematics Non-statutory Guidance*, York: NCC
- NATIONAL CURRICULUM COUNCIL, 1991, *Report on Monitoring the Implementation of the National Curriculum Core Subjects: 1989-90*, York: NCC
- NEW ZEALAND DEPARTMENT OF EDUCATION, 1980, *Sex-Role Stereotyping in Mathematics Textbooks*, Research Report, Wellington: Department of Education
- NIBBELINK, W.H. et al, 1986, 'Sex-Role Assignments in Elementary School Mathematics Textbooks', *Arithmetic Teacher*, 34, 2, pp. 19-21
- NORTHAM, J., 1982, 'Girls and Boys in Primary Maths Books', *Education 3-13*, 10, 1, pp. 11-4
- NORTON, D., 1993, 'Developing Social-functional Numeracy Materials for South African Learners', in The NECC Mathematics Commission, PDME.2: Curriculum Reconstruction for Society in Transition, 2nd-5th April, 1993, Broederstroom, Pre-conference papers

- NOSS, R. & DOWLING, P.C., 1990, 'Mathematics in the National Curriculum: the empty set?', in Dowling, P.C. & Noss, R. (eds), *Mathematics versus the National Curriculum*, Basingstoke: Falmer
- NUSBAUMER, N.R. et al, 1989, 'The Discovery of War as a Social Problem: Teaching as Sociological Practice', *Teaching Sociology*, 17, 3, pp. 316-22
- OFFICE OF POPULATION CENSUSES AND SURVEYS, 1980, *Classification of Occupations*, London: HMSO
- O'HEAR, M.F. & ASHTON, P.J., 1987, 'The Substantive Value of Main Idea Statements in Sociology Textbook', *Forum for Reading*, 18, 2, pp. 46-51
- O'HEAR M.F. et al, 1987, 'Location of Main Ideas in Composition Texts', *Research in the Teaching of English*, 21, 3, pp. 318-26
- O'REILLY, D., 1990, 'Hierarchies in Mathematics: a critique of the CSMS study', in Dowling, P.C. & Noss, R. (eds) *Mathematics Versus the National Curriculum*, Basingstoke: Falmer
- OTTE, M., 1983, 'Textual Strategies', *For the Learning of Mathematics*, 3, 3, pp. 18-28
- OTTE, M., 1986, 'What is a Text?' in Christiansen, B. et al (eds), *Perspectives on Mathematics Education*, Dordrecht: D. Reidel
- PARISH, D., 1991, 'Aboriginal World View in the Educational Context', *The Aboriginal Child at School*, 19, 4, pp. 14-20
- PATTEN, S., 1988, 'English Handbook Selection: A Feminist Choice', *Feminist Teacher*, 3, 3, pp. 23-4
- PEIRCE, C.S., 1931-1958, *Collected Papers*, Cambridge Mass.: Harvard University Press
- PENMAN, D., 1986, 'Confronting Gender Bias in Children's Books', *English for Education*, 20, 1, pp. 2-4
- PERRUCCI, R., 1982, 'The Failure of Excellence in Texts', paper presented at the Annual Conference of the American Sociological Association, San Francisco, September 1982
- PETERSON, S.B. & KRONER, T., 1992, 'Gender Biases in Textbooks for Introductory Psychology and Human Development', *Psychology of Women Quarterly*, 16, 1, pp. 17-36
- PIAGET, J., 1968, *Structuralism*, London: RKP
- PIAGET, J., 1972, *Psychology and Epistemology: towards a theory of knowledge*, Harmondsworth: Penguin
- PIMM, D., 1986, 'Beyond Reference', *Mathematics Teaching*, 116, pp. 48-51
- PIMM, D., 1987, *Speaking Mathematically*, London: RKP
- PIMM, D., 1990, 'Mathematical versus Political Awareness: some political dangers inherent in the teaching of mathematics', in Noss, R. et al (eds) *Political*

Dimensions of Mathematics Education: action and critique, proceedings of the first international conference, London: Department of Mathematics, Statistics and Computing, Institute of Education, University of London

- PIRIE, S., 1981, *Mathematics in Medicine: a report for the Cockcroft Committee*, Nottingham, Shell Centre for Mathematical Education, University of Nottingham
- POTTER., E.F. & ROSSER, S.V., 1992, 'Factors in Life Science Textbooks That May Deter Girls' Interest in Science', *Journal of Research in Science Teaching*, **29**, 7, pp. 669-86
- POULANTZAS, N., 1975, *Classes in Contemporary Capitalism*, London: Verso
- RAO, J.A., 1989, Evaluation of Materials for the Intermediate English Course in Punjab, Pakistan, unpublished MA dissertation, Exeter University
- REGESTER, C., 1991, 'Visual and Verbal Content in U.S. History Textbooks of the 1950s and 1980s', *Reading Improvement*, **28**, 1. pp. 14-25
- REMILLARD, J., 1991a, *Is There an Alternative? An Analysis of Commonly Used and Distinctive Elementary Mathematics Curricula. Elementary Subjects Center, Series No. 31*, E. Lansing: Center for the Learning & Teaching of Elementary Subjects
- REMILLARD, J., 1991b, *Conceptions of Problem Solving in Commonly Used and Distinctive Elementary Mathematics Curricula. Elementary Subjects Center Series, No. 43*, E. Lansing: Center for the Learning & Teaching of Elementary Subjects
- RESTIVO, S., 1984, 'Representations and the Sociology of Mathematical Knowledge' in Belisle, C. et Schiele, B., *Les Savoirs dans les Pratiques Quotidienne*, Lyons: CNRS
- REUSSER, K., 1988, 'Problem Solving Beyond the Logic of Things—contextual effects on understanding and solving word problems', *Instructional Science*, **17**, 4, pp. 309-338
- RICE, I.S., 1987, 'Racism and Reading Schemes. 1986, the current situation', *Reading*, **21**, 2, pp. 92-8
- RICE, I.S., 1988, 'The Portrayal of Ethnic Chinese/Japanese Peoples in Britain's Primary Reading Schemes', *Reading*, **22**, 1, pp. 32-9
- RIST, R., 1970, 'Student Social Class and Teacher Expectations: the self-fulfilling prophecy in ghetto education' *Harvard Educational Review*, **40**, 3, pp. 411-51
- RIVERS, J., 1990, 'Contextual Analysis of Problems in Algebra I Textbooks', Paper presented at the Annual Meeting of the American Educational Research Association, Boston Mass., April 1990
- ROBBINS, D., 1991, *The Work of Pierre Bourdieu: recognizing society*, Milton Keynes: Open University Press
- ROTMAN, B., 1985, 'On Zero', *Mathematics Teaching*, **113**, pp. 24-9

- SAAD, N.S., 1984, Content Analysis of the Civics Education Text Books in the Light of Educational Aims in Intermediate Schools in Iraq, with special attention to the methods followed in achieving them, unpublished PhD, University of Wales, Cardiff
- SALTER, B. & TAPPER, T., 1981, *Education, Politics and the State: the theory and practice of educational change*, London: Grant McIntyre
- SAUSSURE, F. de, 1983, *Course in General Linguistics*, London: Duckworth
- SCOTT-HODGETTS, R., 1986, 'Girls and Mathematics: the negative implications of success', in Burton, L. (ed), *Girls Into Maths Can Go*, London: Holt, Rinehart & Winston
- SCRIBNER, S., 1984, 'Studying Working Intelligence', in Rogoff, B. & Lave, J. (eds), *Everyday Cognition: its development in social context*, Cambridge, Mass.: Harvard University Press
- SEDDON, T., 1983, 'Con-texts', *Radical Education Dossier*, 21, pp. 37-9
- SELANDER, S., 1990, 'Towards a Theory of Pedagogic Text Analysis', *Scandinavian Journal of Educational Research*, 34, 2, pp. 143-50
- SELKE, B.E., 1983, 'History Textbooks: Portraits of Men and Women?', *Southwestern Journal of Social Education*, 13, 1, pp. 13-20
- SEWELL, B., 1981, *Uses of Mathematics by Adults in Daily Life*, London: ACACE
- SHELL CENTRE FOR MATHEMATICAL EDUCATION, nd, Report of Interviews for the Cockcroft Committee, unpublished transcripts, Nottingham: University of Nottingham
- SHUARD, H., 1986, 'The Relative Attainment of Girls and Boys in Mathematics in the Primary Years', in Burton, L. (ed), *Girls Into Maths Can Go*, London: Holt, Rinehart & Winston
- SKOVSMOSE, O., 1985, 'Mathematical Education versus Critical Education', *Educational Studies in Mathematics*, 16, 4, pp. 337-54
- SKOVSMOSE, O., 1988, 'Mathematics as a Part of Technology', *Educational Studies in Mathematics*, 19, 1, pp. 23-41
- SKOVSMOSE, O., 1993, 'Towards a Critical Mathematics Education', in The NECC Mathematics Commission, PDME.2: Curriculum Reconstruction for Society in Transition, 2nd-5th April, 1993, Broederstroom, Pre-conference papers
- SOHN-RETHEL, A., 1973, 'Intellectual and Manual Labour', *Radical Philosophy*, 6, pp. 30-7
- SOHN-RETHEL, A., 1975, 'Science as Alienated Consciousness', *Radical Science Journal*, 2/3, pp. 63-101
- SOHN-RETHEL, A., 1978, *Intellectual and Manual Labour: a critique of epistemology*, London: MacMillan

- SPRADBERY, J., 1976, 'Conservative Pupils? Pupil Resistance to curriculum Innovation in Mathematics', in Whitty, G. & Young, M.F.D., *Explorations in the Politics of School Knowledge*, Driffield: Nafferton
- STRAKA, G.A. & BOS, W., 1989, 'Socialization Objectives of Chinese Primary Schools: Results of a Comparative Textbook Analysis', *Studies in Educational Evaluation*, 15, 2, pp. 257-76
- STURROCK, J., 1979, *Structuralism and Since: from Lévi-Strauss to Derrida*, Oxford: OUP
- STURROCK, J., 1986, *Structuralism*, London: Paladin Grafton
- TADROS, A.A., 1989, 'Predictive Categories in University Textbooks', *English for Specific Purposes*, 8, 1, pp. 17-31
- TAHTA, D., 1985, 'On Notation', *Mathematics Teaching*, 112, pp. 49-51
- TAMMADGE, A., 1987, 'A Mathematics Master in the 1960s', in Howson, G. (ed), *Challenges and Responses: essays to celebrate the twenty-fifth anniversary of the School Mathematics Project*, Cambridge: CUP
- TAXEL, J., 1981, 'The Outsiders of the American Revolution: The Selective Tradition in Children's Fiction', *Interchange on Educational Policy*, 12, 2-3, pp. 206-28
- TAXEL, J., 1983, 'The American Revolution in Children's Fiction: an analysis of literary content, form and ideology', in Apple, M.W. & Weis, L. (eds), *Ideology and Practice in Schooling*, Philadelphia: Temple University Press
- TETREAULT, M.K., 1984, 'Notable American Women: The Case of United States History Textbooks', *Social Education*, 48, 7, pp. 546-50
- THWAITES, B., 1987, 'Foreword' in Howson, G. (ed), *Challenges and Responses: essays to celebrate the twenty-fifth anniversary of the School Mathematics Project*, Cambridge: CUP
- TIETZE, I.N. & DAVIS, B.H., 1981, 'Sexism in Texts in Educational Administration', paper presented at the Annual Meeting of the American Educational Research Association, Los Angeles, April, 1981
- TULL, D., 1991, 'Elementary Textbooks versus the Child: conflicting perceptions of biology', paper presented at the Annual Meeting of the National Association for Research in Science Teaching, Lake Geneva, Wi., April 1991
- TURNER, J., 1987, 'Analytical Theorizing', in Giddens, A. & Turner, J.H. (eds), 1987, *Social Theory Today*, Cambridge: Polity
- TUNSTALL, J., 1983, *The Media in Britain*, London: Constable
- VAUGHN-ROVERSON, C. et al, 1989, 'Sexism in Basal Readers: An Analysis of Male Main Characters', *Journal of Research in Childhood Education*, 4, 1, pp. 62-8
- VENTIMIGLIA, J.C. & DiRENZO, G.J., 1982, 'Sociological Conceptions of Personality', *Social Behavior and Personality*, 10, 1, pp. 25-37

- VITHAL, R., 1993, 'Ethnomathematics:some questions for curriculum reconstruction', in The NECC Mathematics Commission, PDME.2: Curriculum Reconstruction for Society in Transition, 2nd-5th April, 1993, Broederstroom, Pre-conference papers
- VITZ, P.C., 1985, *Religion and Traditional Values in Public School Textbooks: An Empirical Study, Section 1, Part 2 of Equity in Values Education: Do the Values Education Aspects of Public School Curricula Deal Fairly with Diverse Belief Systems? Final Report*, July 1985, New York: Department of Psychology, New York University
- VOGEL, S., 1987, 'Mythology in Introductory Biology', *BioScience*, **37**, 8, pp. 611-4
- VOLOSINOV, V.N., 1973, *Marxism and the Philosophy of Language*, New York: Seminar Press
- VYGOTSKY, L.S., 1978, *Mind in Society: the development of higher psychological processes*, Cambridge Mass.: Harvard University Press
- VYGOTSKY, L.S., 1986, *Thought and Language*, Cambridge Mass.: MIT Press
- WALDEN, R. & WALKERDINE, V., 1982, *Girls and Mathematics: the early years*, Bedford Way Paper no. 8, London: Institute of Education, University of London
- WALDEN, R. & WALKERDINE, V., 1985, *Girls and Mathematics: from primary to secondary schools*, Bedford Way Paper no. 24, London: Institute of Education, University of London
- WALFORD, G., 1983, 'Science Textbook Images and the Reproduction of Sexual Divisions in Society', *Research in Science and Technological Education*, **1**, 1, pp. 65-72
- WALKERDINE, V., 1982, 'From Context to Text: a psychosemiotic approach to abstract thought' in Beveridge, M. (ed) *Children Thinking Through Language*, London: Arnold
- WALKERDINE, V., 1984, 'Developmental Psychology and the Child-centred Pedagogy: the insertion of Piaget into early education' in Henriques, J. et al, *Changing the Subject: psychology, social regulation and subjectivity*, London: Methuen
- WALKERDINE, V., 1988, *The Mastery of Reason*, London: Routledge
- WALKERDINE, V. et al, 1989, *Counting Girls Out*, London: Virago
- WATSON, H., 1987, 'Learning to Apply Numbers to Nature: a comparison of English speaking and Yoruba speaking children learning to quantify', *Educational Studies in Mathematics*, **18**, 4, pp. 339-357
- WATSON, H., 1988, 'Language and Mathematics Education for Aboriginal Australian Children', *Language and Education*, **2**, 4, pp. 255-273
- WILLIAMS, R., 1961, *The Long Revolution*, Harmondsworth: Penguin
- WILLIS, P.E., 1977, *Learning to Labour: how working class kids get working class jobs*, Aldershot: Gower

- WING, A., 1989, School Mathematics Texts: a teacher's interpretive enquiry, unpublished PhD thesis, Southampton University
- WITTGENSTEIN, L., 1958, *Philosophical Investigations*, Oxford: Blackwell
- WITTGENSTEIN, L., 1961, *Tractatus Logico-philosophicus*, London: RKP
- WOLF, A., KELSON, M. & SILVER, R., 1990, *Learning in Context: patterns of skills transfer and training implications*, Moorfoot: Training Agency
- WUNUNG MURRA, W., 1988, "Dhawurpunaramirra" Finding the Common Ground for a New Aboriginal Curriculum', *Curriculum Perspectives*, 8, 2, pp. 69-71
- ZASLAVSKY, C., 1973, *Africa Counts: number and pattern in African culture*, Westport: Lawrence Hill
- ZINCHENKO, P.I., 1979, 'Involuntary Memory and the Goal-directed Nature of Activity' in Wertsch, J.V., (ed) *The Concept of Activity in Soviet Psychology*, New York: M.E. Sharpe Inc